

MIDTERM 1 – REVIEW – PROBLEMS

PEYAM RYAN TABRIZIAN

Problem 1: [Logic] Simplify the following expression

$$((\sim p) \vee q) \vee (p \wedge \sim q)$$

- (a) Using a truth table
- (b) Without using a truth table

Problem 2: [Three little proofs] Show that there is a rational number x and an irrational number y such that x^y is rational.

Problem 3: [Floor/Ceiling] Show that if $x - \lfloor x \rfloor \geq \frac{1}{2}$, then $\lfloor 2x \rfloor = 2 \lfloor x \rfloor + 1$.

Problem 4: [Number Theory]

- (a) Show that if $rs \leq n$, then $r \leq \sqrt{n}$ or $s \leq \sqrt{n}$ (here r and s are positive integers)
- (b) Use (a) to show that if n is not prime, then there is p prime that divides n with $p \leq \sqrt{n}$ (You may use any facts covered in lecture/book/HW)
- (c) Use (b) to show that 113 is prime.

Problem 5: [Number Theory] Show that $n(n+1)(n+2)$ is always divisible by 3

Problem 6: [Induction] A polygon P is said to be convex if for every vertex A and B of P , the line segment joining A and B lies entirely within P . Show that the sum of the interior angles of a polygon with n vertices (with $n \geq 3$) is $180(n-2)$.