Math 453 – Homework 1

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Reading: Chapter 1 and Section 2.2.1 of Chapter 2. I also highly recommend going through the Notation (section A in the appendix). The most important "review" concepts that we'll use for Chapter 2 are: the inequalities (a)−(f) and (i) in B2, the Gauss-Green Theorem (C2), the polar coordinate formula (Theorem 4 in C3), Convolution and properties of mollifiers (C5), the Dominated Convergence Theorem (Theorem 5 in E4) and Lebesgue's Differentiation Theorem (Theorem 6 in E4).

- **Chapter 1:** 5
- **Chapter 2:** 2

Hints and Discussion:

Hint for 5: This problem illustrates how amazing the multi-index notation actually is. Notice how similar this result looks like the Taylor's formula in one variable!

Note: Here $\alpha!$ means $\alpha_1! \, \alpha_2! \cdots \alpha_n!$ and x^α means $x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_n^{\alpha_n}$.

This problem is a neat technique of turning a function of several variables into a function of one variable, sort of like what we did for Laplace's equation! Since g is a function of one variable, you can apply Taylor's formula in **one** variable to g, which says that, for some $\xi \in (0, t)$, we have

$$g(t) = \sum_{m=0}^{k} \frac{t^m}{m!} g^{(m)}(0) + \frac{g^{k+1}(\xi)}{(k+1)!} t^{k+1}$$

After calculating all the derivatives of g, let $t = 1$. You are allowed to use the fact (without proof) from Math 200 that there are $\frac{|\alpha|!}{\alpha!}$ partial derivatives that are equal to a given $D^\alpha f$.

Hint for 2: In my opinion, the easiest way to do this it to apply brute-force by calculating $v_{x_i x_i}$. It's a bit messy, but a great way to practice the chain rule. Recall that a matrix O is orthogonal iff $O^T O = O O^T = I$, which reads as:

$$\sum_{i=1}^{n} o_{ki} o_{ji} = \begin{cases} 1 & \text{if } k = j \\ 0 & \text{if } k \neq j \end{cases}$$

where o_{ki} is the (k, i) entry of O.

Also, I apologize for the linear algebra in this problem, we won't do many linear algebra problems in this course (if at all).