Math 200 – Homework 4

Peyam Tabrizian

Friday, March 3, 2017

This assignment is due on **Friday, March 3, 2017** at 9:50 AM.

Reading: Sections 4.5, 4.6, 4.7, and parts of section 8.4. In section 4.5, you can skip the proofs of the 'Absolute Value' section, but do know the results (which you're hopefully familiar with), and you can skip Theorem 4.5.3. The proof techniques learned in those sections 4.6 and 4.7 are incredibly important, not only for this course, but also in future courses. For section 8.4, you're only responsible for the material presented in lecture about Congruences, GCDs, and (if we have time) Fermat's Little Theorem. More precisely, you're *only* responsible for Theorems 8.4.1, 8.4.3, 8.4.8, 8.4.9, and 8.4.10. Anything else (linear combinations, RSA algorithms, inverses) you're **not** responsible for.

- **Section 4.5:** 25 (QR with $d = 4$), 27 (let $n = \lfloor x \rfloor$), AP1
- **Section 4.6:** 5, 12
- **Section 4.7:** 15, 19 (you may use that if 5 divides n^2, then 5 divides n), 23 (you can assume that $\sqrt{2}$ is irrational), 28, 35, AP2
- **Section 8.4:** 13, AP3

Hint for 4.7.35: To show that a property is true for at *most* one b, suppose that there are b and b' that have that property, and show that $b = b'$. Notice that you have the freedom of choosing any value of r that you wish!

Additional Problem 1: Show that for any positive integers m and n, we have:

$$\left\lceil \frac{n}{m} \right\rceil = \left\lfloor \frac{n + (m-1)}{m} \right\rfloor$$

Note: In lecture on Friday I did the case $m = 3$, so this is just the same problem, but with m instead of 3.

Additional Problem 2: Show that $x = \log_2(3)$ is irrational ($x = \log_2(3)$ means that $2^x = 3$. Feel free to assume that $x > 0$). Think in terms of prime factors.

Additional Problem 3: The **least common multiple** of a and b, denoted by $lcm(a, b)$ is the smallest positive number c such that c is both a multiple of a and of b. For example, $lcm(12, 18) = 36$ because 36 is a multiple of both 12 and 18, and there is no smaller multiple of 12 and 18.

(a) Find $lcm(360, 300)$. You may use the fact that $360 = 2^3 \times 3^2 \times 5$ and $300 = 2^2 \times 3 \times 5^2$.

Hint: This is *really* similar to what we did in lecture with gcd!

(b) Show that if a is of the form $2^{a_1} \times 3^{a_2} \times 5^{a_3}$ and b is of the form $2^{b_1} \times 3^{b_2} \times 5^{b_3}$, then

$$\gcd(a, b) \times lcm(a, b) = ab$$

Hint: What simpler expression is $\min\{a_1, b_1\} + \max\{a_1, b_1\}$ equal to?

Note: This is true in general, not just for the a and b in the form above. The proof is almost identical, except for some book-keeping in case the factors are not the same.

(c) Using (b), if a and b are relatively prime, what simple expression is $lcm(a, b)$ equal to?