

Math 379 – Homework 5

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Friday, October 14, 2016

Problem 1: This problem refers to Example 1 “Rapidly Oscillating Coefficients” (which we’ll finish on Wednesday).

(a) In lecture, we defined/will define

$$\bar{a} = \int_0^1 a + 2w'a + wa'dy,$$

where $w = w(y)$ is the solution of (here $' = \frac{d}{dy}$)

$$\begin{cases} -(aw')' = a' \\ w(0) = w(1) \end{cases} \quad (7)$$

Show that

$$\bar{a} = \left(\int_0^1 \frac{1}{a(y)} dy \right)^{-1}.$$

Hint:

- First solve the equation (7) for w by antidifferentiating (be careful about the limits of integration). You may also assume that $w'(0) = 0$ ¹. If you’re completely stuck, it’s very similar to the last part of the second problem in Homework 1.

¹This is not a very restrictive assumption actually, because by the assumption $w(0) = w(1)$ and Rolle’s Theorem, there is a point x_0 in $(0, 1)$ such that $w'(x_0) = 0$, and you can easily adapt the proof you’ve given with this condition instead of $w'(0) = 0$

- Then go back to the definition of \bar{a} , integrate by parts to write $w a'$ in terms of $w' a$ (why are the boundary-terms 0?), and use your formula for w .
- (b) [This should be quick] Using your formula for \bar{a} from (a), find the general solution of

$$-\bar{a}u_{xx}^0 = f(x)$$

(don't worry about the periodicity for this part).

Note: The reason I put this part is to show you that in fact, one can find an explicit formula for our function u_0 .

Problem 2: This is pure calculus, but it's one of our steps in Example 2 "An Oscillator with Damping"

- (a) Using undetermined coefficients, find a particular solution of

$$w''(t) + w(t) = -\frac{1}{4} \cos(3t)$$

- (b) Using undetermined coefficients, find a particular solution of

$$v''(t) + v(t) = -\frac{3}{4} \cos(t)$$

Note: This is trickier than usual because the inhomogeneous term $\cos(t)$ coincides with the solution of the homogeneous equation, so there is a certain kind of resonance going on!

- (c) Use $-\cos^3(t) = -\frac{3}{4} \cos(t) - \frac{1}{4} \cos(3t)$ to find the general solution of

$$u_1'' + u_1 = -\cos^3(t)$$

- (d) Find the solution of (c) that satisfies $u_1(0) = 0$ and $u_1'(0) = 0$.