MATH 200 – MIDTERM 1 – PRACTICE QUESTIONS

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Here are some practice questions for Midterm 1. I would like to emphasize that you that the actual exam covers not just those practice questions, I'd highly recommend also looking at the homework problems as well as the problems covered during the review session. Also make sure to look at the midterm I gave last semester. Please refer to the study guide if you want to know exactly what to study.

Date: Friday, March 10, 2017.

1. (Note that there might not be a T/F portion on the exam) Label the following statements as **TRUE (T)** or **FALSE (F)**. Any correct answer gives you 2 points, and any incorrect answer gives you 0 points. You do **NOT** get points off for an incorrect answer, and you do **NOT** need to justify your answers.

   (a) If P and Q are propositions, then

   $$\exists x\ (P(x) \wedge Q(x)) \equiv (\exists x\ P(x)) \wedge (\exists x\ Q(x))$$

   (b) If a, b, c are positive integers, then $a|bc \Rightarrow [(a|b) or (a|c)]$

   (c) If a and b are relatively prime, then $lcm(a, b) = ab$

   (d) For any real numbers x and y, $\lceil xy \rceil = \lceil x \rceil \lfloor y \rfloor$

   (e) There is a statement that is logically equivalent to its inverse

2. Write out a truth table for

$$(p \Rightarrow r) \Leftrightarrow (q \Rightarrow r)$$

3. Determine whether the following argument is valid

$$\begin{aligned} & p \Rightarrow (q \ \vee \sim r) \\ & p \Rightarrow p \wedge r \\ \therefore\ & p \Rightarrow r \end{aligned}$$

4. Three little proofs Part 1: Show that $\sqrt{2}$ is irrational.

5. Three little proofs Part 2: Show that there are infinitely many prime numbers.

6. Three little proofs Part 3: Show that there is are irrational numbers x and y such that x^y is rational.

7. Show that if n is an integer, then $(n+1)(n+5)(n+9)$ is always divisible by 3

8. Show that a positive integer n is divisible by 9 if and only if the sum of its digits are divisible by 9.

   Hint: You can write n as $n = a_0 + a_1 10 + a_2 10^2 + \cdots + a_m 10^m$ for some m and some integers a_i between 0 and 9.

9. Show that there are infinitely many composite numbers

10. Show that if $x - \lfloor x \rfloor \geq \frac{1}{2}$, then $\lfloor 2x \rfloor = 2\lfloor x \rfloor + 1$

11. Show that if n is an integer, then

$$\left\lfloor \frac{\left\lfloor \frac{n}{3} \right\rfloor}{3} \right\rfloor = \left\lfloor \frac{n}{9} \right\rfloor$$

12. What is the remainder when 2^{345} is divided by 11?

13. Show that for all integers $n \geq 2$,

$$\sum_{i=1}^{n-1} i(i+1) = \frac{n(n-1)(n+1)}{3}$$

14. Show that for all integers $n \geq 0$,

$$2^n < (n+2)!$$

15. Suppose there are n people at a party and suppose that each shakes hands with all the other people present. Show that $\frac{n(n-1)}{2}$ handshakes must occur.

16. Suppose that at the beginning, there are 3 cells, and after each step, each cell splits into 2. Show by induction on n that after n steps, there are 3×2^n cells.