

## Math 344: Lecture 13:

## Mathematical tutorial: Questions? Email [sjm1@williams.edu](mailto:sjm1@williams.edu)

```
In[*]:= Print[Hyperlink["https://www.youtube.com/watch?v=g1oj7CIqGM8"]]

Print[Hyperlink[

    "http://web.williams.edu/Mathematics/sjmiller/public_html/math/handouts/latex.

    htm"]]

https://www.youtube.com/watch?v=g1oj7CIqGM8
http://web.williams.edu/Mathematics/sjmiller/public_html/math/handouts/latex.htm
```

## Lecture on problem solving and coding

```
In[*]:= Print[

    Hyperlink["https://web.williams.edu/Mathematics/sjmiller/public_html/341Fa21/

    mathematicaprograms/Math341_DiplomaProblems.nb"]]

Print[

    Hyperlink["https://web.williams.edu/Mathematics/sjmiller/public_html/341Fa21/

    mathematicaprograms/Math341_DiplomaProblems.pdf"]]

https://web.williams.edu/Mathematics/sjmiller/public_html/341Fa21/mathematicaprograms/
    Math341_DiplomaProblems.nb
https://web.williams.edu/Mathematics/sjmiller/public_html/341Fa21/mathematicaprograms/
    Math341_DiplomaProblems.pdf
```

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### Long Suits in Bridge

In a hand of bridge, each of the four players is dealt 13 cards. It does not matter what order you get the cards, only which cards you get. NOTE:  $nCr = \binom{n}{r}$

What is the probability you are dealt at least 7 cards in a suit?

```

In[1]:= (* Want the probability of at least 7
in same suit in a hand of 13 cards*)
(* create a simple deck
good enough for this problem *)
deck = {}; (* initializes deck to be empty *)
For[i = 1, i ≤ 4, i++,
  For[j = 1, j ≤ 13,
    j++, deck = AppendTo[deck, i]];
] (* end of the i loop *)
Print[deck]

{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4}

```

```

In[60]:= weightedatleastseven[numdo_] := Module[{},
  success = 0; (* counts how often happens *)
  For[n = 1, n ≤ numdo, n++,
    {
      hand = RandomSample[deck, 13];
      (* chooses a hand of 13 *)
      For[s = 0, s ≤ 5, s++, suit[s] = 0];
      (* initialize each suit count to be 0 *)
      For[c = 1, c ≤ 13, c++,
        {
          x = hand[[c]];
          (*this calls in the c-th element*)
          suit[x] = suit[x] + 1;
          For[s = 1, s ≤ 4, s++,

```

```

    If[suit[s] ≥ 7, success = success + 1]];
(* this gives a WEIGHTED count -
   get one point for 7 in a row,
   2 for 8 in a row, .... Even worse as can
   get multiple credit for 7 in a row! Exit
   the loop once you find 7 in a row or
   better put this outside the c loop!*)
  ]];
(* this is where
   should put the s For loop *)
  ]]; (* end of the n loop *)
Print["Observed success rate is ",
      100.0 success / numdo, "%."];
theory =
  4.0 Sum[ Binomial[13, k] Binomial[39, 13 - k] ,
          {k, 7, 13}] / Binomial[52, 13];
Print["Theoretical success rate is about ",
      100.0 theory, "%."];
] (* end of the module *)

```

```
In[61]= Timing[weightedatleastseven[10000]]
```

```
Observed success rate is 7.81%.
```

```
Theoretical success rate is about 4.03213%.
```

```
Out[61]=
```

```
{0.34375, Null}
```

```

In[62]:= atleastseven[numdo_] := Module[{},
  success = 0; (* counts how often happens *)
  For[n = 1, n ≤ numdo, n++,
    {
      hand = RandomSample[deck, 13]; (* chooses a hand of 13 *)
      For[s = 0, s ≤ 5, s++, suit[s] = 0];
      (* initialize each suit count to be 0 *)
      For[c = 1, c ≤ 13, c++,
        {
          x = hand[[c]]; (*this calls in the c-th element*)
          suit[x] = suit[x] + 1;
        }];
      For[s = 1, s ≤ 4, s++,
        If[suit[s] ≥ 7, success = success + 1]];
    }
  ]; (* end of the n loop *)
  Print["Observed success rate is ",
    100.0 success / numdo, "%."];
  theory =
    4.0 Sum[ Binomial[13, k] Binomial[39, 13 - k] , {k, 7, 13}] /
    Binomial[52, 13];
  Print[
    "Theoretical success rate is about ", 100.0 theory, "%."];
] (* end of the module *)

```

```

In[63]:= atleastseven[100000]
Observed success rate is 4.018%.
Theoretical success rate is about 4.03213%.

```

```

In[40]:= (* darth vader problem - fail with prob p,
die after first fail *)
darthvader[numdo_, p_, numfailsallowed_] :=
  Module[{}],

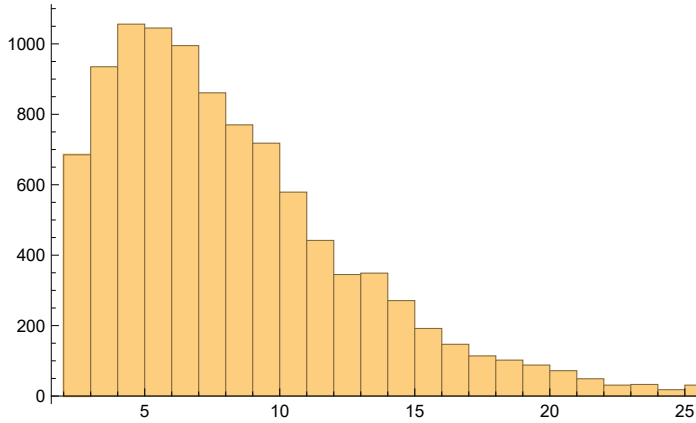
```

```

(* p is the prob of failing,
numdo is number of iterations *)
servicetime = {}; (* how long live *)
For[n = 1, n ≤ numdo, n++,
{
  havenotfailed = numfailsallowed;
  tasksdone = 0;
  While[havenotfailed > 0,
    {
      x = Random[];
      (* if x ≤ p, failed, else live *)
      tasksdone = tasksdone + 1;
      If[x ≤ p,
        havenotfailed = havenotfailed - 1];
    }]; (* end of while loop *)
  servicetime =
    AppendTo[servicetime, tasksdone];
  }]; (* end of the n loop *)
Print[Histogram[servicetime]];
Print[1.0 Mean[servicetime]];
Print[1.0 StandardDeviation[servicetime]];
] (* end of module *)

```

```
In[43]:= darthvader[10000, .25, 2]
```



7.9232

4.86153

## The Double Sixes Game

You have two fair die.

On each turn you can roll one or both of the die.

The goal is to have both show a 6.

Thus once one of the die lands on a 6 you can stop rolling it.



Questions:

- How many rolls do you expect before you have double sixes?
- What is the probability you win on your first turn? On your second? On your  $n^{\text{th}}$ ?

```
In[68]:= f[n_] := 2 (5 / 6) ^ n - (25 / 36) ^ n
g[n_] := 1 - f[n] (* probability succeed by n *)
success[n_] := g[n] - g[n - 1];
(* probability succeed at n *)

In[71]:= doublesixes[numdo_] := Module[{},
  count = {};
  For[m = 1, m ≤ numdo, m++,
    {
      firstdie = 0; seconddie = 0; rolls = 0;
      While[firstdie + seconddie < 12,
        {
          rolls = rolls + 1;
          die1 = RandomInteger[{1, 6}];
          die2 = RandomInteger[{1, 6}];
          If[die1 == 6, firstdie = 6];
          If[die2 == 6, seconddie = 6];
        }];
      count = AppendTo[count, rolls];
    }];
  theory = {};
  For[k = 1, k ≤ 30, k++, theory = AppendTo[theory, {k + .5, success[k]}]];
  Print[Show[Histogram[count, Automatic, "Probability"], ListPlot[theory]]];
]
```

