

Math 200 – Homework 12

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This assignment is due on **Friday, May 12, 2017** at 9:50 AM.

Reading: Sections 9.7, 9.8, 9.9, 10.1. In Section 9.8, skim the section on probability axioms (although hopefully they shouldn't be too surprising). For Bayes' Theorem, on the final, I will only ask you for the case of two mutually disjoint events ($n = 2$), like I did in lecture. I won't have time to discuss probability trees (page 614), but please look at them. Don't worry about the definition of independence of three or more events (pages 620 and 621), but know how to use them (for instance, understand how to do Example 9.9.9). In Section 10.1, there are a *lot* of definitions, but unfortunately that is the nature of graph theory, so make sure to understand (and memorize) all the definitions.

Note: I also posted a 'Non-Homework' on my website, just in case you need more practice for sections 10.2, 10.5, and 10.6 (which will be on the final). It will never be due and not be graded.

- **Section 9.7:** 39, AP
- **Section 9.8:** 15, 22
- **Section 9.9:** 7, 25, 29
- **Section 9.1 (!):** 20 (this concerns the Monty Hall Problem)
- **Section 10.1:** 20, 23, 28, 48

Additional Problem: Another nice property of Pascal's triangle is that not only is the triangle symmetric, but also it attains its maximum around its axis of symmetry. For instance $\binom{6}{r}$ has its maximum at $r = 3$, and $\binom{5}{r}$ has its maximum at $r = 2$ and $r = 3$. Prove this, namely show that given n , $\binom{n}{r}$ has its maximum at $r = \frac{n}{2}$ if n is even, and $r = \frac{n \pm 1}{2}$ if n is odd.

Hint: Show that $\binom{n}{r}$ is increasing for $r \leq \frac{n}{2}$. For this, show that the successive ratios $\frac{\binom{n}{r+1}}{\binom{n}{r}}$ are all greater than 1, as long as $r < \frac{n-1}{2}$. And then use that the triangle is symmetric about the axis $r = \frac{n}{2}$, which shows that $\binom{n}{r}$ must be decreasing for $r \geq \frac{n}{2}$ and therefore must have a maximum at $r = \frac{n}{2}$ if n is even, and at $r = \frac{n \pm 1}{2}$ if n is odd.

Hint for 9.8.22: First do the case where the first coin lands heads H, then do the case TH , and then the case TTH , etc.

Note: Problem 48 in 10.1 is an instance of the (famous) four color theorem, which says that any graph can be colored using four colors in a way that no adjacent vertices have the same color. Its significance lies in the fact that it's the first major theorem that has been proved using a computer. You basically reduce the theorem to a set of 1936 examples, and let the computer do the colorings of all the examples. It gave rise to a great debate between mathematicians who believe that a proof by computer is a valid proof technique and those who believe that it's not.