Math 200 – Homework 5

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Monday, March 13, 2017

This assignment is due on **Monday, March 13, 2017** at 9:50 AM.

Note: The first midterm will take place on Friday, March 10, from 9 AM to 9:50 AM. It will cover everything from Section 1.1 up to and including Section 5.3 (Mathematical Induction II), as well as parts of section 8.4. Note in particular that Section 5.4 (Strong Induction) will **not** be on the first midterm. I will send out a study guide and a practice exam sometime during the week-end. I'd highly recommend doing this homework assignment before the midterm since some of the sections on this assignment will be on the exam.

Reading:

- Section 5.1: Again, just skim this one; hopefully you're already familiar with sequences from calculus. Skip the sections "Sequences in Computer Programming" and "Application"
- Sections 5.2, 5.3: Those sections are **very** important, so make sure to thoroughly read it. Induction is a new proof technique that a lot of students had trouble with in the past. That said, it a powerful proof-technique that you will see over and over again in future math and Computer Science courses.

(TURN PAGE for the actual assignment; Note that there are hints on pages 3-4)

- **Section 8.4:** AP1
- **Section 5.1:** Nothing
- **Section 5.2:** 12, 14
- **Section 5.3:** 22, 23, 29, 34, 39, AP2, AP3

Additional Problem 1: Calculate 2016^{2016} **mod** 11

Additional Problem 2: Let $U_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n} = \sum_{i=1}^{n} \frac{1}{i}$. Show that for $n \geq 1$,

$$U_{2^n} \geq 1 + \frac{n}{2}$$

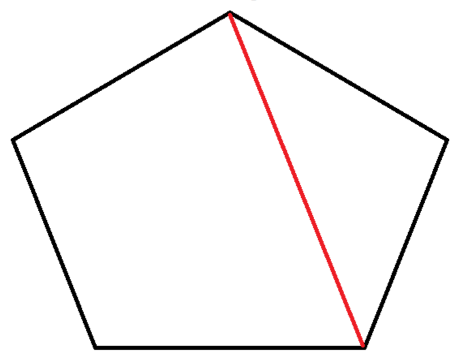
Note: In particular, since $1 + \frac{n}{2} \to \infty$ as $n \to \infty$, and by some book-keeping, this shows that the harmonic series $\sum_{i=1}^{\infty} \frac{1}{i}$ diverges! (you don't have to show that)

Additional Problem 3: (hopefully this should be fun :)) Suppose there are n people in a group, each aware of a scandal no one else in the group knows about. The people communicate by telephone; when two people in the group talk, they share information about all scandals each knows about. For example, on the first call, two people share information, so by the end of the call, each of these people knows about two scandals. The **gossip problem** asks for $G(n)$, the minimum number of telephone calls that are needed for all n people to learn about all the scandals.

(a) Find $G(1), G(2), G(3), G(4)$ (you don't need to show that it's the minimum)

(b) Use mathematical induction to show that $G(n) \leq 2n - 4$ for $n \geq 4$.

200 - Spring 2017/Homework/Polygon.png



Hints:

(AP1) This is similar to what I did in lecture on Friday.

(5.3.23) For (a), you could (but don't have to) use $(a+b)^3 = a^3 + 3a^2b + 3ab^3 + b^3$

(5.3.29) Imagine you show up in a room of n people; How many people do you have to greet?

(5.3.34) Careful! In (b), you are **fixing** n (say $n = 3$) and are doing induction on m.

(5.3.39) You are allowed to use that the sum of the angles of a triangle is 180 degrees. The idea is to pick one vertex of the polygon and form a triangle using that vertex and its two neighbors, as in the picture above.

(AP2) For P_{n+1}, decompose the sum as:

$$\left(1 + \frac{1}{2} + \cdots + \frac{1}{2^n}\right) + \left(\frac{1}{2^n+1} + \frac{1}{2^n+2} + \cdots + \frac{1}{2^n+2^n}\right)$$

Notice that each term in the second pair of parentheses is $\geq \frac{1}{2^n+2^n}$, how many such terms are there? Therefore, the whole term in the second pair of parentheses is $\geq$ Number of terms $\times \frac{1}{2^n+2^n}$.

(AP3) Just to clarify this problem a bit: For the case $n = 3$, suppose there are 3 people, Ashley, Bob, and Claudia, and Ashley knows that 'Kim Kardashian got robbed in Paris', Bob knows that 'Brad Pitt and Angelina Jolie got divorced', and Claudia knows that 'Charlie Puth joins the Voice'. Here's a possible scenario: Ashley calls Bob and tells him about the Kardashian news ('Hey, have you heard about Kim Kardashian?') and Bob says 'Oh, I haven't heard that, but have you heard about Brangelina?,' so both Ashley and Bob know about Kardashian/Brangelina, and then Bob calls Claudia and tells her about Kardashian/Brangelina, and Claudia tells him about Charlie Puth. So Bob and Claudia now know about all the 3 gossips. And finally Claudia calls Ashley and tells her about the all the 3 news, so Ashley also knows about all the news. Since there were 3 phone calls in total, $G(3) = 3$ (technically you'd have to show that there can't be less than 3 phone calls, since it's the minimum number of phone calls, but you don't have to do that).

Now for the actual hint: Suppose you're Person 1 who knows about the gossip a_1. Then call Person 2 who knows gossip a_2, and so both you and that person know gossip a_1 and a_2. Then notice that Person $2, \cdots, n + 1$ form a group of n people, and finally have Person 2 (who knows all the gossips) call you.