

# MATH 200 – FINAL EXAM – STUDY GUIDE

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The Math 200 – Final Exam will take place from Saturday, May 13 until Sunday, May 21 (inclusive). It is a **self-scheduled exam**, which means that you can pick it up any time you want from the second floor of Hopkins Hall, and take it there for a total of 150 minutes (= 2h30). It covers Section 1.1 (Variables) up to and including Section 10.6 (Rooted Trees). It will be a closed book and closed notes-exam, and no cheat-sheets will be allowed. Unless otherwise noted below, you are responsible for **everything** we've learned, including what is in the book, what you've learned in lecture, and what you've done in the homework.

This is a study guide for the exam, and it is really meant to be what it is, namely a *guide*. This list is not meant to be exhaustive, although I've tried to put everything that's important and not important). For your convenience, I've also included the study guides for midterms 1 and 2, so there is some repeat.

**Main concepts:** Logic (Chapters 2 and 3), Number Theory (Chapter 4), Induction and Recursion (Chapter 5), Sets and Functions and Relations (Chapters 6, 7, and 8), Combinatorics and Probability (Chapter 9), Graph Theory (Chapter 10)

## 1. CHAPTER 1: SPEAKING MATHEMATICALLY

- You can basically skip this chapter since it's just an introduction, *except* how to show something is or is not a function.
- Don't forget about AP1 and AP2 on HW1.

## 2. CHAPTER 2: THE LOGIC OF COMPOUND STATEMENTS

- Sections 2.1 and 2.2: Everything in those two sections is very important, so make sure to thoroughly read it. In particular, know how to construct truth tables and show that two statements are equivalent (or not) using truth tables. Moreover, I could ask you about how to

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show equivalence using the laws of logic, like they do in example 2.1.14.

- Section 2.3: Ignore everything from “Additional Valid Argument Forms: Rules of Inference,” to the end of the section. In particular, you do **not** need to know the knights/knaves problem and/or the CSI Williamstown logic puzzle I’ve covered in lecture.
- You do **not** need to memorize the table on page 61. All I’m asking you from this table is to know what Modus Ponens and Modus Tollens is.
- Sections 2.4 and 2.5: Skip!
- Although this is more relevant to Chapter 9, don’t forget about AP3 in HW1.

### 3. CHAPTER 3: THE LOGIC OF QUANTIFIED STATEMENTS

- Sections 3.1 and 3.2: Again, everything in this chapter is very important, **except** you can skip any section/example that has to do with Tarski’s world
- Section 3.3: Skip the section on Prolog.

### 4. CHAPTER 4: ELEMENTARY NUMBER THEORY AND METHODS OF PROOF

- Section 4.1: you don’t need to memorize the lists on pages 155 – 158, but of course they are very useful tips!
- Section 4.2: The most important thing in this section is the concept of a rational number; once you understand that, you should hopefully be able to understand the rest of the chapter. Also remember how to convert any number with a repeating decimal into a rational number like I did in lecture (or you did on the HW).
- Section 4.3: Skip the proofs of theorems 4.3.1 and 4.3.2 and 4.3.4, but know their statements; but you do need to know how to prove Theorem 4.3.4 by strong induction!
- Remember AP2 on HW3.
- Section 4.4: Skip the section on “Absolute Value” in section 4.4
- Don’t worry about what I did in lecture about how to show that if  $x$  is rational, then  $x$  has a repeating decimal.
- Section 4.5: You are responsible for this section, even though I didn’t fully cover it in lecture, but you can skip the proof of theorem 4.5.3 if you want to.
- Section 4.6: This section is very important, and make sure to thoroughly read it

- Section 4.7: You are definitely responsible for knowing the three little proofs I covered in lecture, namely: The proof that  $\sqrt{2}$  is irrational, the proof that there are infinitely many prime numbers, and the proof that there are two irrational numbers  $x$  and  $y$  such that  $x^y$  is rational. You can pick whichever version you prefer, either the book-one or the one I presented in lecture (but if you choose the book version, you'll need to prove Proposition 4.7.3 as well!)
- Section 4.8: Skip!
- Remember what I covered in lecture about congruences, gcd, relatively primeness, and Fermat's Little Theorem. Know the statement of Fermat's Little Theorem, and how to use it! Those are certainly valid topics I could ask on the exam! Ignore the Chinese Remainder Theorem. More precisely, know Theorems 8.4.1, 8.4.2, 8.4.3 (skip the proof), 8.4.9 (skip the proof) and 8.4.10 (skip the proof). Good practice would be, for example AP1 on HW5, or Problem 2 on Midterm 1 of last semester.

## 5. CHAPTER 5: SEQUENCES, MATHEMATICAL INDUCTION, AND RECURSION

- Section 5.1: Skim it, unless you don't know what a sequence is. If you do read it, know what the summation/product notation is, but you can definitely skip the section "Sequences in Computer Programming" and "Application."
- Sections 5.2 - 5.4: You are responsible for knowing the coin-change example, although this is probably easier to prove using strong induction. Also you need to know the section about the sum of a geometric sequence, even if I haven't covered it in lecture. Also know the example of trominoes. Remember AP2 and AP3 in HW 5 (in particular the gossip problem!) In particular don't expect every induction proof to be about sums, and also look at AP 1 in HW 6; I could ask you about more exotic situations like the handshake-problem or the polygon problem on your Homework. And remember the standing-in-line problem from Midterm 1 from last semester. For strong induction, there are two classical in this section: The coin-changing problem, and the fact that any integer  $n > 1$  can be written as a product of primes. Please thoroughly understand those examples, although I could ask you for more exotic examples like the ones on the practice exam for Midterm. You can ignore everything starting from page 272 on!
- Section 5.5: Skip!

- Section 5.6: Pay particular attention to the Fibonacci example and the Tower of Hanoi-example. Good exam questions would be variations on those problems, just like for instance 5.6.20 or 5.6.23 that were on Homework 11, or Problem 1 on Midterm 2 from this semester. I may possibly ask you about different examples as well. However, ignore Examples 5.6.7 and 5.6.8, as well as the section on recursive definitions of sums and products
- Section 5.7: the only important concepts are the definition of an arithmetic sequence (top of page 307) and the definition of a geometric sequence (page 308). Everything else, although important, isn't very appropriate to ask on an exam, so just skim it. I could potentially ask you about problems like 5.7.15, though, and I could ask you to verify that a formula is correct using induction (like Example 5.7.7). Definitely skip Examples 5.7.4, 5.7.6, and 5.7.8
- Section 5.8: In this section, you are responsible for:
  - (a) The case of two distinct roots
  - (b) The case of a single root (Theorem 5.8.5 in the book)
 You should also be able to do this using initial conditions, like for example in 5.8.8. I will **NOT** ask you about complex roots. You are **NOT** responsible for AP2 on HW 6.
- Section 5.9: You can skip everything **except** Examples 5.9.6 and 5.9.8 (which I've actually covered in lecture when I did chapter 7)

## 6. CHAPTER 6: SETS

- Section 6.1: Know every piece of terminology in that section. You can skip the last section (labeled Optional)
- Section 6.2: You do **NOT** need to know the *names* of the set identities in Theorem 6.2.2 **EXCEPT** the Commutative Laws, the Associative Laws, the Distributive Laws, and the De Morgan's Laws. But certainly know how to use them. And of course remember that to prove two sets are equal, you have to show that each of them is contained in the other.
- Section 6.3: You can ignore the proof of Theorem 6.3.1 if you understood the proof I gave in lecture (with  $n = 3$ ).
- Section 6.4: Skip the part about Boolean Algebras. You're only responsible for Russel's paradox and the Halting Problem and whatever paradoxes are on the homework. Finally, you are not responsible for understanding the Axiom of Choice / Banach-Tarski paradox that I did in lecture.

## 7. CHAPTER 7: FUNCTIONS

- Section 7.1: Ignore examples 7.1.9 and 7.1.10, as well as the section on Boolean functions.
- Section 7.2: Again, a lot of important terminology! Ignore the section Application: Hash Functions, and examples 7.2.6, 7.2.10 and 7.2.14.
- Section 7.3: Everything in this section is important! You should also know how to show that the composition of one-to-one functions is one-to-one and that the composition of onto functions is onto,
- Section 7.4: This section is probably the most important section of Chapter 7 because it puts everything we learned in that section together, so pay particular attention to it. Ignore Example 7.4.1, the proof of Theorem 7.4.3, and the section “Application: Cardinality and Computability.” Don’t forget about AP 1 - AP3 in HW 9. You **ARE** responsible for knowing about the Grand Hotel Peyam (Hilbert’s Hotel) Problem. Remember that I also sent you a link with a video explaining it. **Know how to prove that  $\mathbb{R}$  is uncountable!** For 7.4.35, again you don’t need to know how to come up with the set  $A$ , but do know how to use it!
- Also know the definitions of  $f(C)$  and  $f^{-1}(C)$  for sets, as in 7.1.42, 7.1.43, 7.3.47 (those definitions are valid even if  $f$  doesn’t have an inverse!). Remember for instance Problem 3 on Midterm 2 from last semester, or Problem 4 on Midterm 2 from this semester.

## 8. CHAPTER 8: RELATIONS

- Section 8.1: Ignore the section “N-ary relations and Relational Databases” and Example 8.1.6.
- Section 8.2: Ignore Example 8.2.1 and the section “The Transitive Closure of a Relation.”
- Section 8.3: Ignore Examples 8.3.3, 8.3.4, 8.3.7, 8.3.9. Don’t forget about AP 4 and AP 5 on HW 9. The proof of Lemmas 8.3.2 and 8.3.3 is a good practice of the techniques in Chapter 6.
- Don’t forget about Problem 4 in Midterm 2 from last semester and Problem 2 in Midterm 2 from this semester.

## 9. CHAPTER 9: COUNTING AND PROBABILITY

- Section 9.1: Ignore Example 9.1.5. By the way, I’m assuming that you know what a deck of card looks like (Example 9.1.1) and what a pair of dice looks like (Example 9.1.2). Don’t forget about the Monty Hall Problem!

- Section 9.2: Ignore Examples 9.2.5 and 9.2.6. Everything else is fair game, in particular drawing a possibility-tree, the multiplication-rule, and counting the number of  $r$ -permutations of a set of  $n$  elements.
- Section 9.3: Ignore Examples 9.3.4, 9.3.5. In particular, know the rule about the probability of the complement of an event (page 543). Know the Inclusion/Exclusion Rule for **two** events (the first half of Theorem 9.3.3). I will **not** ask you about the Inclusion/Exclusion rule for 3 or more events, so you can skip Example 9.3.7. Don't forget about the Additional Problem on HW 10.
- Section 9.4: The book's presentation of the pigeonhole principle is a bit awkward, so all I'm asking you to understand is what I covered in lecture. Don't forget about the Additional Problem on HW 11.
- Section 9.5: Ignore the section "The Number of Partitions of a Set into  $r$  subsets," but don't forget about Problem 9.5.11 (the Poker Problem)
- Section 9.6: Ignore Example 9.6.4
- Section 9.7: Everything in this section is important. In particular, you should know how to prove the theorems in both ways (algebraic proofs and combinatorial proofs). Don't forget about the additional problems in Homework 11.
- Section 9.8: Skim the section on probability axioms
- Section 9.9: I will only ask you about Bayes' Theorem (Theorem 9.9.1) in the case  $n = 2$ , i.e. the version I did in class. Don't worry about the definition of independence of *three* or more events (pages 620 and 621), but know how to use them (for instance, understand how to do Example 9.9.9).
- In the lecture about probability paradoxes, you're only supposed to know the section on Bayes' Theorem and the Monty Hall Problem, but I may ask you for other paradoxical scenarios, like the Gambler's ruin on the practice final.

## 10. CHAPTER 10: GRAPH THEORY

- Section 10.1: Know the definitions of a graph, loops, and parallel, simple graph, complete graph on  $n$  vertices, subgraph, degree. You **don't** need to know the definition of adjacent, connects, incident on, isolated, complete bipartite graphs (ignore Example 10.1.10). You can ignore the proofs of Theorem 10.1.1 and of Proposition 10.1.3 (but know their statements).
- Section 10.2: Again, lots of definitions! Know the definition of walk, closed walk, circuit, connected, Euler circuit, Hamiltonian

Circuit. You can ignore the definitions of trail, path, simple circuit, connected component. Ignore Lemma 10.2.1. You can ignore the proofs of Theorems 10.2.2 and 10.2.3 if you want (but know their statements), ignore Corollary 10.2.5 and Proposition 10.2.6. Know how to find an Euler Circuit (or show one does not exist, see Theorem 10.2.4) and know how to find a Hamiltonian circuit. Ignore the section on Euler Trails (Corollary 10.2.5 and Example 10.2.7). I will not ask you to show that a graph does not have a Hamiltonian circuit, but I could ask you to show that a graph does not have an Euler circuit. Ignore Example 10.2.9.

- Sections 10.5 and 10.6: I will only cover those two sections very briefly, but know all the definitions in that section, including tree, forest, terminal vertex/leaf, internal vertex/branch, rooted tree, level, height, children, parent, sibling, ancestor, descendant, binary tree, left/right child, full binary tree, left subtree, right subtree. Ignore the proofs of Lemma 10.5.1 and Theorem 10.5.2 (but know their statements), ignore Examples 10.5.3 and 10.5.4, ignore Lemma 10.5.3, ignore the proofs of Theorems 10.5.4, 10.6.1 and 10.6.2 (but know their statements), ignore Example 10.6.2. I will **NOT** ask you about tree induction on this exam, so you can ignore, for example, question 9 on the final from last semester.
- Good exam questions include, but are not limited to, constructing graphs or trees with certain properties (or showing that such graphs do not exist), and finding Euler/Hamiltonian circuits. For more practice, look at the Non-Homework posted on my website.