Astronomy 111 **Fall 2017**

Homework Assignment #7

(remember: no ragged edges; write on one side of page only; leave room for comments/corrections)

due in class Thu. 12/7

1. By what factor does the mass of an object increase if it travels at:
   a) 25% the speed of light?
   b) 99.99% the speed of light?

2. Newton's Laws tell us that the kinetic energy of a moving object is ½mv^2. What is the kinetic energy of a 1 kg mass moving at $v=10^4$ m/sec (about 1/3 the orbital speed of the Earth)? Compare this with its rest mass energy, and comment.

3. The expanding universe is carrying distant objects away from each other at a rate proportional to their separations. We use the Doppler effect observed in spectra of distant galaxies and quasars to calculate recession speeds. For the most distant objects recession speeds approach *c*, and therefore, the relativistic Doppler shift expression must be used. We define the *redshift*, z, as the fractional change in wavelength:

$$z \equiv \frac{\Delta\lambda}{\lambda} = \left[\frac{1+\mathrm{v}/c}{1-\mathrm{v}/c}\right]^{1/2} - 1$$

   a) The most distant quasar currently known is ULAS J1120+0641, discovered with the UK Infrared Telescope on Mauna Kea. It has a redshift of 7.1. Calculate its radial velocity in terms of v/c.

   b) Show that in the limit of v/c << 1, the above expression for z reduces to the familiar Doppler shift equation. (*Hint*: Use a binomial expansion and think about which terms can be neglected.)

4. As the binary pulsar loses energy through gravitational radiation, why do the components speed up and why does the period decrease?

5. Suppose that the Earth were collapsed to the size of a small ball bearing, becoming a black hole.

   a) What would be the orbital period of the Moon, orbiting at a distance of $4\text{x}10^5$ km?

   b) What would be the orbital period of a spacecraft orbiting at a distance of 6000 km, the current radius of the Earth?

   c) What would be the orbital period of a mini-spacecraft orbiting at a distance of 0.1 m above the black hole? Compare the orbital speed in this case to *c*.

6. Why is time dilation in a gravitational field equivalent to a gravitational redshift? (*Hint:* Consider the definition of frequency, and how the value of the frequency is affected by a redshift.)

7. Calculate the wavelength we would observe from Earth for Hα (λ_{rest}=6563 Å) emitted by hydrogen atoms 100 km above the Schwarzschild radius of a 15 $M_{\odot}$ black hole. In what region of the spectrum would we observe it?

8. A recently discovered quasar called SDSSJ0100+2802 (abbreviated J0100) is one of the most powerful objects known, with a total luminosity L = 2×10^{41} W.

   a) Use Eddington Limit arguments to estimate the mass of the black hole in J0100.

   b) Briefly explain the physical basis of the Eddington Limit (what happens if a source violates it?). Is your estimate of black hole mass from part (a) an upper or lower limit?

   c) If the black hole is 10% efficient at converting infalling mass into energy, at what rate is mass falling into the black hole in J0100? Express your answer both in terms of kg/sec and solar masses per year.
   [adapted from C. Reynolds]

9. Read the paper documenting the detection of gravitational radiation (http://sites.williams.edu/kkwitter/files/2016/11/GW_BHkk_post.pdf). It's a technical article (with *lots* of authors), but skim it and especially read the portions I have highlighted. In one paragraph, describe what happened to this system in your own words. Compare the initial masses of the components and the final merged mass. Explain why the final mass is less than the sum of the initial masses. What was the fate of the "missing" mass, and how much energy did it contain?