Astronomy 111 **Fall 2017**

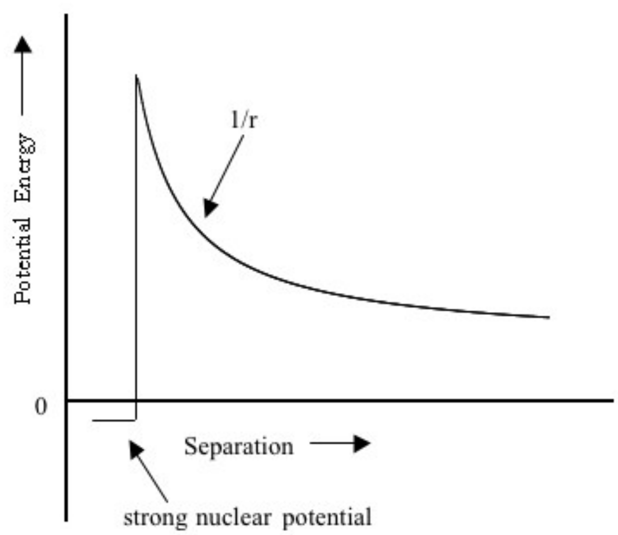
Homework Assignment #4

(remember: no ragged edges; write on one side of page only to leave room for comments/corrections)

due in class Thu. 11/2

1. (*based on a problem in Carroll & Ostlie*)
For the calculations you will do below, $e=1.6 \times 10^{-19}$ in SI units, $k=1.38 \times 10^{-23}$ J/K, and $h=6.62 \times 10^{-34}$ J-sec. We covered the basic concept in class about quantum mechanical tunneling being necessary in order for fusion to occur in the sun and other stars. Here you will actually calculate that this is true:

Consider two protons in the core of the sun. The height of the potential energy barrier, as shown below, goes as $1/r$ for large separation (Coulomb repulsion), and becomes negative (attractive) at small separations, as the strong nuclear force dominates over the Coulomb repulsion.



If we assume that the energy required to overcome the Coulomb barrier is provided by the thermal (kinetic) energy of the gas, and that all protons are non-relativistic, then we can estimate the temperature, $T_{classical}$, as follows. If we equate the average kinetic energy of an incoming proton to the energy of the Coulomb barrier at r, we find $\frac{3}{2}kT_{classical} = \frac{\alpha e^2}{r}$, where $\alpha = 1/4\pi\varepsilon_o$, ($9\times10^9$ in SI units), and r is the separation. We then find that $T_{classical} = \frac{2\alpha e^2}{3kr}$.

a) Assume a separation equal to that of the proton radius (10^{-15} m). Calculate $T_{classical}$ and compare it with the actual central temperature of the Sun, and comment.

Now, if we assume that for quantum tunneling to occur a proton must be within one deBroglie wavelength (λ=h/p, where h is Planck's constant and p is the momentum) of the target. We can rewrite the kinetic energy in terms of momentum: $\frac{1}{2}m\text{v}^2 = \frac{p^2}{2m}$. If we set the distance of closest approach equal to one proton wavelength and let the barrier height equal the original kinetic energy, we find $\frac{\alpha e^2}{\lambda} = \frac{p^2}{2m} = \frac{\left(h/\lambda\right)^2}{2m}$.

b) Solve the above equation for λ and substitute $r = \lambda$ in the equation for $T_{classical}$, which now becomes $T_{quantum}$. Solve for $T_{quantum}$ and compare it with the actual central temperature of the Sun, and comment.

2. (a) The Earth's atmosphere is (very nearly) in hydrostatic equilibrium. What does this imply about the pressure as a function of altitude?

(b) The pressure at sea level is 1 atmosphere = 10^5 Pascals. (The Pascal is the SI unit of pressure, equal to 1 newton/square meter.) Use the equation of hydrostatic equilibrium ($dP/dR = -g\rho$), to calculate dP (the change in atmospheric pressure) as one ascends to Denver, whose altitude is 1600 m. You will need to know that the mass density, ρ, of the atmosphere at sea level is roughly 1 kg/m^3, and that g is the acceleration of gravity at sea level, which you can look up. Assume that g and ρ are constant. What percentage of sea-level pressure is Denver's atmospheric pressure? Compare this answer with what you get from using the more exact scale height expression:

$$P_R = P_0 e^{-R/7.64}$$

where R is in km. Did we make much of an error assuming g and ρ are constant?

(c) Do the same calculations for Mauna Kea, whose altitude is 4300 m. Are you surprised that oxygen is sometimes needed for observers at the summit?

Kutner

Ch. 9: Question 1

Problems 6 (your answer should be in kg/sec), 10

Ch. 15: Question 1