MATH 200 – MIDTERM 2 – STUDY GUIDE

PEYAM RYAN TABRIZIAN

The second Math 200 – Midterm will take place on **Friday, April 21, 2017**, from 9 AM to 9:50 AM in our usual lecture-room. It is **not** cumulative and covers Section 5.4 (Strong Mathematical Induction) up to and including Section 8.3 (Equivalence Relations). It will be a closed book and closed notes-exam, and no cheat-sheets will be allowed. Unless otherwise noted below, you are responsible for **everything** we've learned, including what is in the book, what you've learned in lecture, and what you've done on Homework $6, 7, 8$, and 9 (so I'd highly recommend doing homework 9 beforehand).

This is a study guide for the exam, and it is really meant to be what it is, namely a *guide*. This list is not meant to be exhaustive, although I've tried to put everything that's important and not important)

Main concepts: Strong Induction (Chapters 5), Recursion (Chapter 5), Sets (Chapter 6), Functions (Chapter 7), Cardinality (Chapter 7), Relations (Chapter 8)

1. Chapter 5: Sequences, Mathematical Induction, and Recursion

Just to re-emphasize: I will not ask questions about regular induction, but strong induction is fair game for the exam.

- Section 5.4: There are two classical strong induction examples in this section: The coin-changing problem, and the fact that any integer $n > 1$ can be written as a product of primes. Please thoroughly understand those examples, although I could ask you for more exotic examples like the ones on the practice exam or in the review session, or like AP1 on HW 6 (the quotient-remainder theorem one). You can ignore everything starting from page 272 on!

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- Section 5.5: Skip!

- Section 5.6: Pay particular attention to the Fibonacci example and the Tower of Hanoi-example. Good exam questions would be variations on those problems, just like for instance 5.6.20 or 5.6.23. I may possibly ask you about different examples as well. Ignore Examples 5.6.7 and 5.6.8, as well as the section on recursive definitions of sums and products

- Section 5.7: the only important concepts are the definition of an arithmetic sequence (top of page 307) and the definition of a geometric sequence (page 308). Everything else, although important, isn't very appropriate to ask on an exam, so just skim it. I could potentially ask you about problems like 5.7.15, though, and I could ask you to verify that a formula is correct using induction (like Example 5.7.7). Definitely skip Examples 5.7.4, 5.7.6, and 5.7.8

- Section 5.8: You are responsible for the following two cases:
  (a) The case of two distinct roots
  (b) The case of a single root (Theorem 5.8.5 in the book)
  You should also be able to do this using initial conditions, like for example in 5.8.8. You are **NOT** responsible for AP2 on HW 6, and I will not ask you about the case where the roots are complex numbers.

- Section 5.9: You can skip everything **except** Examples 5.9.6 and 5.9.8 (which I've actually covered in lecture when I did chapter 7)

2. Chapter 6: Sets

The material in Chapters 6 and 7 are fundamental in case you want to take upper-division math classes, so make sure to thoroughly study those two chapters.

- Section 6.1: Know every piece of terminology in that section. You can skip the last section (labeled Optional)

- Section 6.2: You do **NOT** need to know the *names* of the set identities in Theorem 6.2.2 **EXCEPT** the Commutative Laws, the Associative Laws, the Distributive Laws, and the De Morgan's Laws. But certainly know how to use them. And of course remember that to prove two sets are equal, you have to show that each of them is contained in the other.

- Section 6.3: You can ignore the proof of Theorem 6.3.1 if you understood the proof I gave in lecture (with n = 3).

- Section 6.4: Skip the part about Boolean Algebras. Youre only responsible for Russels paradox and the Halting Problem and whatever paradoxes are on the homework. Also, for 6.4.26, you do not need to understand how I came up with the set B (I would give you the definition if necessary). Finally, you are not responsible for understanding the Axiom of Choice / Banach-Tarski paradox that I did in lecture.

3. Chapter 7: Functions

- Section 7.1: Ignore examples 7.1.9 and 7.1.10, as well as the section on Boolean functions. Don't forget about the definitions of images and inverse images, like 7.1.42, 7.1.43, 7.4.47 (those definitions are valid even if f does not have an inverse!)

- Section 7.2: Again, a lot of important terminology! Ignore the section Application: Hash Functions, and examples 7.2.67.2.10 and 7.2.14.

- Section 7.3: Everything in this section is important! You should also know how to show that the composition of one-to-one functions is one-to-one and that the composition of onto functions is onto (see the practice exam)

- Section 7.4: This section is probably the most important section of Chapter 7 because it puts everything we learned in that section together, so pay particular attention to it. Ignore Example 7.4.1, the proof of Theorem 7.4.3, and the section "Application: Cardinality and Computability." Don't forget about the Additional Problem in HW 8, and about AP2 and AP3 in HW 9, but you can ignore AP 1 (it's too inappropriate for an exam). You're also responsible for knowing about the Grand Hotel Peyam (Hilbert's Hotel) Problem. Remember that I also sent you a link with a video explaining it. **Know how to prove that $\mathbb{R}$ is uncountable**!

4. Chapter 8: Relations

Just to re-emphasize: Equivalence *classes* will not be on the exam!

- Section 8.1: Ignore the section "N-ary relations and Relational Databases" and Example 8.1.6.
- Section 8.2: Ignore Example 8.2.1 and the section "The Transitive Closure of a Relation."
- Section 8.3: The *only* thing you'll need to know from that section is the definition of equivalence relation on the bottom of page 462; everything else will be on the final. 8.3.21 (1), 8.3.30 (1), AP4 and AP5(a) in HW 9 are fair game for this midterm, but I will **NOT** ask you about equivalence classes.