# WCMC Math Challenge Questions - Summer 2018

**Instructions.** Try as many of the following questions as you can. There is no time limit, but you should do these questions **on your own without any help from people, books, internet, or any other sources**. We are more interested in how you approach the questions and how you communicate your reasoning than in how many correct answers you obtain. For each solution you submit, please include a clear and complete explanation of your answer. Please typeset your answers. Send any questions you have to Allison.Pacelli@williams.edu.

1. There's a small village at the base of a mountain with a population of 7777. There are at least 2 people in the village that do not have dogs. Also, given any three people in the village, at least one of the three has a dog. Do we have enough information to determine exactly how many people in the village have dogs? Explain why not, or find the exact number. Explain your reasoning.

2. Mr. Z, construction boss, has 3 boxes of building supplies, containing nails and screws. One is labeled 'nails," another is labeled 'screws," and the last is labeled 'nails and screws." We know that a tired employee labeled each box incorrectly. Taking only one item out of one box, how can we label the boxes correctly for Mr. Z?

3. Find all integers $M$ that satisfy all of the following properties. Explain your answer fully.

(i) If 3 divides $M$, then $20 \leq M \leq 40$.

(ii) If 9 does not divide $M$, then $20 \leq M \leq 30$.

(iii) 7 divides $M$.

4. There's a box of five hats: two blue and three white. Andy, Kate, and James each place a hat on his or her head, while blindfolded. One by one, each child removes his blindfold and (without using a mirror) gets one opportunity to guess the color of the hat on his own head. If any of the three guesses correctly, everyone gets to go to the park! Andy, Kate, and James are each very logical, and know that the others are as well.

First, James removes his blindfold. He sees the hats that the others are wearing, but admits that he is unable to discern his own hat color.

Next, Kate removes her blindfold, and sadly reveals that she too is not able to determine the color of her own hat.

Finally, Andy pipes up and says "I can answer with my blindfold on! I know what color hat I am wearing."

*What color is Andy's hat, and how does he know? Explain fully.*

5. Andy encounters a strange island, where every creature has either green, purple, or blue hair on his head. He's told by a reliable source that those with green hair always tell the truth, those with purple hair always lie, and those with blue hair make statements that are alternately true and false (though the order of which statements are true and which are false is unknown). One day, Andy meets three of the islanders, but each is wearing a very big hat which completely covers his hair. Andy asks each of the three the color of their hair, and the responses are as follows:

A: I have green hair.

B: I have purple hair.

C: 1. They are both lying. 2. I have blue hair.

Assuming that each of the three has a different color hair, determine the hair color of each of A, B, and C. Explain your answer fully. If we don't assume that each has a different hair color, is it possible to determine the hair color of each islander? Explain fully.

6. There are 100 doors, and 100 people lined up in a row. All the doors are closed. The first person goes through and opens all the doors. The second person goes through and changes the position (open to closed or closed to open) of every second door. The third person goes through and changes the position of every third door. And so on. After all 100 people have gone through the doors, which doors are open and which are closed? Explain fully.

7. Five pirates of different ages ($A$, $B$, $C$, $D$, and $E$ oldest to youngest) have a treasure of 100 gold coins. On their ship, they decide to split the coins using this scheme:

The oldest pirate proposes how to share the coins, and ALL pirates (including the oldest) vote for or against it.

If 50% or more of the pirates vote for it, then the coins will be shared that way. Otherwise, the pirate proposing the scheme will be thrown overboard, and the process is repeated with the pirates that remain. As pirates tend to be a bloodthirsty bunch, if a pirate would get the same number of coins if he voted for or against a proposal, he will vote against so that the pirate who proposed the plan will be thrown overboard.

A) Assuming that all 5 pirates are intelligent, rational, greedy, and do not wish to die, (and are rather good at math for pirates) what will happen?

B) Come up with another mathematically interesting extension to this problem that you could investigate. You don't need to do the problem, only pose the question.