

Math 200 – Homework 11

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This assignment is due on **Friday, May 5, 2017** at 9:50 AM.

Reading: Sections 9.4, 9.5, 9.6. The book’s presentation of The Pigeonhole Principle is a bit awkward, so all I’m asking you to understand is what I covered in lecture. In Section 9.5, ignore the section “The Number of Partitions of a Set into r subsets,” and in Section 9.6, ignore Example 9.6.4.

Note: Problem 11 in section 9.5 is very, **very** long, that’s why there are ‘only’ 9 problems. I’d suggest you to play some poker to get a feel for it :) Also, for part (i) of that problem, just tell me how to get the answer from your answers from (a)-(h), don’t actually calculate it.

- **Section 9.4:** 11, 30, AP
- **Section 9.5:** 11, 13, 20
- **Section 9.6:** 3, 14, 18

Additional Problem: Use the pigeonhole principle to show that if $f : \{1, 2, \dots, n\} \rightarrow \{1, 2, \dots, n\}$ is one-to-one, then f is also onto.

Hint: Suppose f is not onto and show f is not one-to-one. Think of your outputs $f(1), \dots, f(n)$ as pigeons and of your elements $1, 2, \dots, n$ in your codomain B as your holes. Since f is not onto, its range is not all of B , so one of the holes in B **must** be empty, so you’re really putting n pigeons into fewer than n holes! Conclude.

Note: One can also show (but you don't have to do this) that if $f : \{1, 2, \dots, n\} \rightarrow \{1, 2, \dots, n\}$ is onto, then f is one-to-one. So for **finite sets of the same size**, one-to-one is equivalent to onto! Wow!!! Note that this is not true for infinite sets, there are functions $f : \mathbb{Z} \rightarrow \mathbb{Z}$ that are one-to-one but not onto, and vice-versa.

More Hints:

Hint for 9.4.11: Think of the sets $\{1, 2\}, \{3, 4\}, \dots, \{2n - 1, 2n\}$ as your pigeonholes.

Hint for 9.4.30: Think of it in terms of the worst-case scenario. Suppose you're only getting 4 pennies from each year \dots .

Hint for 9.5.11: Look at how they do Example 9.5.8. Some of the solutions are in the back of the book if you're stuck.

Hint for 9.5.20(c) Think of CR as a new letter and of ON as a new letter. Then the word MILLIMI(CR)(ON) only has 9 letters. Also doesn't the word HULLA-BALOO (from 9.5.19) make you giggle?