

Math 200 – Homework 6

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This assignment is due on **Friday, March 17, 2017** at 9:50 AM. Sections 5.4 (You can ignore everything from page 272 on), 5.6 (ignore the section on compound interest), 5.7, 5.8, 5.9

Note: I won't have much time (if at all!) to go over sections 5.7 and 5.9 in depth, but please read them. I won't ask about 5.9 on the exam but section 5.9 is very important if you're majoring in Computer Science

- **Section 5.4:** 10, 25(a)(b) (regular induction), AP1

- **Section 5.6:** 20(a), 23(a)

Note: The way I understand problem 20 is that you can only move the disks one space at a time (so if you want to move a disk from A to C , it would take 2 moves, not one). Also the goal isn't necessarily to move the entire tower from A to C ; you could move it from A to B if you want to.

- **Section 5.7:** 15 (also prove by induction that your formula is correct)

- **Section 5.8:** 13, 14, AP2

- **Section 5.9:** 6

Additional Problem 1: Use strong induction to prove the following (special case of) the quotient remainder theorem with $d = 5$: For any integer $n \geq 0$, there exist integers q and r with $0 \leq r < 5$ such that

$$n = 5q + r$$

Hint: This is just like the Bank of Peyamerica problem (see the Strong Induction lecture), but you're changing the amount n into 5 dollar bills now. Also beware that you have to prove many base cases here. To see how many you have to prove, try out the induction proof you wrote down for $n = 1, n = 2, \dots$ to see where the inductive step breaks down.

Additional Problem 2: The purpose of this problem (not on the exam) is to give you fun variations of difference equations!

(a) [Higher-order difference equations] Find the general solution to

$$u_{n+3} = 6u_{n+2} - 11u_{n+1} + 6u_n$$

Hint: The auxiliary equation is $r^3 - 6r^2 + 11r - 6 = (r-1)(r-2)(r-3) = 0$

(b) [Systems of difference equations] Verify that

$$\begin{cases} u_n = A(2^n) + B(-1)^n \\ v_n = 4A(2^n) + B(-1)^n \end{cases}$$

solves the *system* of difference equations

$$\begin{cases} u_{n+1} = -2u_n + v_n \\ v_{n+1} = -4u_n + 3v_n \end{cases}$$

Note: This actually gives us a more realistic version of the Pokemon-game introduced in lecture, where you can think of u_n the HP of Pikachu after round n and v_n the HP of Bulbasaur after round n .