MATH 200 – MIDTERM 1

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Name: _____

Instructions: Welcome to Midterm 1! You have 50 minutes to take this exam, for a total of 100 points. **Do not open the exam until instructed to do so**. Remember that you are not only graded on the correctness of your answer, but also on the clarity and completeness of your proofs. Write in complete sentences whenever you can. If you need to continue your work on the back of the page, clearly indicate so, or else your work will be discarded. May your luck be prime! :)

Honor Code: I promise not to communicate or collaborate with anyone during the exam, and I will not use any books or notes or cheat sheets or personal electronic devices (**including calculators**).

Signature: _____

1		20
2		25
3		30
4		25
Total		100

Date: Friday, March 10, 2017.

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1. *(20 points)* Fill out the following truth table (feel free to add more columns if that helps you)

p	q	r	$p \Rightarrow (q \wedge r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

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2. *(25 points)* Show that if p is prime, then $\sqrt{p}$ is irrational.

   Note: You are allowed to use (without proof) that if p is prime and p divides n^2, then p divides n (where n is an integer)

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3. *(30 points)* Show that, for all integers $n \geq 1$, we have:

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} \geq \sqrt{n}$$

Hint: At some point, you'll need to use $n(n+1) = n^2 + n \geq n^2$.

Note: In this problem, you are not only graded on the correct answer, but also on the way your write down your answer, so make sure to include all your steps.

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4. *(25 points)* Show that if x is a real number and m is a positive integer such that $x - \lfloor x \rfloor \geq \frac{m-1}{m}$, then

$$\lfloor mx \rfloor = m \lfloor x \rfloor + (m - 1)$$

Note: The quotient-remainder theorem does not help, so don't even go there...

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