MATH 200 – MIDTERM 2

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Name: ____

Instructions: Welcome to Midterm 2! You have 50 minutes to take this exam, for a total of 100 points. **Do not open the exam until instructed to do so**. Remember that you are not only graded on the correctness of your answer, but also on the clarity and completeness of your proofs. Write in complete sentences whenever you can. If you need to continue your work on the back of the page, clearly indicate so, or else your work will be discarded. Each problem is worth the same number of points, so **do not spend <u>too</u> much time on each problem**! May your luck be uncountable! :)

Honor Code: I promise not to communicate or collaborate with anyone during the exam, and I will not use any books or notes or cheat sheets or personal electronic devices (**including calculators**).

Signature: ____

1		25
2		25
3		25
4		25
Total		100

Date: Friday, April 21, 2017.

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1. *(25 points)* It's Caroline's birthday today (happy birthday!!!), and she wants to celebrate with a bang!

   Definition: A n-cake is a cake with 2 layers of size n, topped with 2 layers of size $n-1, \cdots$, topped with 2 layers of size 1 (so it's like the tower of Hanoi problem, but with two disks for each size)

   Suppose there are 3 poles A, B, C, and Caroline needs to move an n-cake from pole A to pole C. The layers are transferred one by one from one pole to another, and at no time may a larger layer be placed on top of a smaller layer, but it's ok if a layer is placed on top of a layer of the *same* size.

   Let c_n be the minimum number of moves needed to transfer an n-cake from one pole to another. Find a recurrence relation for c_n.

   Note: In this problem, you do **NOT** have to justify your answer.

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2. *(25 points)* Define a relation $\sim$ on $\mathbb{Z}$ by:

$$m \sim n \text{ if and only if } m - n \text{ is even}$$

Show that $\sim$ is an equivalence relation on $\mathbb{Z}$.

Note: You may use **without proof** any facts that you know about even integers.

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3. *(25 points)* Let $A = \{1, 2, 3, 4\}$, and define $F : \mathcal{P}(A) \to \mathbb{N}$ by

$$F(X) = \text{ the number of elements in } X$$

For example, $F(\{1, 3\}) = 2$ because $\{1, 3\}$ has 2 elements.

(a) Is F one-to-one? Why or why not?

(b) Is F onto $\mathbb{N}$? Why or why not?

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4. *(25 points)* Suppose $f : A \to B$ and C and D are subsets of B. Show that

$$f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$$

Hint: For any set S, $x \in f^{-1}(S)$ means $f(x) \in S$.

Note: For sake of time, you do **NOT** need to justify every step.

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