Math 453 – Homework 2

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Monday, February 20, 2017

This assignment is due on **Monday, February 20, at 10:50 AM**.

Note: Remember that there is **no** class on Friday, February 17, because of Winter Carnival (and because it's my dad's 88th birthday, so that's a good reason to cancel class!)

Reading: Section 2.2.1 of Chapter 2, as well as the "Review of Prerequisites" that I've done in lecture. I'll hopefully start on Section 2.2.2 and maybe on 2.2.3 as well, but I won't assign problems from that section yet!

Problem 1: Suppose that $f \in C_c(\mathbb{R}^n)$ and that $g \in L^1(\mathbb{R}^n)$. Show that $f \star g \in C(\mathbb{R}^n)$.

Note: The notation means that f is continuous with compact support, and $\int_{\mathbb{R}^n} |g(x)|\,dx < \infty$, and you have to show that $f \star g$ is continuous on $\mathbb{R}^n$.

Hint: f is uniformly continuous!

Problem 2: **Carefully** fill in the details of the proofs of Theorems 2 (Integration by parts formula) and 3 (Green's formula) on page 712. Formulas (i) and (ii) in Theorem 3 are absolutely crucial in this course, so make sure you remember them!

(TURN PAGE for Problem 3)

Problem 3: (I told you it's gonna show up again :P) This exercise is great practice of the review techniques that I presented in lecture, especially the integration by parts formula and the polar coordinate formula:

Suppose that $u = u(x)$ and $v = v(x)$ are solutions to the following system of PDE in $\mathbb{R}^n$:

$$\begin{cases} \Delta u = v \\ -\Delta v = u \end{cases}$$

Assume that there exists a constant $C > 0$ such that for all $x \in \mathbb{R}^n$ the following holds:

$$|u(x)| \leq \frac{C}{|x|^n}, |v(x)| \leq \frac{C}{|x|^n}, |Du(x)| \leq C, |Dv(x)| \leq C$$

Show that $u = v = 0$ in $\mathbb{R}^n$.

Hint: Fix $r > 0$, multiply the first equation by v and the second equation by u, add the two resulting equations and integrate on $B(0, r)$. Then use the integration by parts formula (or formula (iii) in Theorem 3 on page 712) and finally let r go to ∞. You need to show that the left-hand-side of your (integral) equation goes to 0 as $r \to \infty$. For this, use the definition of the normal derivative and your assumptions. Finally, remember that if $\int f = 0$ where $f \geq 0$, then in fact $f \equiv 0$.