FINAL EXAM – REVIEW – PROBLEMS

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Problem 1: Draw an example of such a graph (or show that no such example exists)

(a) A binary tree of height 3 and 11 leaves
(b) A full binary tree with an odd number of leaves
(c) A full binary tree with as many branches as leaves
(d) A simple graph with a Hamiltonian circuit but no Euler circuit
(e) A simple graph with an Euler circuit but no Hamiltonian circuit
(f) A graph with vertices of degrees $1, 2, 3, 4, 5, 6$

Problem 2: What is the expected value of the number of aces in a 5-card poker hand?

Problem 3: Suppose you have a bag of 12 crooked coins (6 red and 6 blue). The red ones come up heads $\frac{3}{5}$ of the time and the blue ones come up tails $\frac{2}{3}$ of the time. Suppose you reach into the bag, pull out a coin at random, and toss it. The coin comes up tails. What is the probability that you pulled out a blue coin?

Problem 4: Suppose that Café Peyam has 6 different kinds of pies (Apple, Blueberry, Chocolate, Dutch Caramel, Elderflower, Fudge). How many selections of 20 pies contain at most 10 apple pies and at most 6 blueberry pies?

Problem 5: Show that there are at least two numbers in the list

$$1!, 2!, \cdots, n!, (n+1)!$$

such that $i! - j!$ with $i \neq j$ is a multiple of n

Problem 6: Show that that there is a rational number x and an irrational number y such that x^y is irrational.

Problem 7: Find 2017^{2017} **mod** 13 (note that 2017 is prime)

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Problem 8: Suppose you have a staircase with n stairs, and you can climb it up by either moving up one stair or two stairs at a time (or a combination of the two). Let c_n be the number of different ways to climb the staircase. Find a recurrence relation for c_n.

Problem 9: Show that the set of real numbers in $(0,1)$ whose decimal expansion contains only the digits 3 and 7 (for example 0.3737773...) is uncountable

Problem 10: Define an equivalence relation $\sim$ on $\mathbb{R}$ by $x \sim y$ iff $|x| = |y|$. Find all the equivalence classes of $\sim$

Problem 11: Show that if $f : A \to B$ is one-to-one and S and T are subsets of A, then $f(S) \cap f(T) \subseteq f(S \cap T)$.