Math 453 – Homework 5

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Friday, March 10, 2017

This assignment is due on **Friday, March 10, at 10:50 AM**.

Note: The **take-home midterm** will be given out on Wednesday, March 15, at 11:50 AM and will be due on Friday, March 17 at noon. It will cover up to *and including* whatever we will cover on Monday, March 13. Note that class on Friday, March 17 will be cancelled, and there will be no homework due that day. There will be a special TA Session on Tuesday, March 14, from 8 to 9 pm in 204 Clark, and no TA Session on Thursday, March 16.

Reading: Section 2.3.1

Chapter 2: 12, 13, 14, and the following Problem:

Additional Problem: Let $f : \mathbb{R} \to \mathbb{R}$ be given, let F be an antiderivative of f, $\mathcal{A} = \left\{ u \in C^2(\overline{W}) \mid u = g \text{ on } \partial W \right\}$, and suppose that $u \in \mathcal{A}$ minimizes the following energy

$$I[u] = \int_W \frac{1}{2} |Du|^2 - F(u) dx$$

Mimic the proof of the converse of Dirichlet's principle for Laplace's equation to find a PDE that u must solve. Note that here $F(u)$ is F with input u, not F multiplied by u.

(TURN PAGE for hints)

Hint for 12: Unlike what the hint tells you, you don't have to use (a) to do (b) (unless you find it useful)

Hint for 13: Remember that you're in the case $n = 1$. Note that in (a), $z = \frac{x}{\sqrt{t}}$. For (b), what the hint is trying to say is this: Notice that if u is a solution to the heat equation, then so is u_x. In particular, if you differentiate the solution you found in (a), you will find a solution of the heat equation. The only issue is finding c, but for this choose c such that the integral of the solution you found in (a) from $-\infty$ to ∞ is equal to 1. A change of variable is useful here. In the end, you should find the fundamental solution for the case $n = 1$. Ignore the hint about the initial condition, unless you find it useful.

Hint for 14: This problem should be shorter than you think: Let $v(x,t) = u(x,t)h(x,t)$, where h is an **easy** exponential function depending on c that you have to find beforehand (try out a couple of guesses and don't try things that are too complicated), and show that v solves the heat equation with possibly a new inhomogeneous term. Then use formula 17 on page 51. Happy searching!