MIDTERM 2 – REVIEW – PROBLEMS

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Problem 1: [Strong Induction] A polygon P is said to be convex if for every vertex A and B of P, the line segment joining A and B lies entirely within P. Show using **strong induction** that the sum of the interior angles of a convex polygon with n vertices (where $n \geq 3$) is $180(n-2)$.

Problem 2: [Recursion] A single pair of bunnies (male and female) is born at the beginning of the year. Assume the following conditions hold:

(1) No bunnies ever die.
(2) Bunnies are not fertile during their first **two** months of their life, but thereafter give birth to three new male/female pairs at the end of every month.

Let s_n be the number of bunny pairs at the end of month n, with $s_0 = 1$. Find a recurrence relation for s_n, and explain how you got your answer.

Problem 3: [Set theory/Functions] Show that for any $f : A \to B$, and any $S, T \subseteq B$,

$$f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$$

Problem 4: [Functions] Define $f : \mathbb{Z} \to \{0, 1, 2\}$ by $f(n) = n \bmod 3$ (the remainder when n is divided by 3).

(a) Is f one-to-one?
(b) Is f onto $\{0, 1, 2\}$?

Problem 5: [Relations] Define a relation $\sim$ on $\mathbb{Z}^+ \times \mathbb{Z}^+$ by $(a, b) \sim (c, d)$ if and only if $ad = bc$. Show that $\sim$ is an equivalence relation.

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