MATH 200 – MIDTERM 1 – STUDY GUIDE

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The first Math 200 – Midterm will take place on **Friday, Marh 10, 2017**, from 9 AM to 9:50 AM in 105 Bronfman (our usual lecture-room). It covers Section 1.1 up to and including Section 5.3 (Mathematical Induction II), as well as Section 8.4. In particular, note that *strong* induction will **not** be on the exam. It will be a closed book and closed notes-exam, and no cheat-sheets will be allowed. Unless otherwise noted below, you are responsible for **everything** we've learned, including what is in the book, what you've learned in lecture, and what you've done in the homework.

This is a study guide for the exam, and it is really meant to be what it is, namely a *guide*. This list is not meant to be exhaustive, although I've tried to put everything that's important and not important)

Main concepts: Logic (Chapters 2 and 3), Number Theory (Chapter 4), and Induction (Chapter 5).

1. Chapter 1: Speaking Mathematically

- This section is mainly meant to be an introduction to the basic concepts in this class, so don't spend too much time on it!

- I think the two main concepts here are the notion of a set and the notion of a function, so *if* I ask you something about this chapter, it's most likely to show that something is or is not a function (although I could ask you about sets as well)

- Don't forget about the three additional problems on HW 1!

2. Chapter 2: The logic of compound statements

- Everything in sections 2.1 and 2.2 is very important, so make sure to thoroughly read it. In particular, know how to construct truth tables and show that two statements are equivalent (or not) using truth tables. Moreover, I could ask you about how to show equivalence

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using the laws of logic, like they do in example 2.1.14.

- For section 2.3, ignore everything from "Additional Valid Argument Forms: Rules of Inference," to the end of the section. In particular, you do **not** need to know the kights/knaves problem and/or the CSI Williamstown logic puzzle I've covered in lecture.

- You do **not** need to memorize the table on page 61. All I'm asking you from this table is to know what Modus Ponens and Modus Tollens is.

- Skip Sections 2.4 and 2.5.

3. Chapter 3: The Logic of Quantified Statements

- Again, everything in this chapter is very important, **except** you can skip any section/example that has to do with Tarski's world, and you can skip the section on Prolog in section 3.3.

4. Chapter 4: Elementary Number Theory and Methods of Proof

- This (and induction) is the **highlight** of this midterm! You really need to thoroughly understand it and know how to prove that a statement is true or false!

- In section 4.1, you don't need to memorize the lists on pages 155 – 158, but of course they are very useful tips!

- The most important thing in section 4.2 is the concept of a rational number; once you understand that, you should hopefully be able to understand the rest of the chapter. Also remember how to convert any number with a repeating decimal into a rational number like I did in lecture (or you did on the HW).

- In section 4.3, skip the proofs of theorems 4.3.1 and 4.3.2 and 4.3.4, but know their statements.
- Remember the two Additional Problems in HW3.

- Skip the section "Absolute Value" in section 4.4

- Don't worry about what I did in lecture about how to show that if x is rational, then x has a repeating decimal.

- In section 4.5, you can skip the proof of theorem 4.5.3 if you want to.

- Section 4.6 is very important, and make sure to thoroughly read it

- **Make sure to know (= memorize!!!) the three little proofs that I covered in lecture, namely:**

  (1) Show that $\sqrt{2}$ is irrational

  (2) Show that there are infinitely many prime numbers

  (3) Show that there are x and y irrational such that x^y is rational.

  In fact, a very valid question on the exam could be for instance "Show that there are infinitely many prime numbers!" You can pick whichever version you prefer, either the book-one or the one I presented in lecture (but if you choose the book version, you'll need to prove Proposition 4.7.3 as well!) I could also ask you a variation on those, like the problem on Midterm 1 from last semester, or the problem on the Review Session.

- Skip section 4.8.

- Remember the three additional problems on HW 4.

- For section 8.4: Either just review whatever I covered in lecture about congruences, gcd, relatively primeness, and Fermat's Little Theorem, or look at Theorems 8.4.1, 8.4.3, 8.4.8, 8.4.9, and 8.4.10. Don't forget to also look at Additional Problem 1 on HW 5, that's also a very standard exam question.

5. Chapter 5: Sequences, Mathematical Induction, and Recursion

- Skim section 5.1, unless you don't know what a sequence is. If you do read it, know what the summation/product notation is, but you can definitely skip the section "Sequences in Computer Programming" and "Application:"

- **Sections 5.2 and 5.3 are also the highlight of the midterm, and probably the heart of the course.** In particular, you need to know not only simple induction proofs involving sums and inequalities, but also more exotic examples like trominoes, and also *all* the problems that are on HW 5 (like the handshake-problem, the polygon problem, or the gossip problem). Also look at the 'standing in line problem' from the midterm I gave last semester. That's why I *highly* recommend doing HW 5 *before* the midterm. That said, you *can* skip the coin-changing example for now, because it'll be easier to prove using strong induction. Also you need to know the section about the sum of a geometric sequence, even if I'm not going to cover it in lecture.

- Again, strong induction (Section 5.4) will **not** be on the exam (but will be on the next).