MATH 200 – MIDTERM 1

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Instructions: Welcome to Midterm 1! You have 50 minutes to take this exam, for a total of 100 points. **Do not open the exam until you're instructed to do so**. No books or notes or cheat sheets or personal electronic devices (including calculators) are allowed, and you may not communicate or collaborate with anyone during the exam. Remember that you are not only graded on the correctness of your answer, but also on the clarity and completeness of your proofs. Write in full sentences whenever you can. If you need to continue your work on the back of the page, clearly indicate so, or else your work will be discarded. Good luck, and may modus ponens be with you! :)

1		10
2		25
3		20
4		30
5		15
Total		100

Date: Wednesday, October 19, 2016.

1. *(10 points, 2 pts each)*

Label the following statements as **TRUE (T)** or **FALSE (F)**. Any correct answer gives you 2 points, and any incorrect answer gives you 0 points. You do **NOT** get negative points for an incorrect answer, and you do **NOT** need to justify your answers.

Warning: Do not spend more than 5 minutes on this question, because there is plenty of work to do on the remaining 4 problems!

_____ (a) If $P = P(x, y)$ is a predicate, then

$$[\exists y \, \forall x \, P(x, y)] \Rightarrow [\forall x \, \exists y \, P(x, y)]$$

_____ (b) If P and Q are propositions, then

$$\sim (P \wedge Q) \equiv ((\sim P) \wedge (\sim Q))$$

_____ (c) The following argument is valid

$$\begin{aligned} P &\Rightarrow Q \\ \sim Q & \\ \therefore \sim P & \end{aligned}$$

_____ (d) The negation of "All students know discrete math" is "No student knows discrete math".

_____ (e) The rule $f : \mathbb{Q} \to \mathbb{Z}$ given by $f\left(\frac{a}{b}\right) = ab$ is a function.

2. *(25 points)*

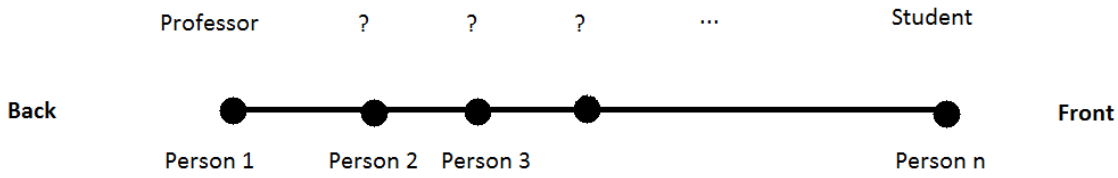
   (a) *(10 points)* Prove that that if n^2 is divisible by 5, then n is divisible by 5. You may use any facts discussed in lecture or in the book or in the homework.

   (b) *(15 points)* Using (a), prove that $\sqrt{5}$ is irrational.

3. *(20 points)* Find the remainder when 2017^{2017} is divided by 13.

   Hint: You may use the facts that $2017 = 166 \times 12 + 3$ and that $2017 = 155 \times 13 + 2$ (you may or may not need to use that 2017 is prime)

4. *(30 points)* Suppose n people (where $n \geq 2$) stand in line (say at Paresky for lunch) and the person at the very front of the line is a student and the person at the very back of the line is a professor, as in the picture below. Show by induction that somewhere in the line there is a student *directly* in front of a professor. Clearly indicate where you use your induction hypothesis.



Hint: For the inductive step, argue in terms of whether the second person in line is a professor or a student.

Note: In order to save you time, you do **not** have to *explicitly* write out what P_n and P_{n+1} are, as long as it's clear what you're doing; that said, you are still graded on how write down a correct induction proof.

5. *(15 points)*

Warning: This problem is slightly harder, and I do not expect a lot of people to get it right!

Show that for all real numbers x,

$$\left\lfloor \frac{\lfloor \frac{x}{3} \rfloor}{3} \right\rfloor = \left\lfloor \frac{x}{9} \right\rfloor$$

Hint: Use the quotient remainder theorem and a proof by cases. In order to save you some time, you only need to show me how to do it for one of the cases.