Quantum logic and perfect state transfer with superconducting phase qubits

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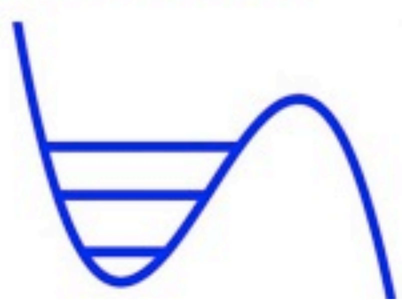
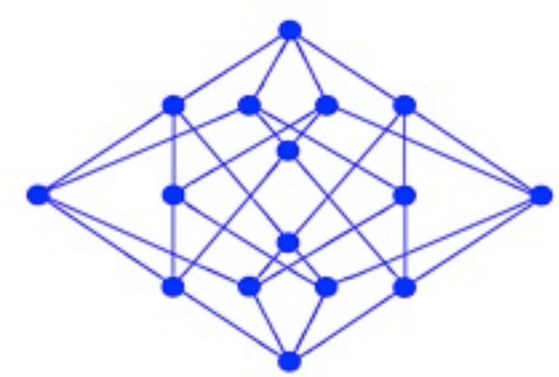
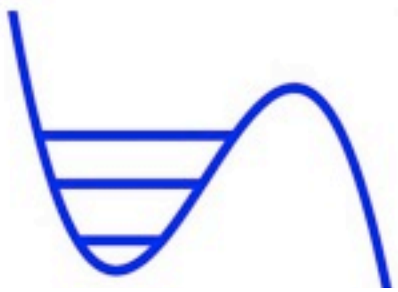
Department of Physics
Williams College



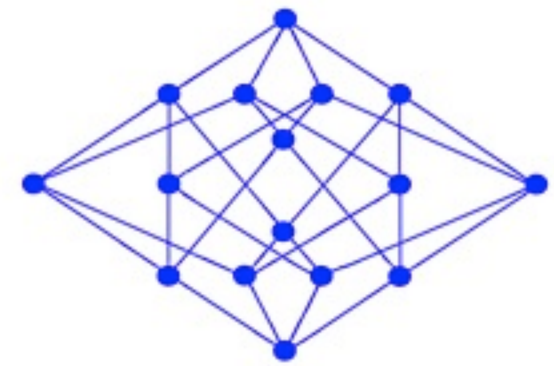


NIST Physics Laboratory

National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce



Outline

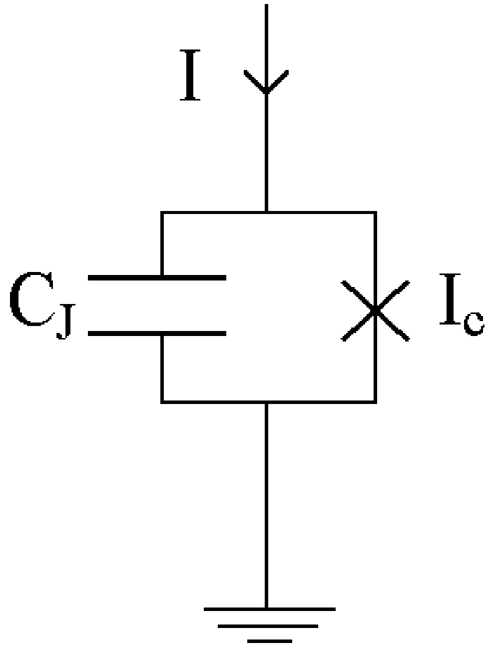


- Phase qubits
- Quantum logic gates
  - Swap gate and higher levels
  - Nonadiabatic phase gate
- Perfect state transfer
  - Phase qubit hypercube
  - Long-range couplings, decoherence, disorder
- Conclusions

F.W. Strauch *et al.*,
Phys. Rev. Lett. **91**, 167005 (2003)

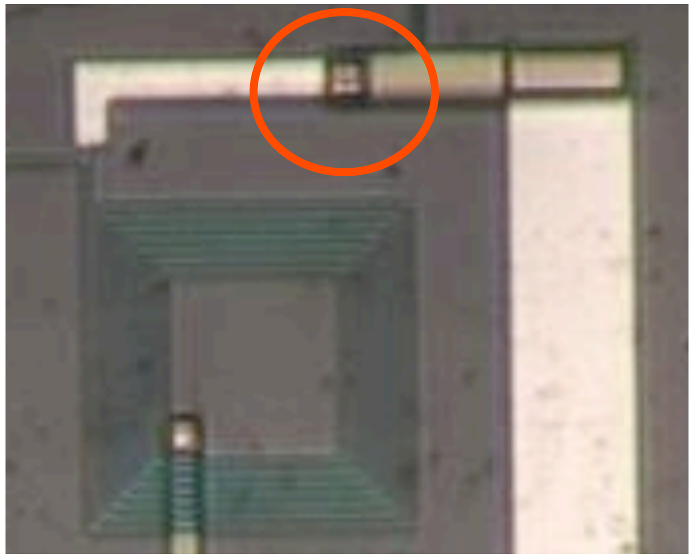
F.W. Strauch and C. J. Williams,
Phys. Rev. B **78**, 094516 (2008)

Phase Qubit

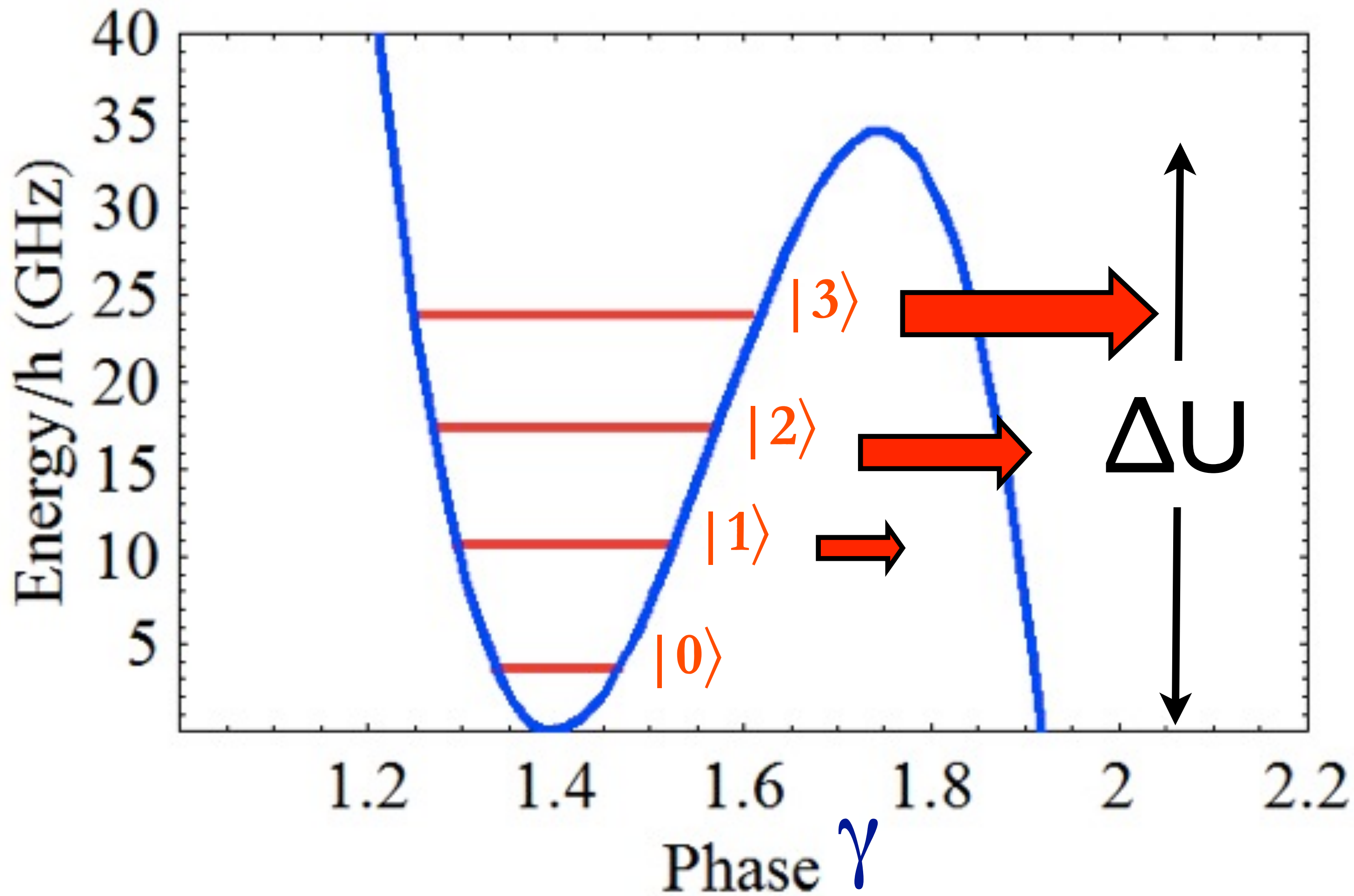


Artificial Atom, controlled by wires.

Josephson Junction



$$-\frac{\hbar^2}{2C(\Phi_0/2\pi)^2}\frac{d^2\Psi}{d\gamma^2}-\frac{\Phi_0}{2\pi}(I_c\cos\gamma+I_b\gamma)\Psi=E\Psi$$

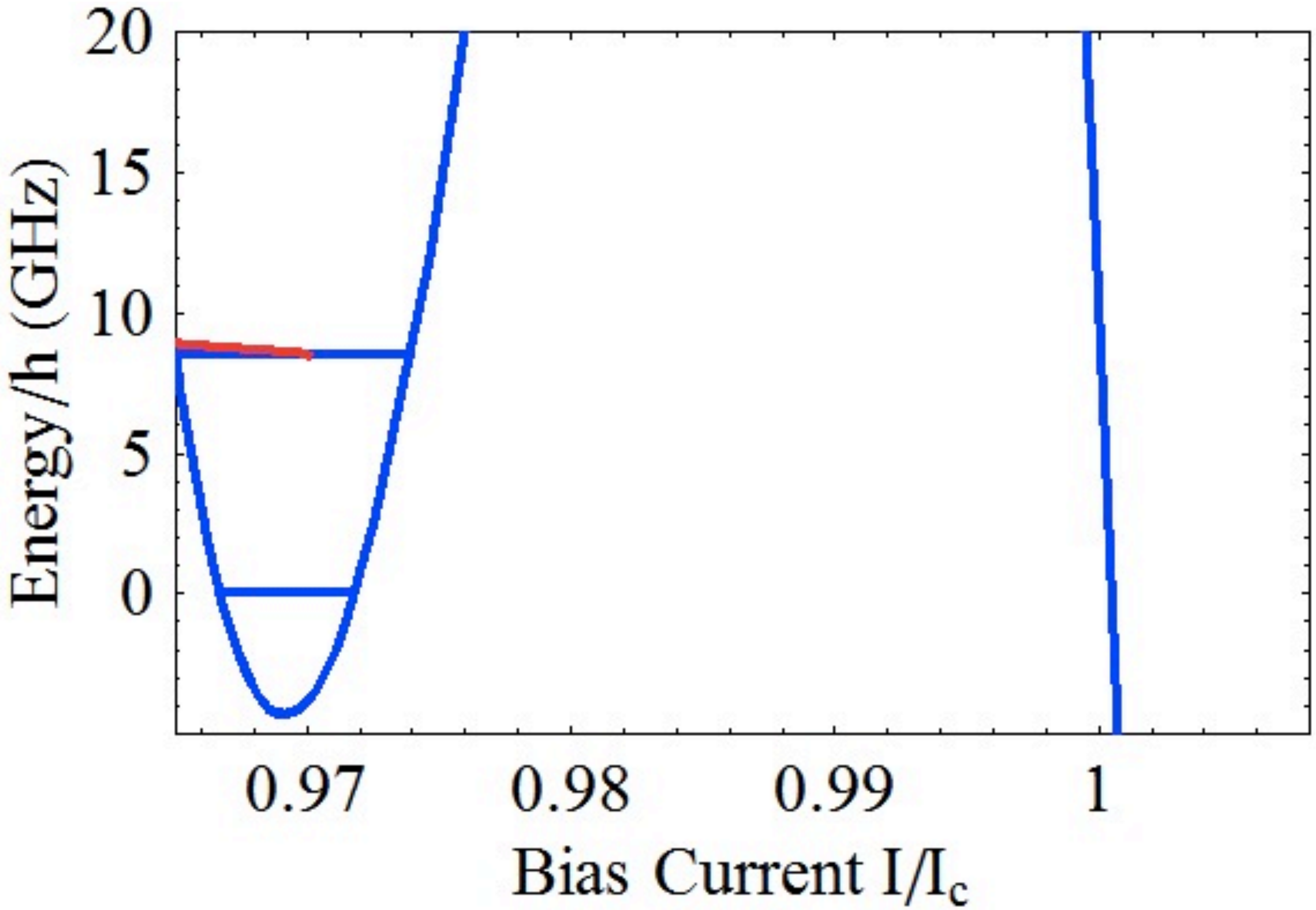


Tunable quantum oscillator involving the superconducting current of *billions of Cooper pairs*

Distinct spectroscopic transitions between energy levels can be probed by microwaves.

Tunable Oscillator

$$-\frac{\hbar^2}{2C(\Phi_0/2\pi)^2}\frac{d^2\Psi}{d\gamma^2}-\frac{\Phi_0}{2\pi}(I_c\cos\gamma+I_b\gamma)\Psi=E\Psi$$



Sweep of bias current allows experimental control of energy levels.

Tunable Oscillator

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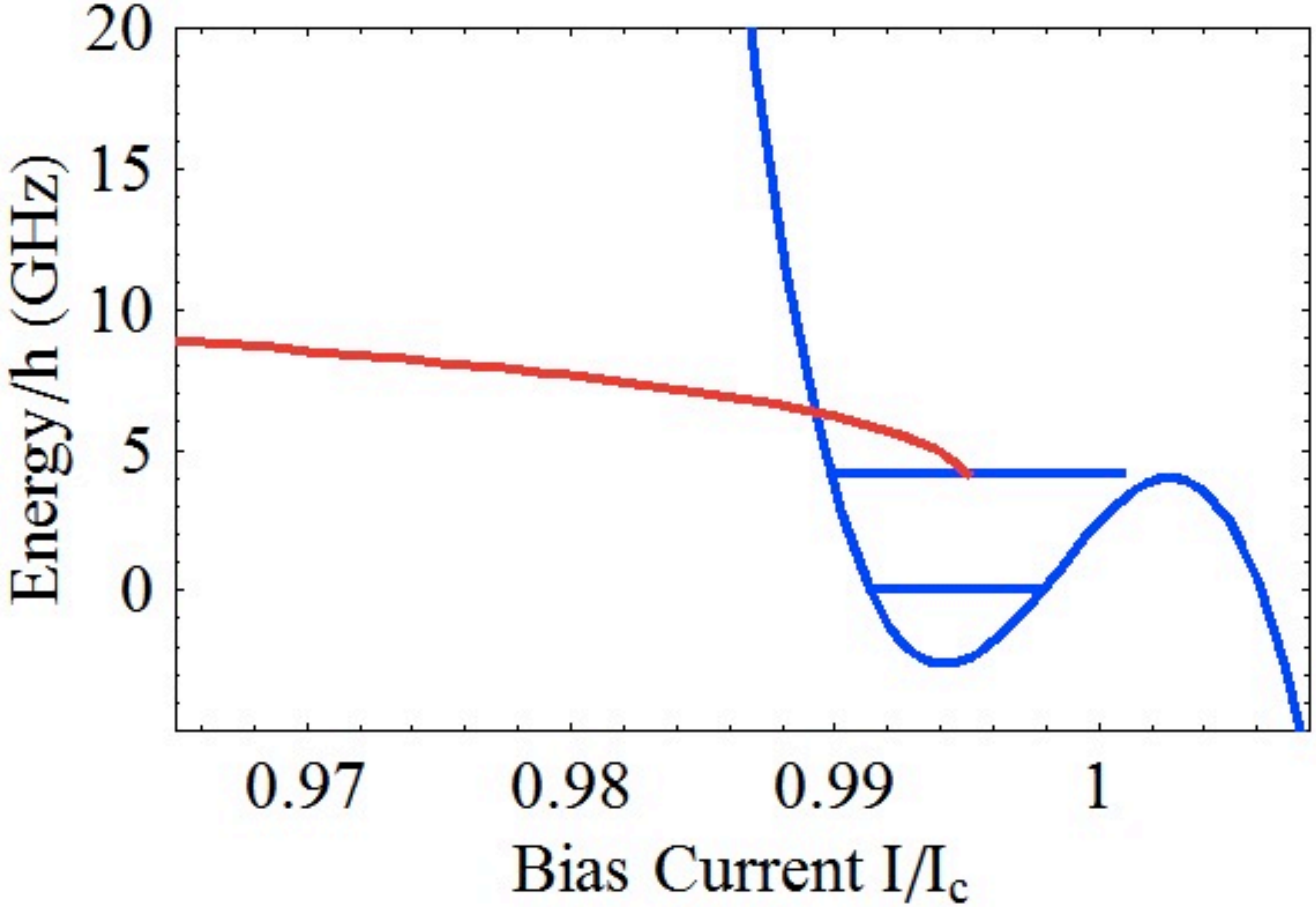

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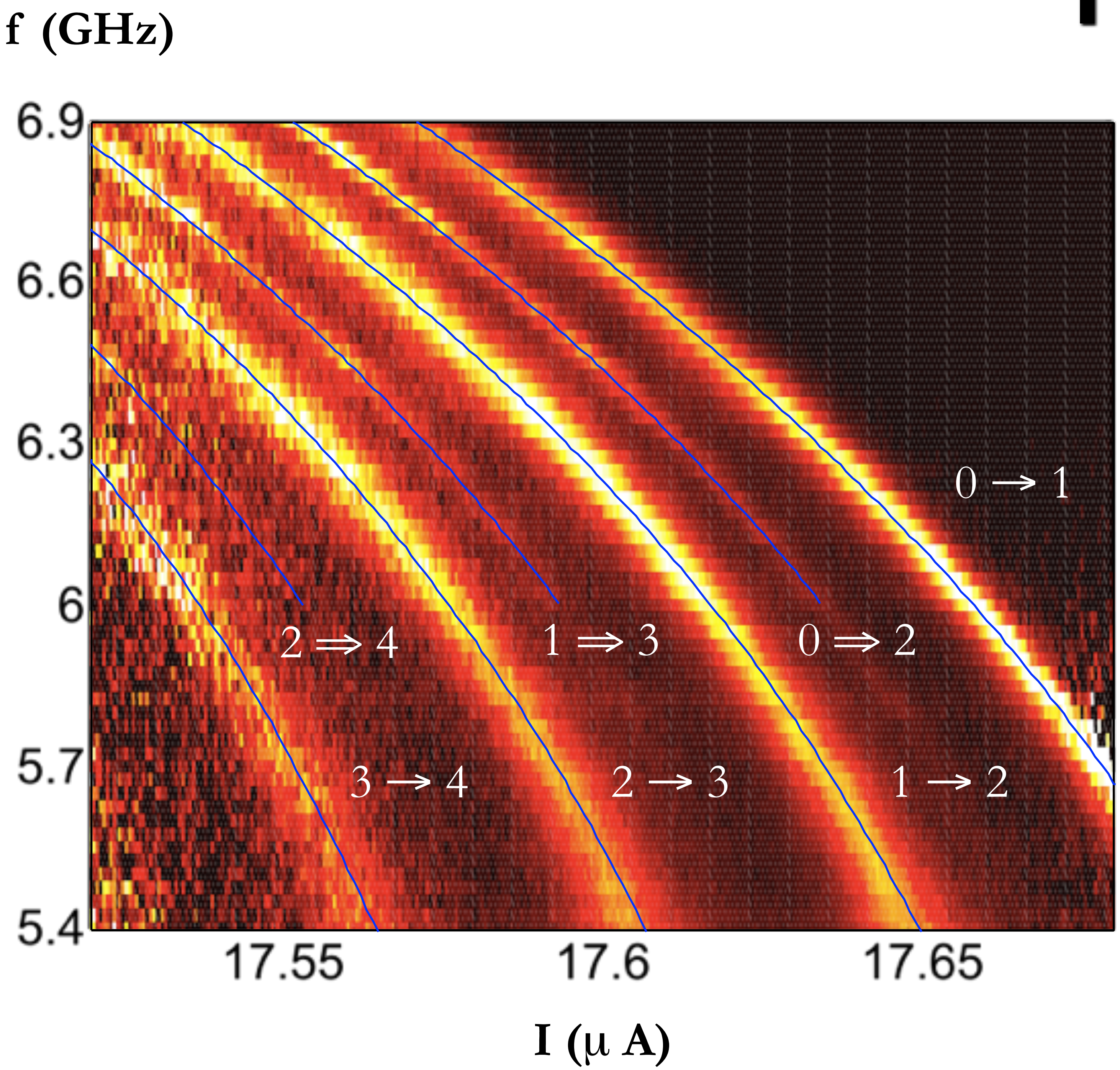
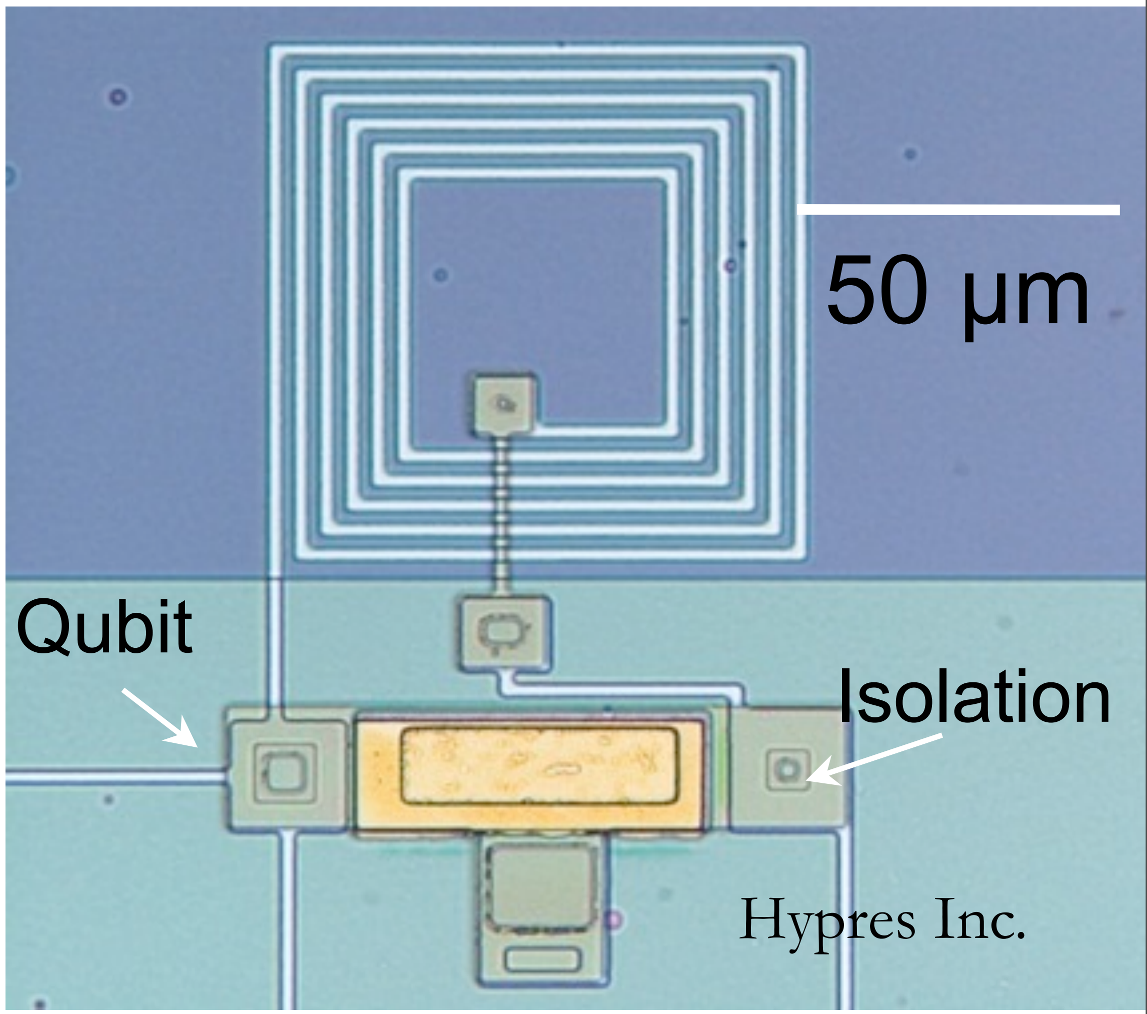
Multi-Photon Spectroscopy

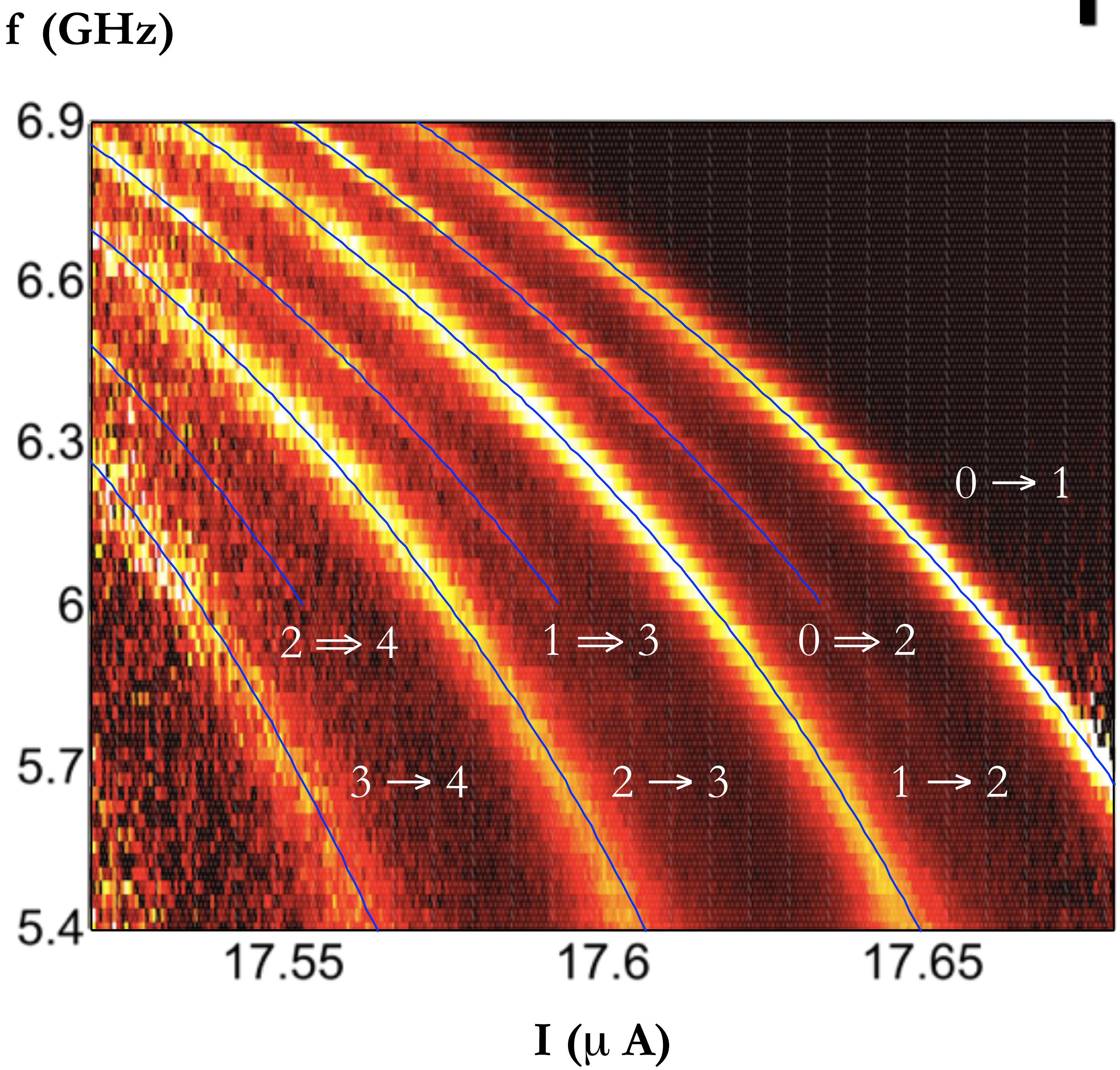


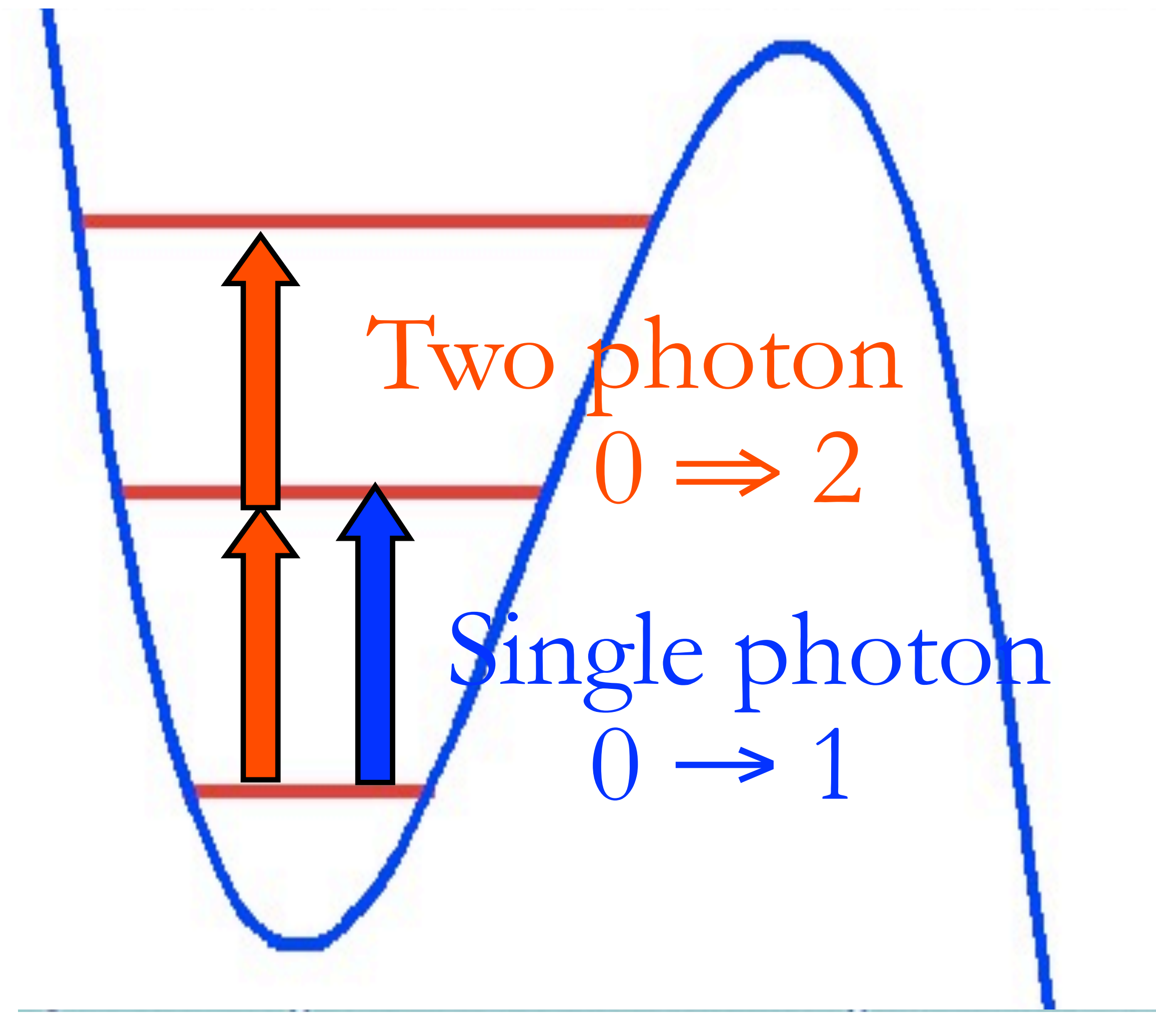


Sudeep Dutta et al. (Univ. Maryland)

Each microwave transition is an excitation of the junction with an increased tunneling rate. Bright indicates a large number of tunneling events, dark a small number of events.

Multi-Photon Spectroscopy





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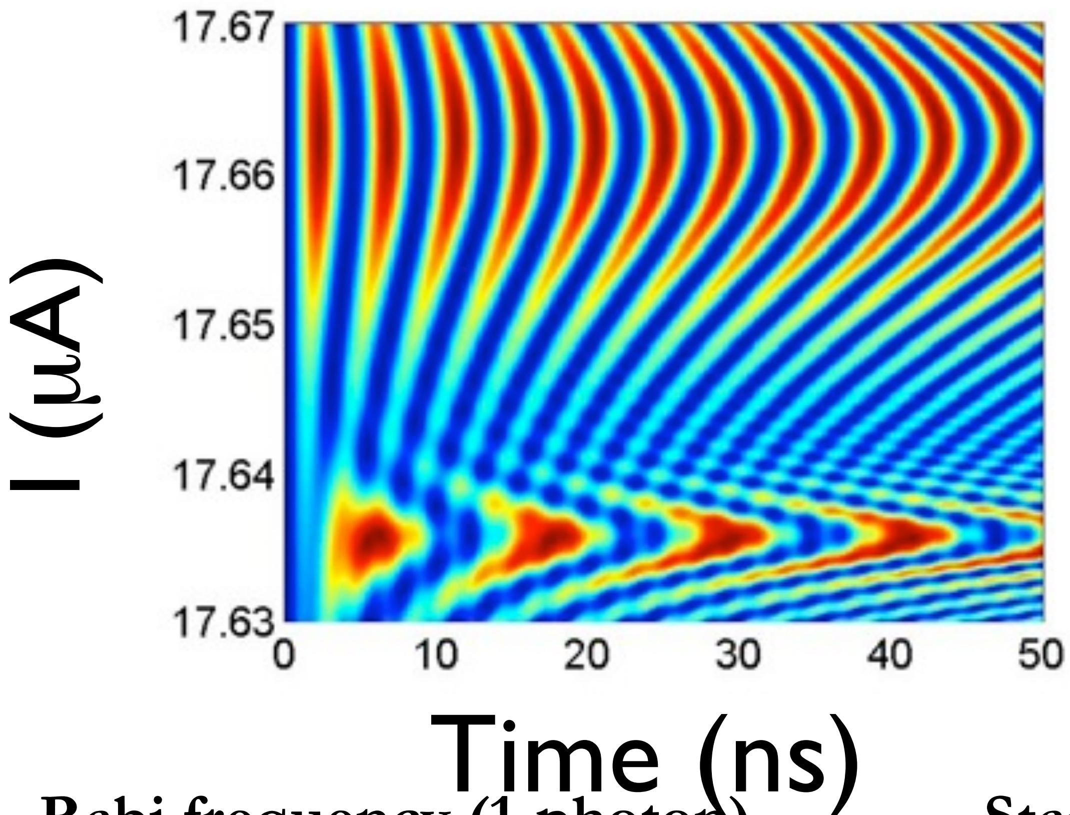
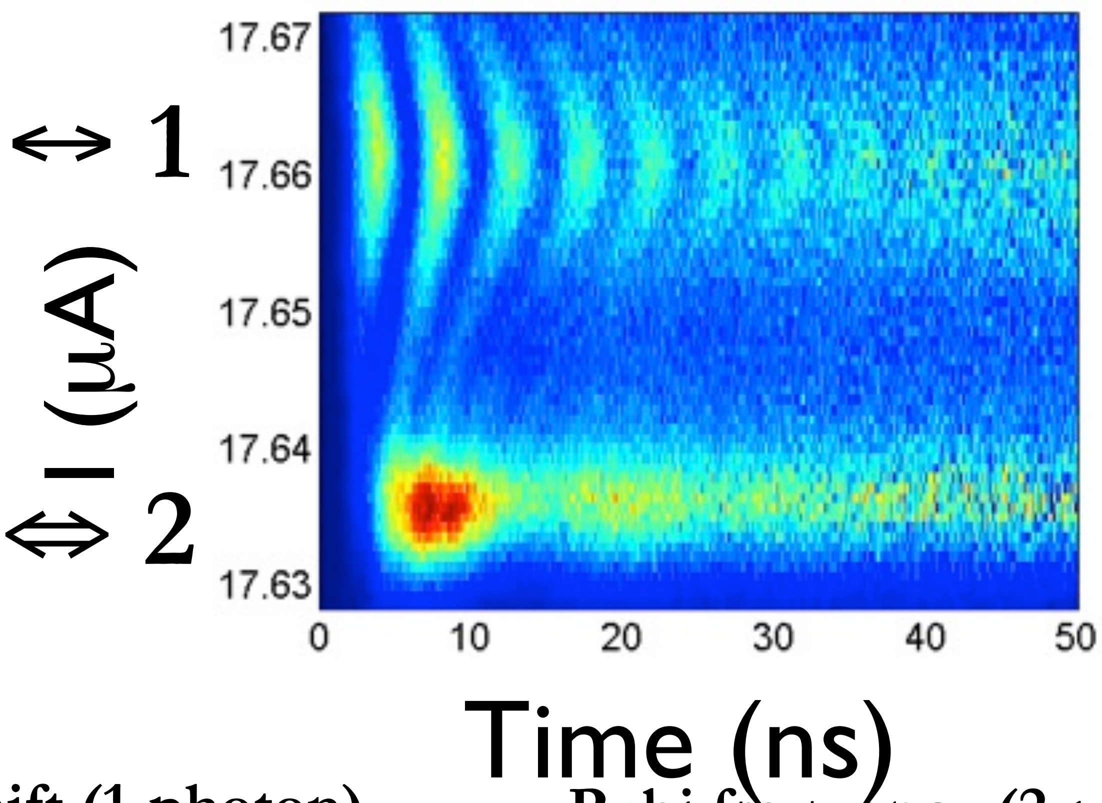
Multi-Photon Rabi Oscillations

Theory (without decoherence)



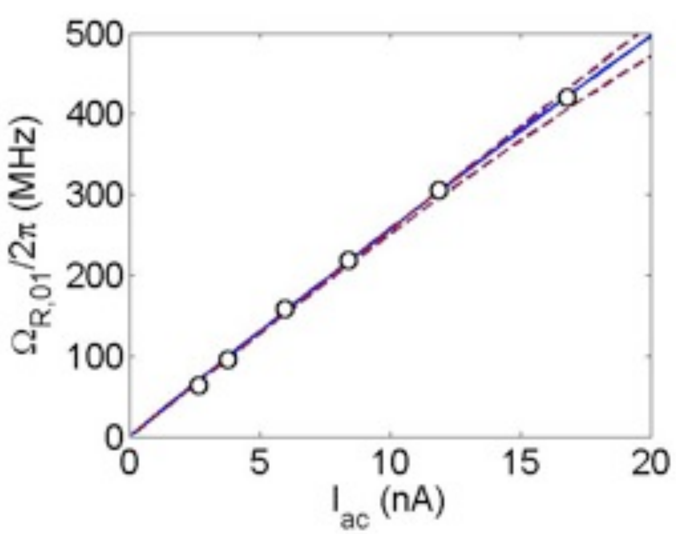
Experiment



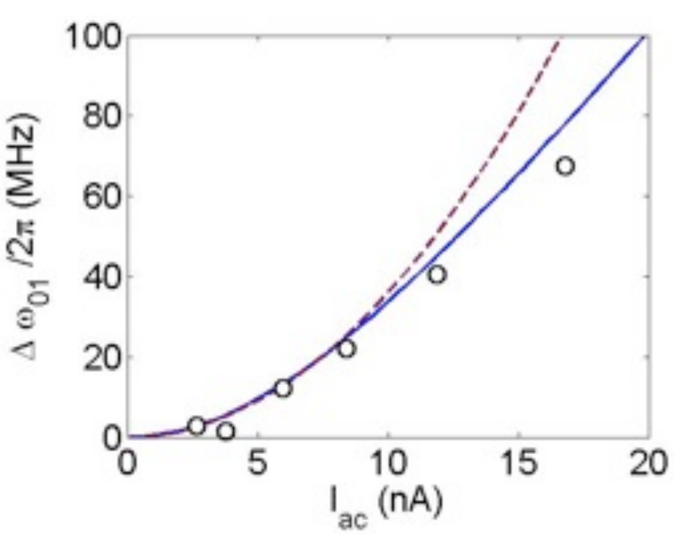
0 ↔ 1

0 ⇔ 2

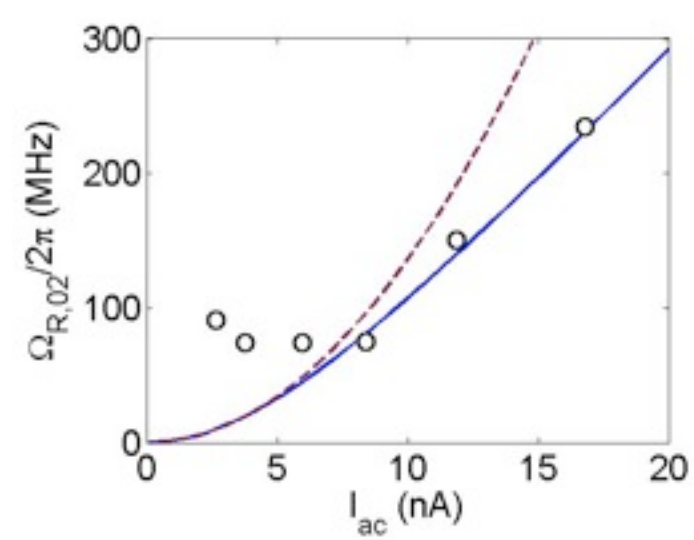
Rabi frequency (1-photon)



Stark shift (1-photon)



Rabi frequency (2-photon)

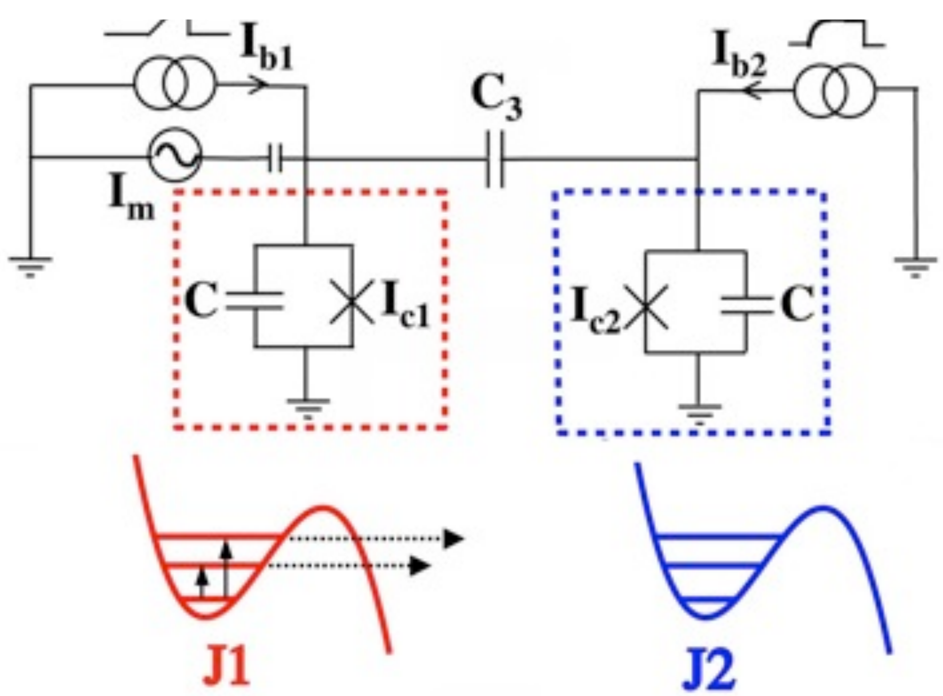


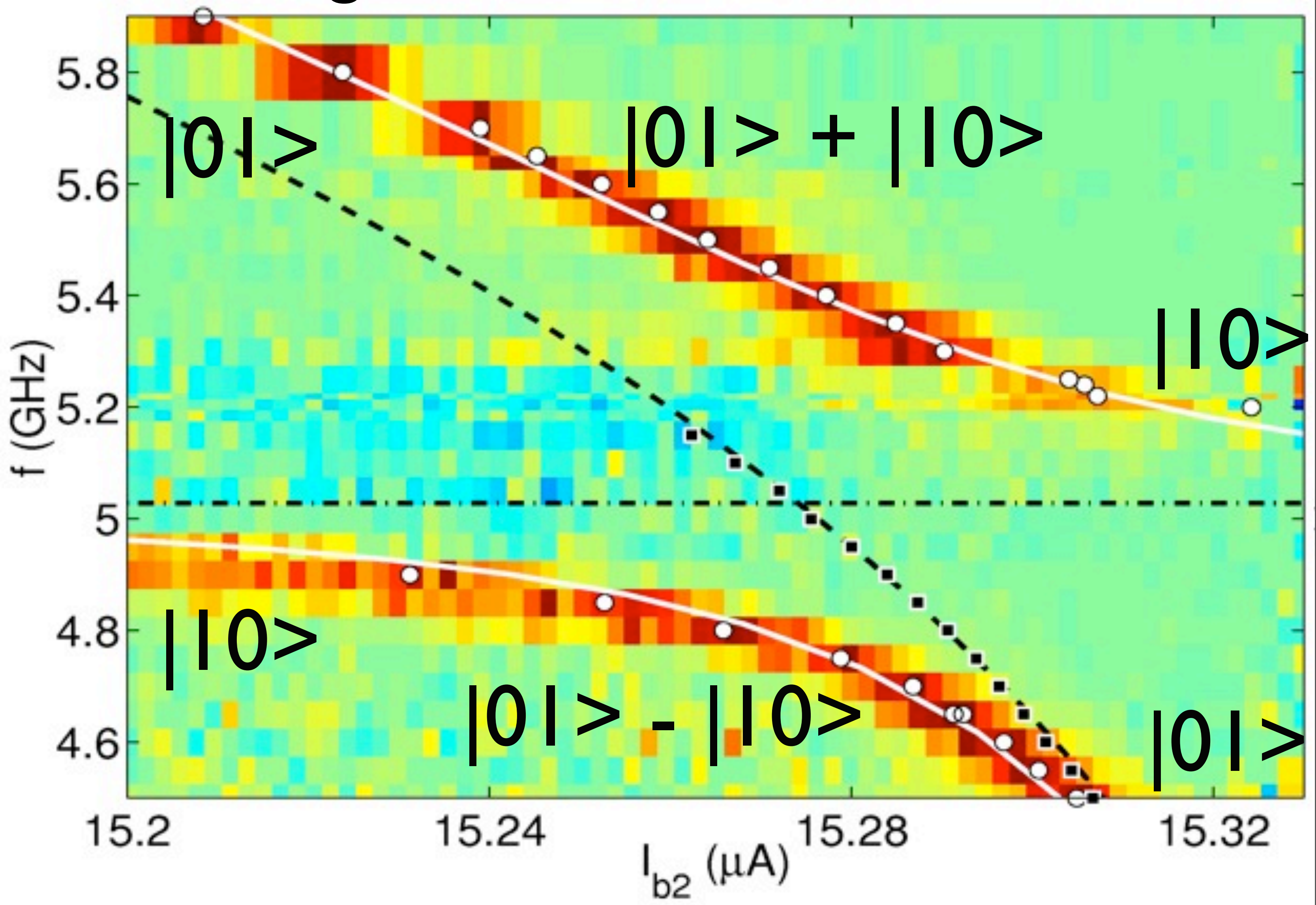
F. W. Strauch *et al., IEEE Trans. Appl. Supercond.* **17,** 105-108 (2007)

Artificial Molecules

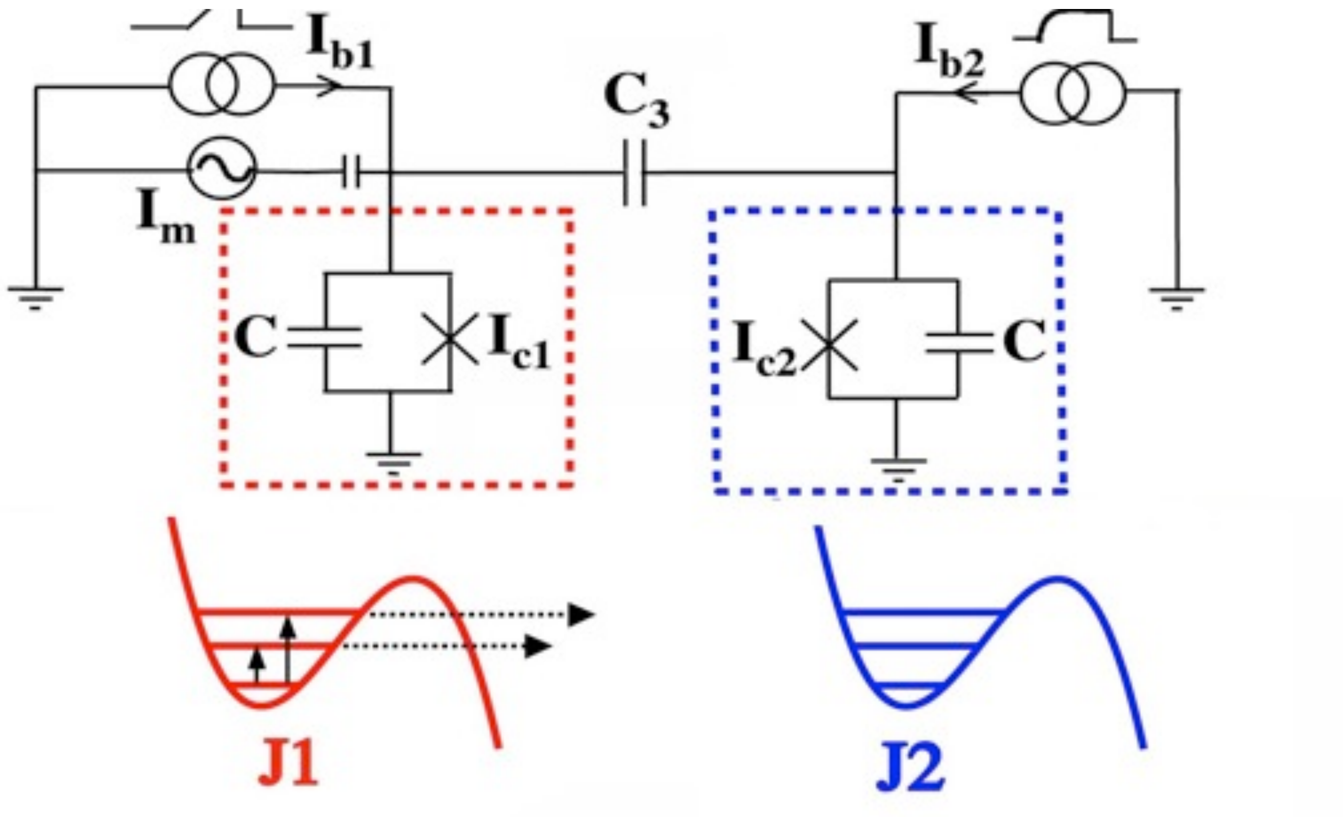
"Entangled Macroscopic Quantum States in Two Superconducting Qubits"

A. J. Berkley, H. Xu, R. C. Ramos, M. A. Gubrud, F. W. Strauch et al., *Science*, **300**, 1548 (2003).





Capacitively Coupled Josephson Junctions



$$E_C = \frac{e^2}{2C_J} \; , \; E_J = \frac{\hbar I_C}{2e}$$

$$J_1 = I_1 / I_C \, , \; J_2 = I_2 / I_C$$

$$\boxed{\zeta = C/(C + C_J)}$$

$$H = 4E_C(1+\zeta)^{-1}(p_1^{\;2} + p_2^{\;2} + 2\zeta p_1 p_2)$$
$$- E_J(\cos\gamma_1 + J_1\gamma_1 + \cos\gamma_2 + J_2\gamma_2)$$

Energy Spectrum

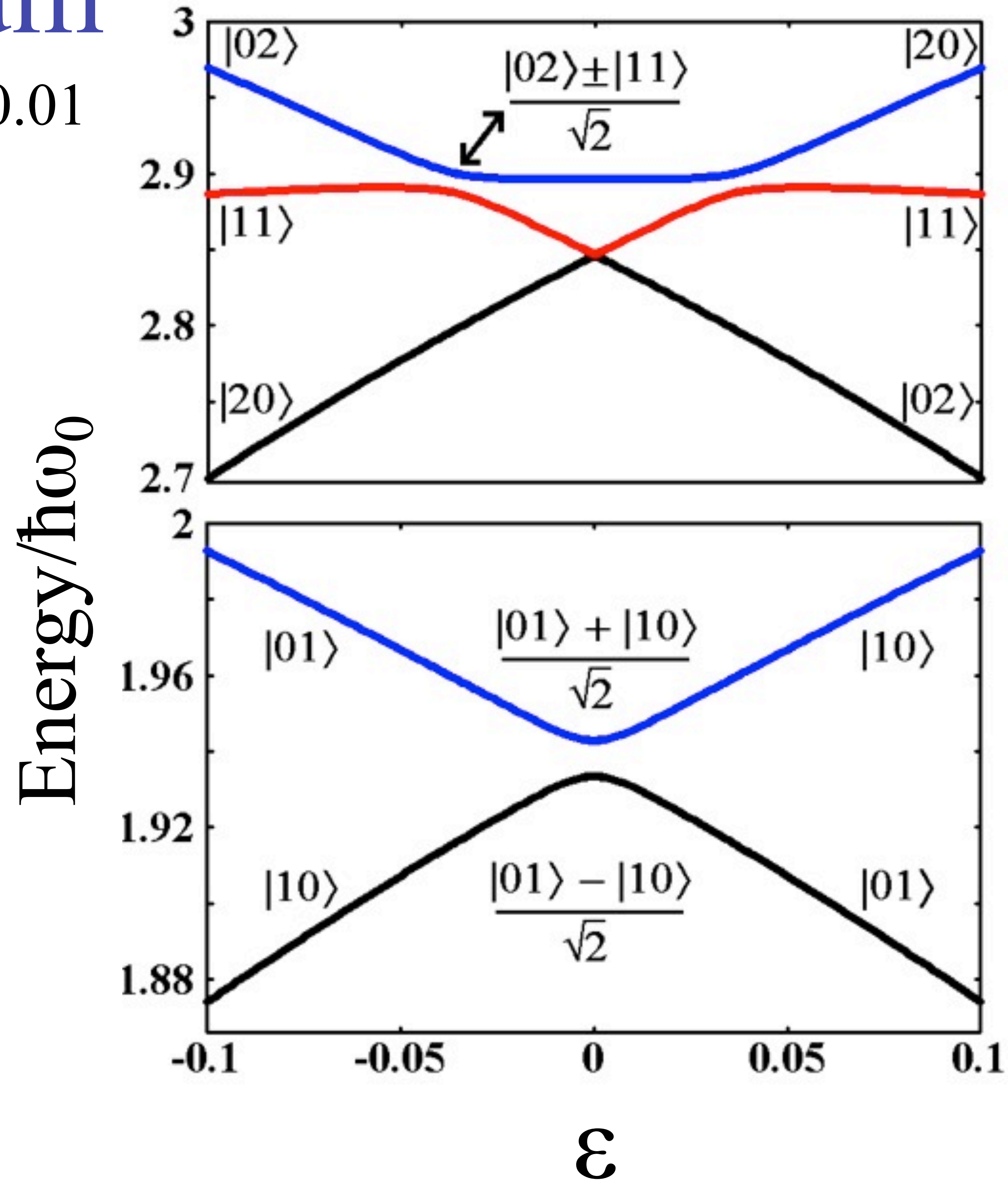
$N_s = 4,\ \zeta = 0.01$

$$\sqrt{1 - J_{1,2}} = \sqrt{1 - J_0}\,(1 \pm \varepsilon)$$

$$\hbar\omega_0 = (8E_C E_J)^{1/2}(1 - J_0^2)^{1/4}$$

$$\omega_1 \approx \omega_0\,(1+\varepsilon/2)$$
$$\omega_2 \approx \omega_0\,(1-\varepsilon/2)$$

- Energy states are unentangled away from avoided level crossings.
- Entanglement is maximized at the avoided level crossings.



Gate Design

- **Control**: Interactions controllable (tuned on and off) through bias currents for small coupling.

  (e.g. $\zeta = 0.01$)

- **Dynamical conditions**: Characteristic ramp time must satisfy

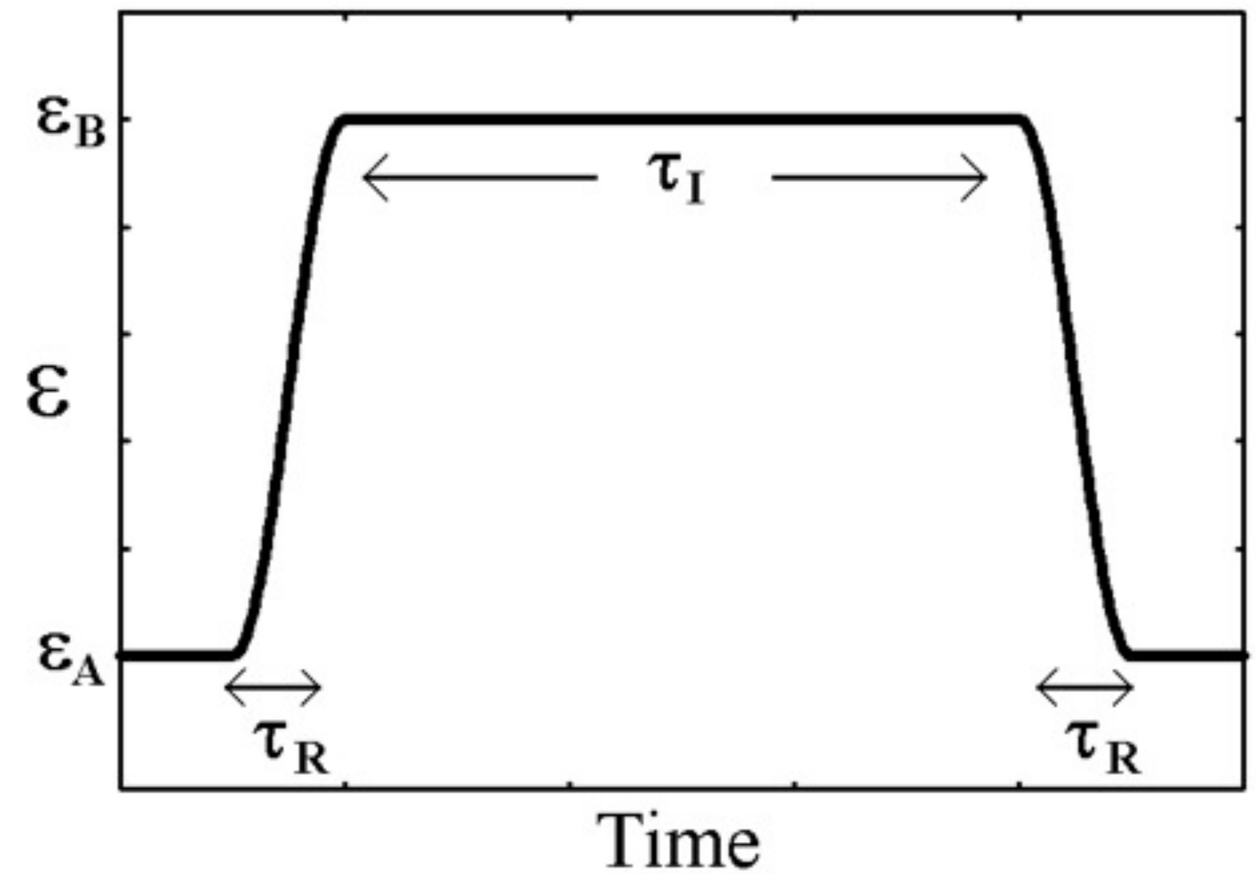
$$\frac{2\pi}{\omega_0} < \tau_R < \frac{1}{\zeta}\frac{2\pi}{\omega_0} \approx 100 \times \frac{2\pi}{\omega_0}$$

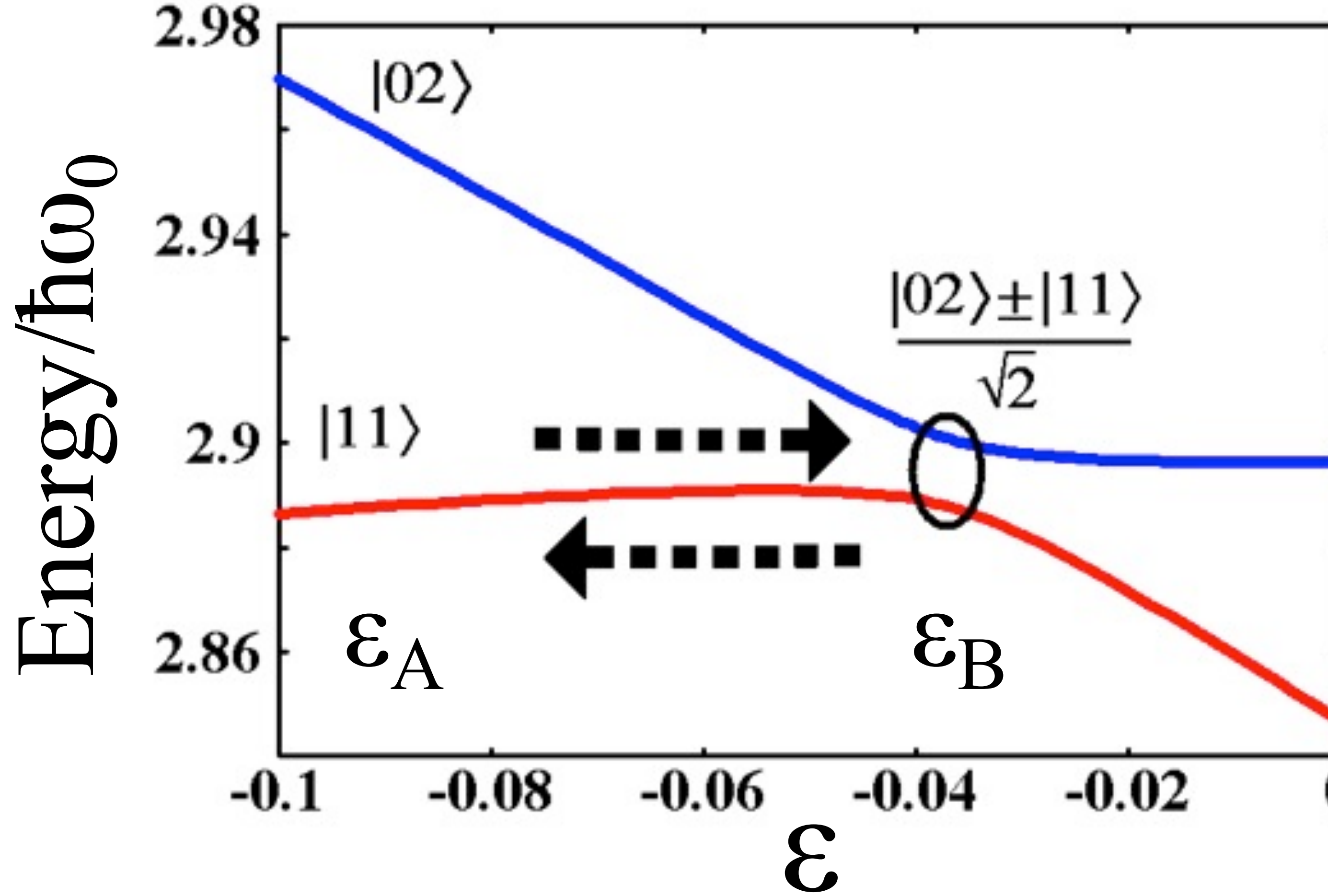
- **Leakage**: Both *tunneling* and evolution through the auxiliary states $|02\rangle$ and $|20\rangle$ must be taken into account.

$$N_s = \Delta U/\hbar\omega \geq 4$$

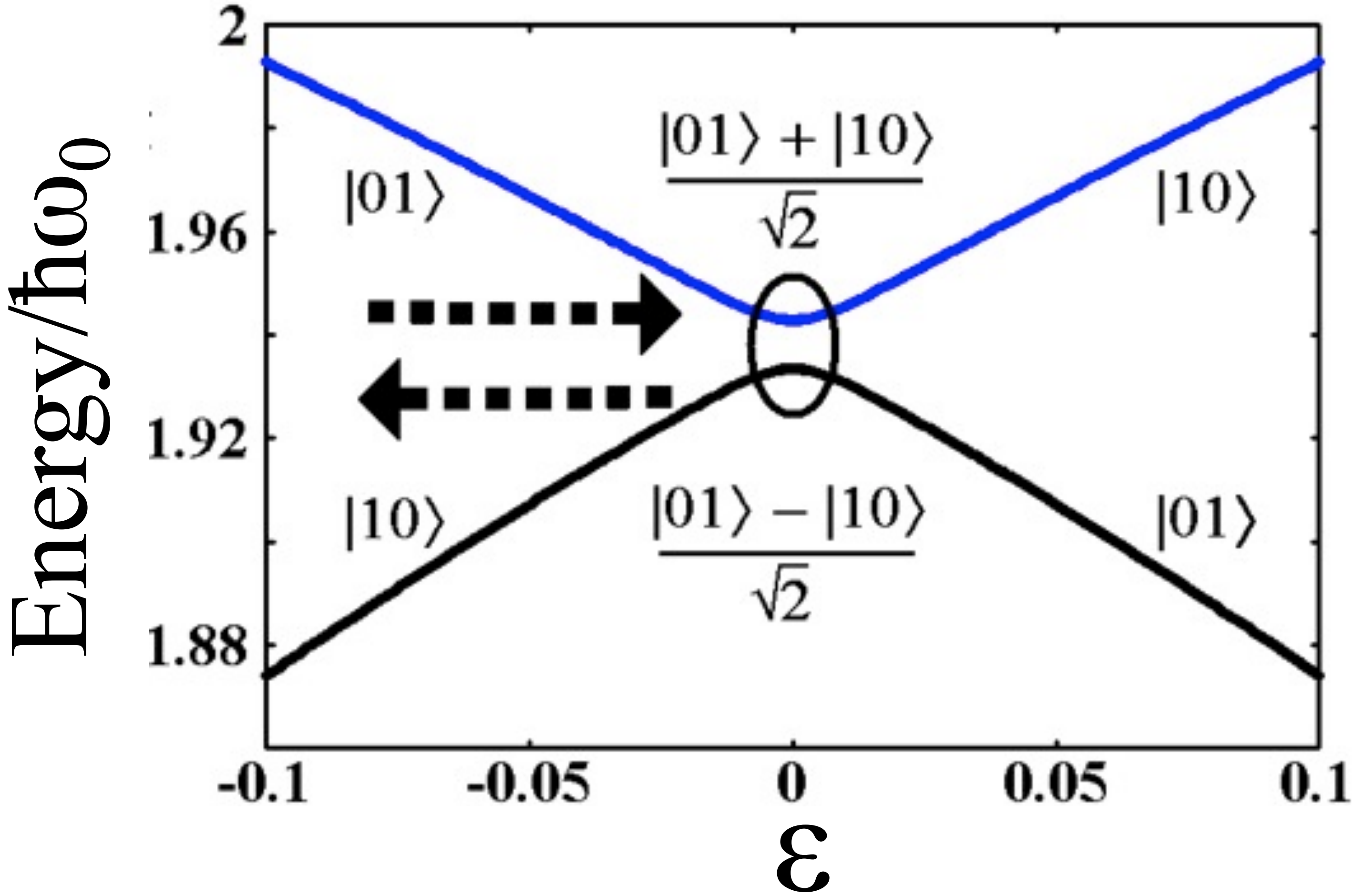
Gate Operation

- Start from detuned junctions
- Ramp bias currents, in time τ_R, from ε_A to ε_B.
- Wait for time τ_I.
- Detune the junctions.





Swap-Like Operation

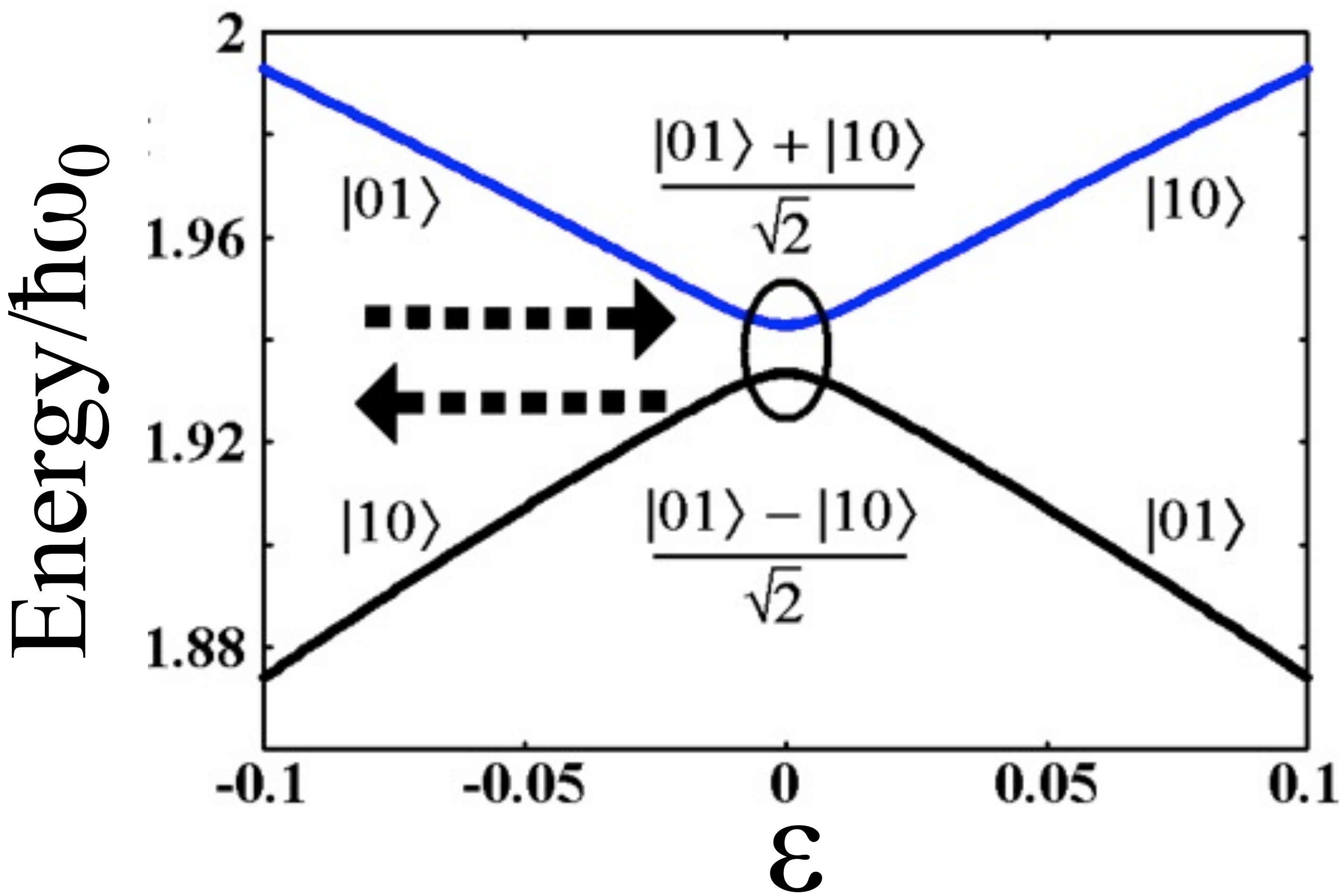


$$|01\rangle \Rightarrow \cos\theta_1|01\rangle - i\sin\theta_1|10\rangle$$
$$|10\rangle \Rightarrow \cos\theta_1|10\rangle - i\sin\theta_1|01\rangle$$

$$N_s \cong 5.16,\ \zeta = 0.01$$
$$\theta_1 = \pi/2 \quad \theta_2 \cong \pi/4$$

$$U_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_1 & -i\sin\theta_1 & 0 \\ 0 & -i\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 0 & e^{-i\theta_2} \end{pmatrix}$$

Swap-Like Operation



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Why these numbers?

Energy Spectrum

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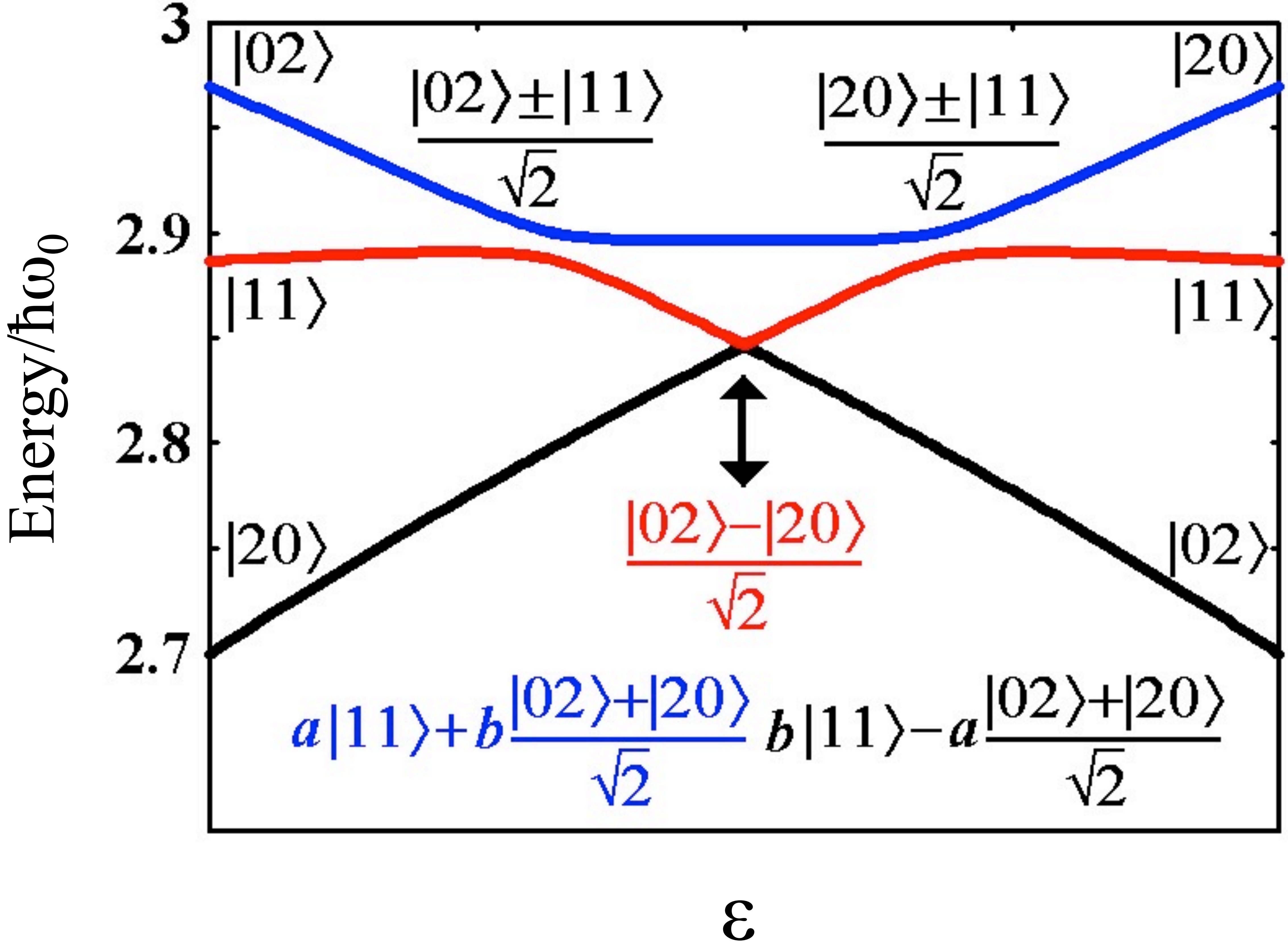

Auxiliary Level Dynamics

A high fidelity swap gate requires consideration of the auxiliary levels:

$$E_{02} = E_{20}$$

$$\mathcal{H} = \begin{pmatrix} E_{02} & 0 & \tilde{g} \\ 0 & E_{20} & \tilde{g} \\ \tilde{g} & \tilde{g} & E_{11} \end{pmatrix}$$

$$\tilde{g} = 8E_c(1+\zeta)^{-1}\langle 02|p_1 p_2|11\rangle \approx 2^{-1/2}\zeta\hbar\omega_{01}$$

$$\Delta E = E_{11} - E_{02} = \hbar(\omega_{01} - \omega_{12})$$

Energy shift of $|11\rangle$ is second-order, but state mixing is first order:

$$\tan\theta = \frac{1}{2\sqrt{2}}\left(\sqrt{(\Delta E/\tilde{g})^2 + 8} - (\Delta E/\tilde{g})\right) \approx \frac{\sqrt{2}g}{\Delta E}$$

$$\begin{aligned} |\Psi_-\rangle &= 2^{-1/2}\cos\theta(|02\rangle + |20\rangle) - \sin\theta|11\rangle \\ |\Psi_0\rangle &= 2^{-1/2}(|02\rangle - |20\rangle) \\ |\Psi_+\rangle &= 2^{-1/2}\cos\theta(|02\rangle + |20\rangle) + \sin\theta|11\rangle \end{aligned}$$

$$E_\pm = E_{02} + \frac{1}{2}\left(\Delta E \pm \sqrt{\Delta E^2 + 8g^2}\right) \approx \begin{cases} E_{11} + 2g^2/\Delta E \\ E_{02} - 2g^2/\Delta E \end{cases}$$

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$$\begin{aligned} p_{11} &= |\langle 11|e^{-i\mathcal{H}t/\hbar}|11\rangle|^2 \\ &= \cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta\cos\Omega t \end{aligned}$$

$\hbar\Omega = (E_+ - E_-)/2$

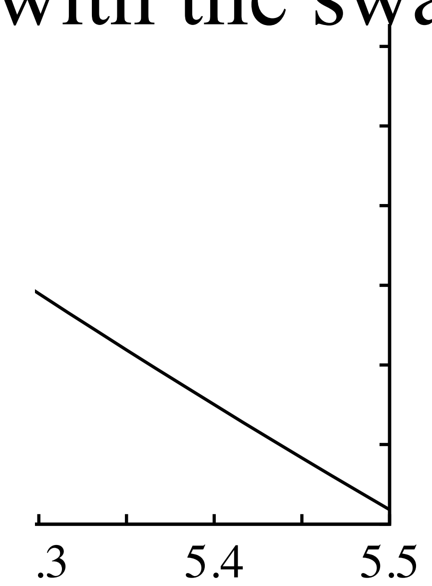
Optimizing the Swap Gate

$$\begin{aligned} p_{11} &= |\langle 11|e^{-i\mathcal{H}t/\hbar}|11\rangle|^2 \\ &= \cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta\cos\Omega t \end{aligned}$$

Average error is:

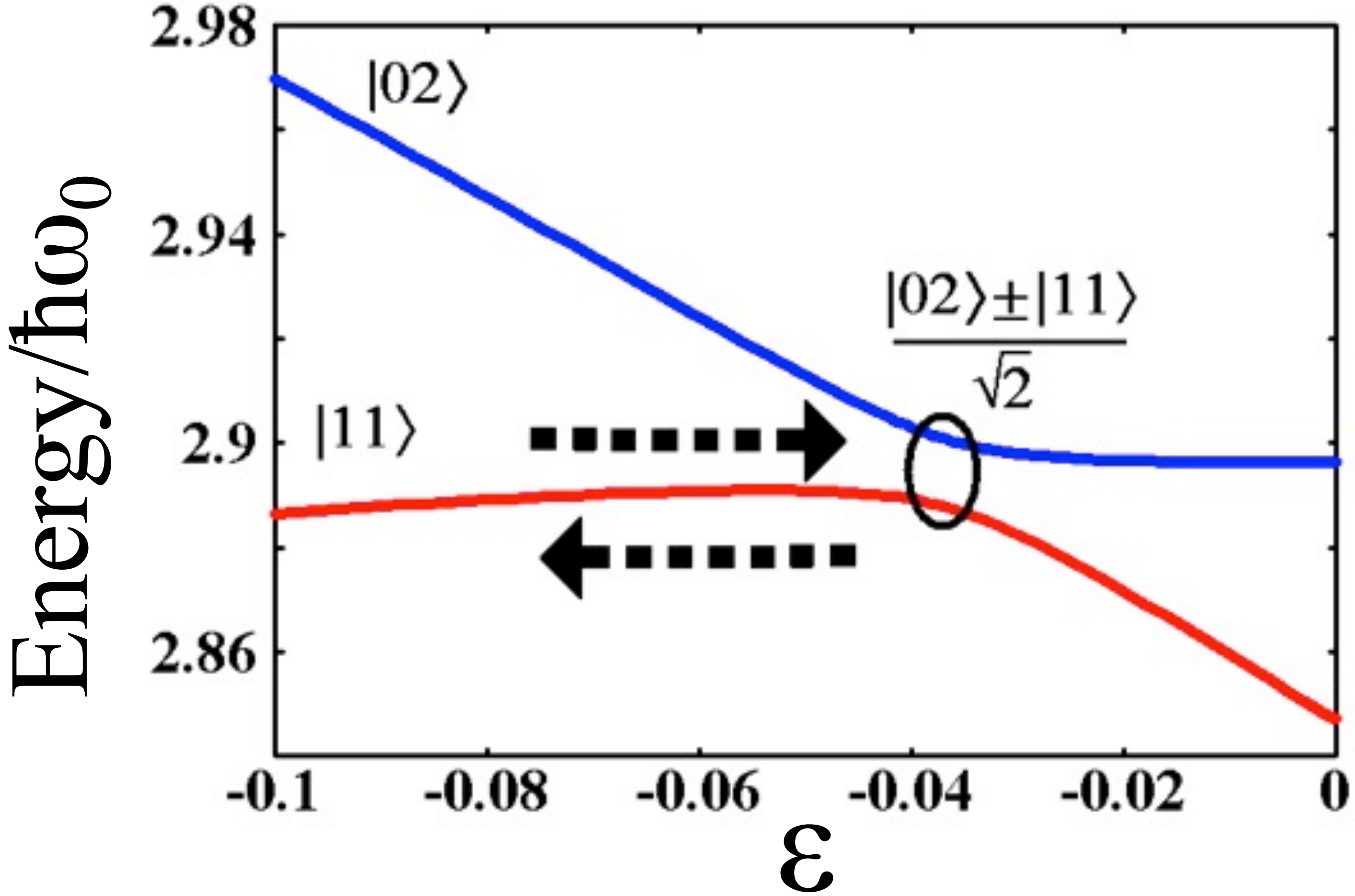
$$1 - \langle p_{11}\rangle \approx 2\theta^2 \approx \frac{4g^2}{\Delta E^2} \begin{array}{l} \approx \ \ 4\% \text{ for } \zeta = 0.01 \text{ and } N_s = 3 \\ \approx 14\% \text{ for } \zeta = 0.01 \text{ and } N_s = 5 \end{array}$$

The error can be minimized by synchronizing the oscillations of p_{11} with the swap oscillations, by tuning both qubits' energies (through N_s):



For N_s = 5.16, the |11⟩ oscillations are four times as fast as the swap oscillations.

Phase Gate Operation

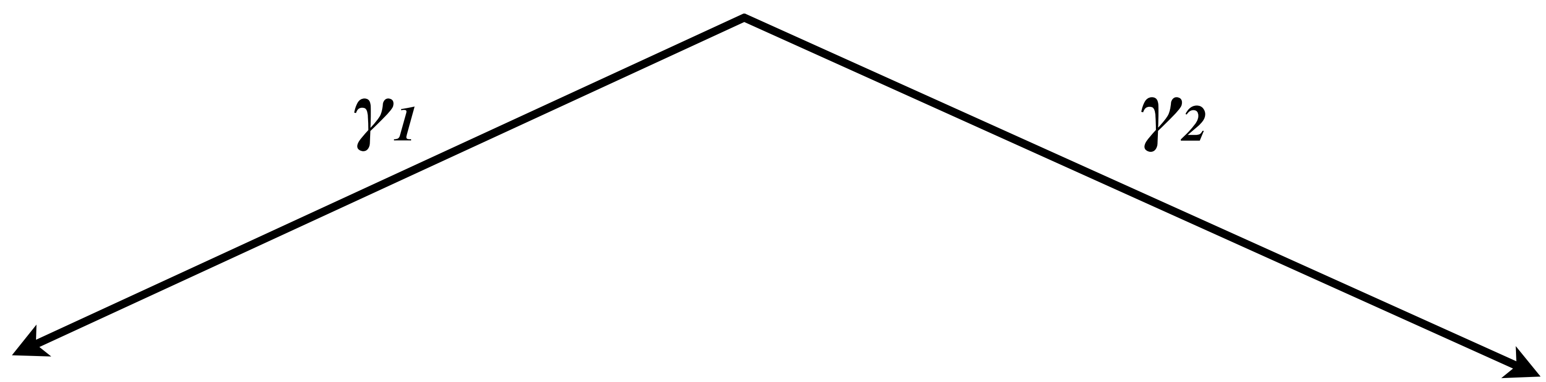


$$\left|11\right\rangle \Rightarrow \left|02\right\rangle \Rightarrow -\left|11\right\rangle$$

$$U_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

This avoided level crossing is isolated, so the other two-qubit states $|00\rangle$, $|01\rangle$ and $|10\rangle$ are unaffected.

Nonadiabatic Phase Gate



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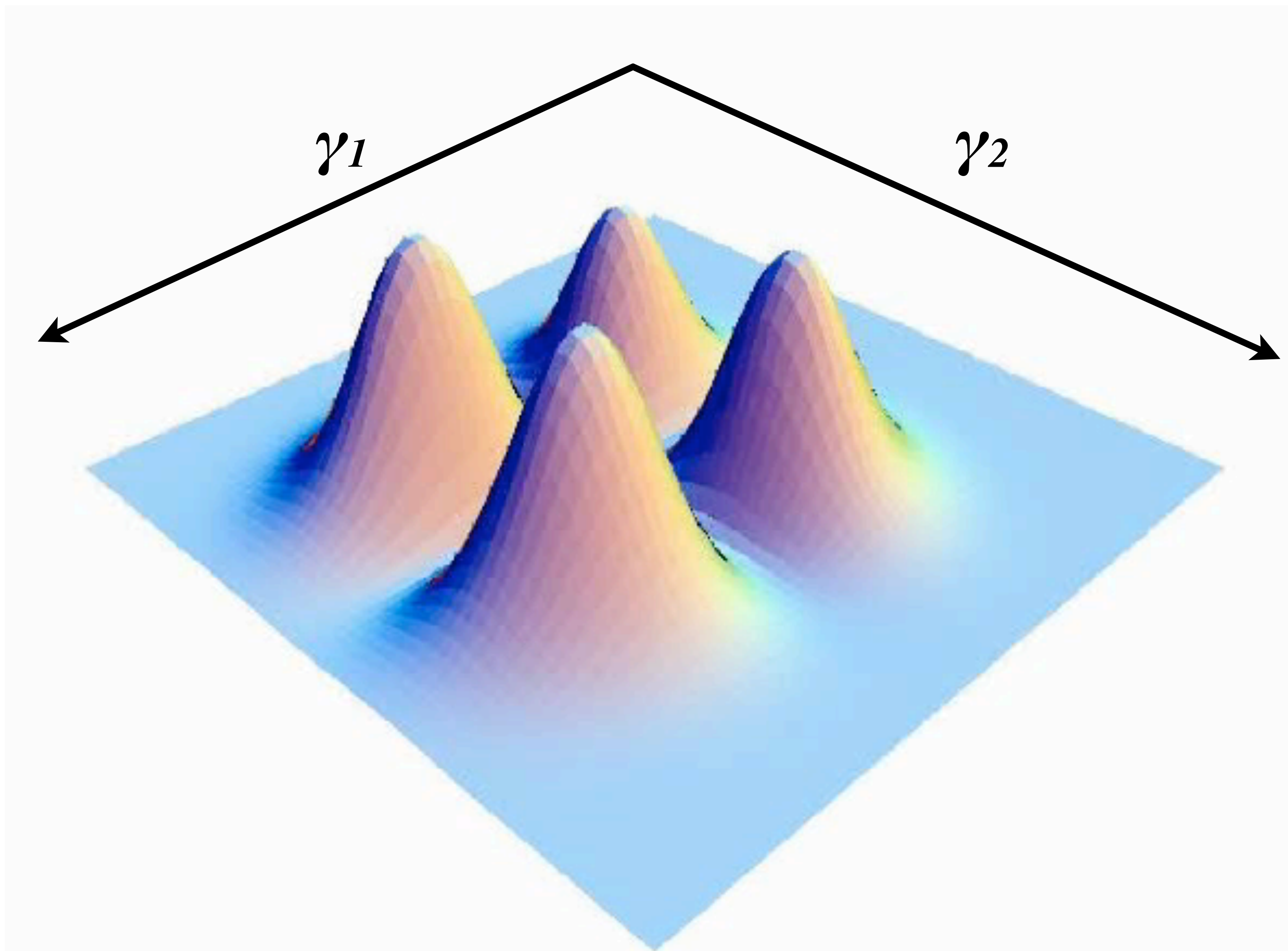
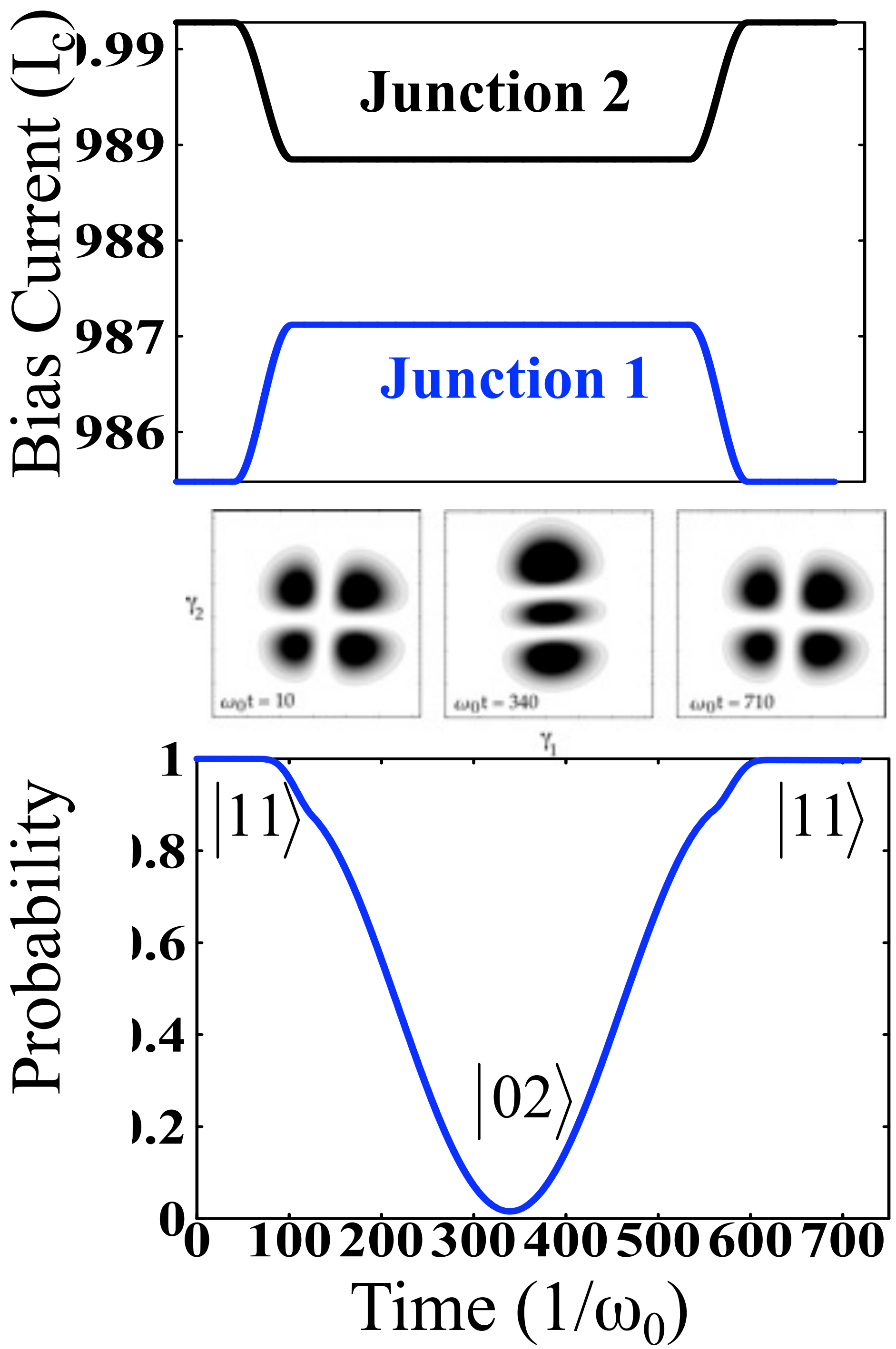


$$|11\rangle \Rightarrow |02\rangle \Rightarrow -|11\rangle$$

Quantum Logic Gates

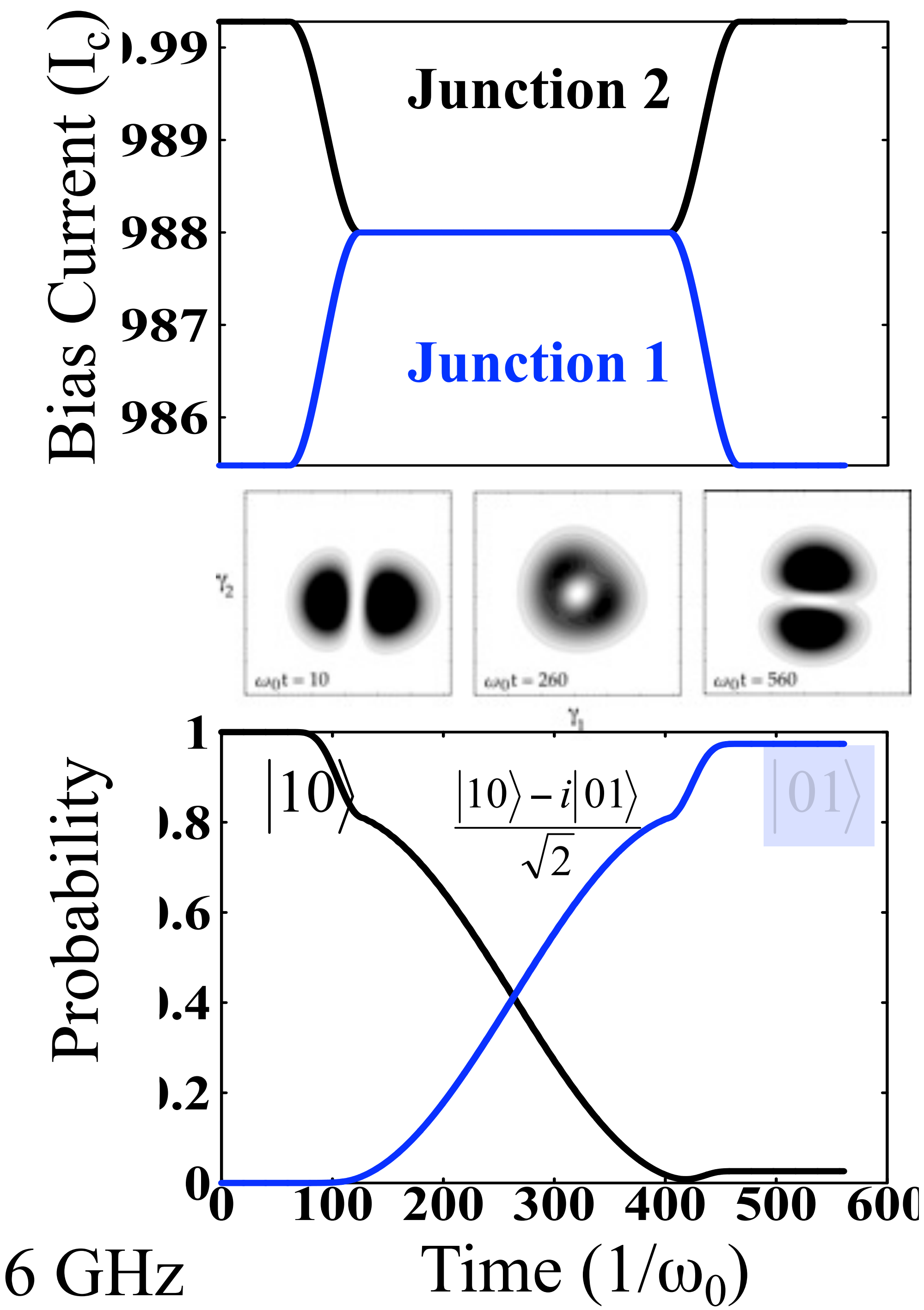
Phase Gate

F = 0.996, T_{gate}=14.85 ns



Swap Gate

F = 0.972, T_{gate}=10.7 ns

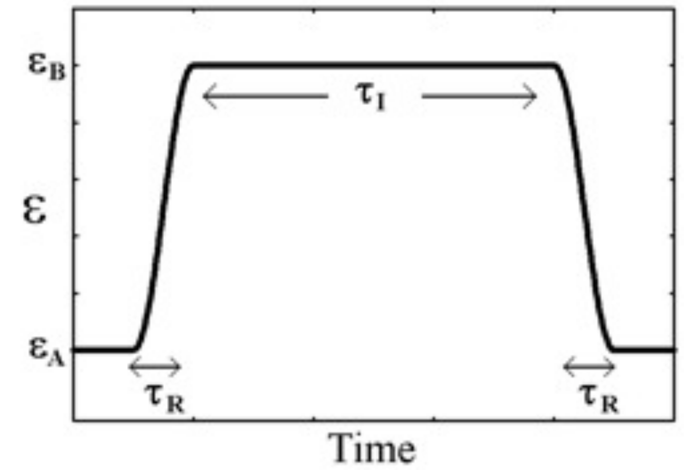


$\omega_0/2\pi$= 6 GHz

Optimized Logic Gates

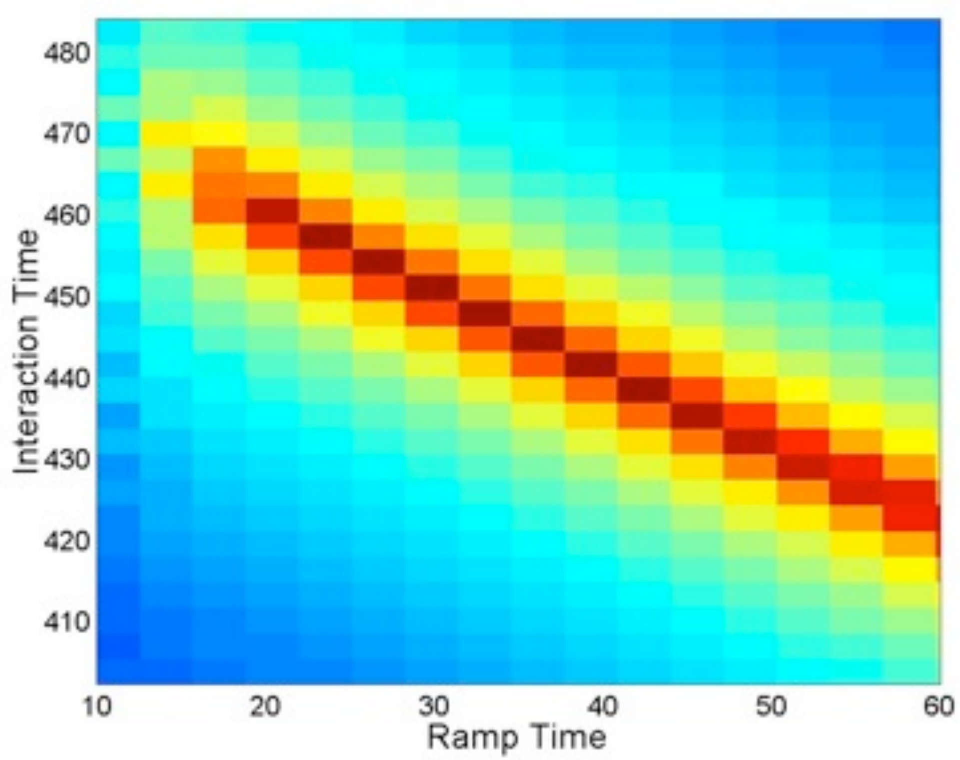
τ_R: **Ramp Time**

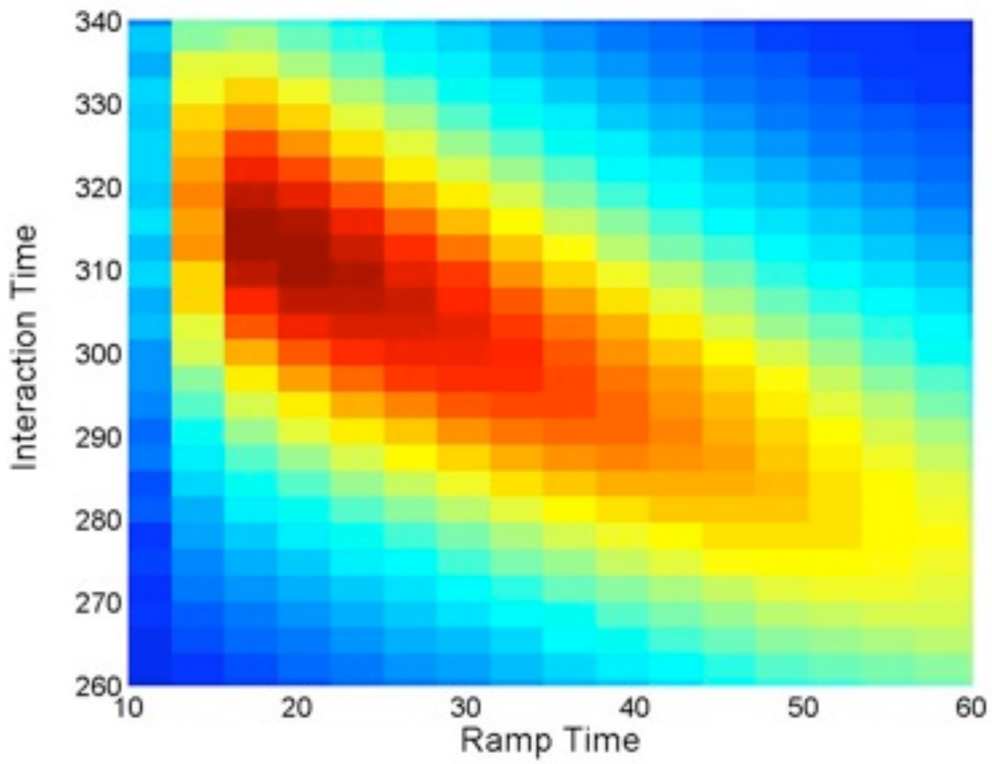
τ_I: **Interaction Time**



Phase Gate
$F_{max} \sim 0.9999$

Swap Gate
$F_{max} \sim 0.99$



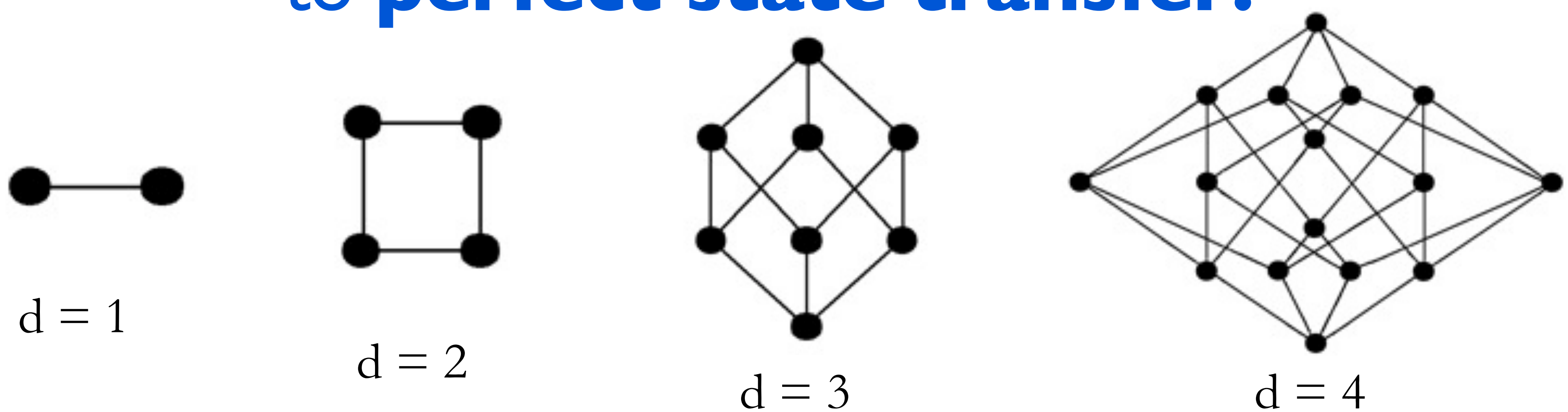


Quantum State Transfer

One can transfer the state of a single qubit from site A to site B using a set of permanently coupled qubits with Hamiltonian:

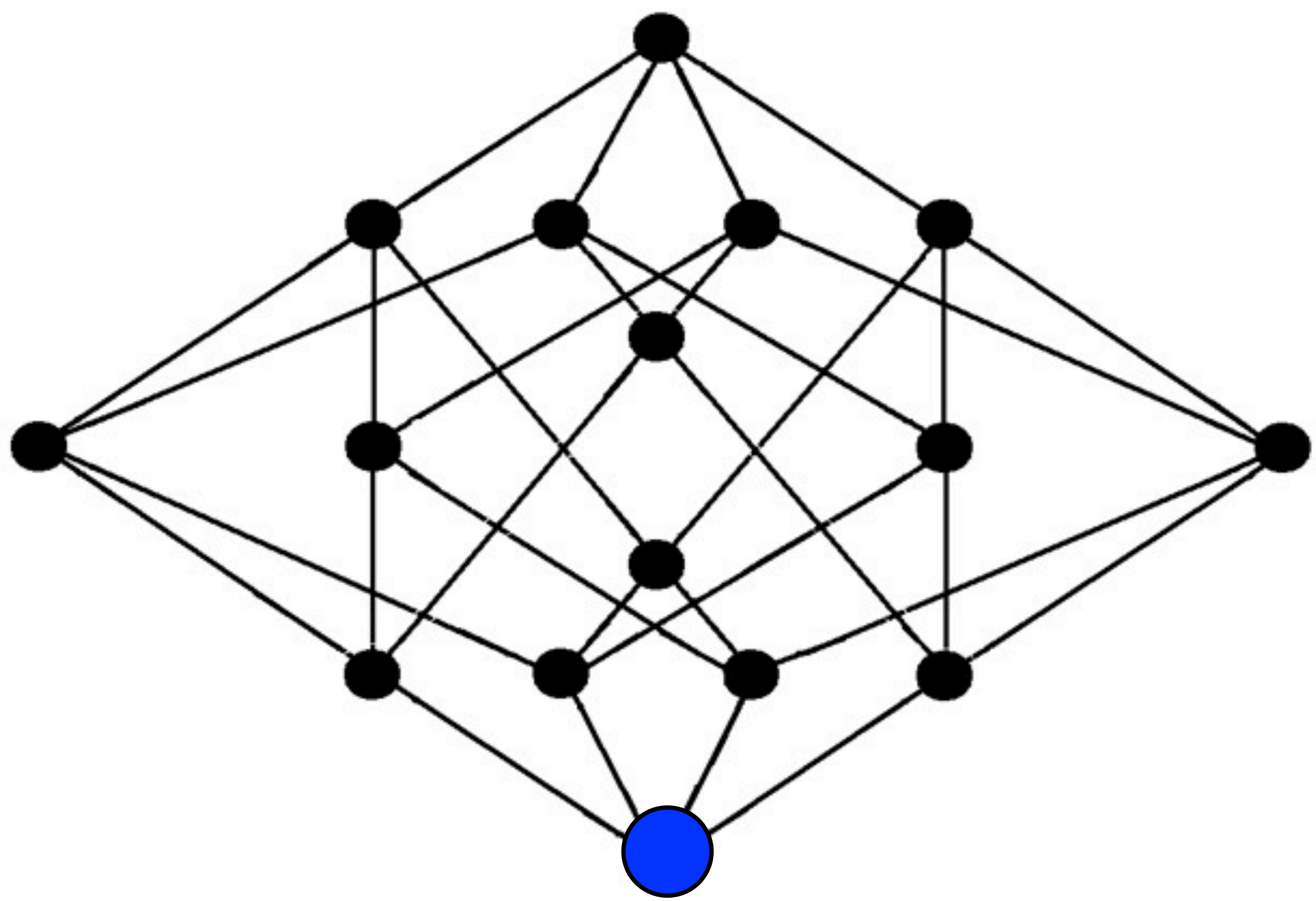
$$H = -\tfrac{1}{2}\sum_j \hbar\omega_j \sigma_j^z + \sum_{jk} \hbar\Omega_{jk}(\sigma_j^+\sigma_k^- + \sigma_k^+\sigma_j^-)$$

Dynamics of a single excitation (with $\omega = 0$) maps onto a tight-bonding model with $H = \hbar\Omega$, where the coupling matrix Ω is proportional to the adjacency matrix of the coupling graph. Certain coupling schemes such as the **hypercube** (M. Christandl *et al.*, Phys. Rev. Lett. **92**, 187902 (2004)) lead to **perfect state transfer:**



Hypercube State Transfer

- Each vertex represents a qubit. Quantum states travel along **all paths simultaneously** in superposition with **full constructive interference,** yielding **perfect state transfer.**



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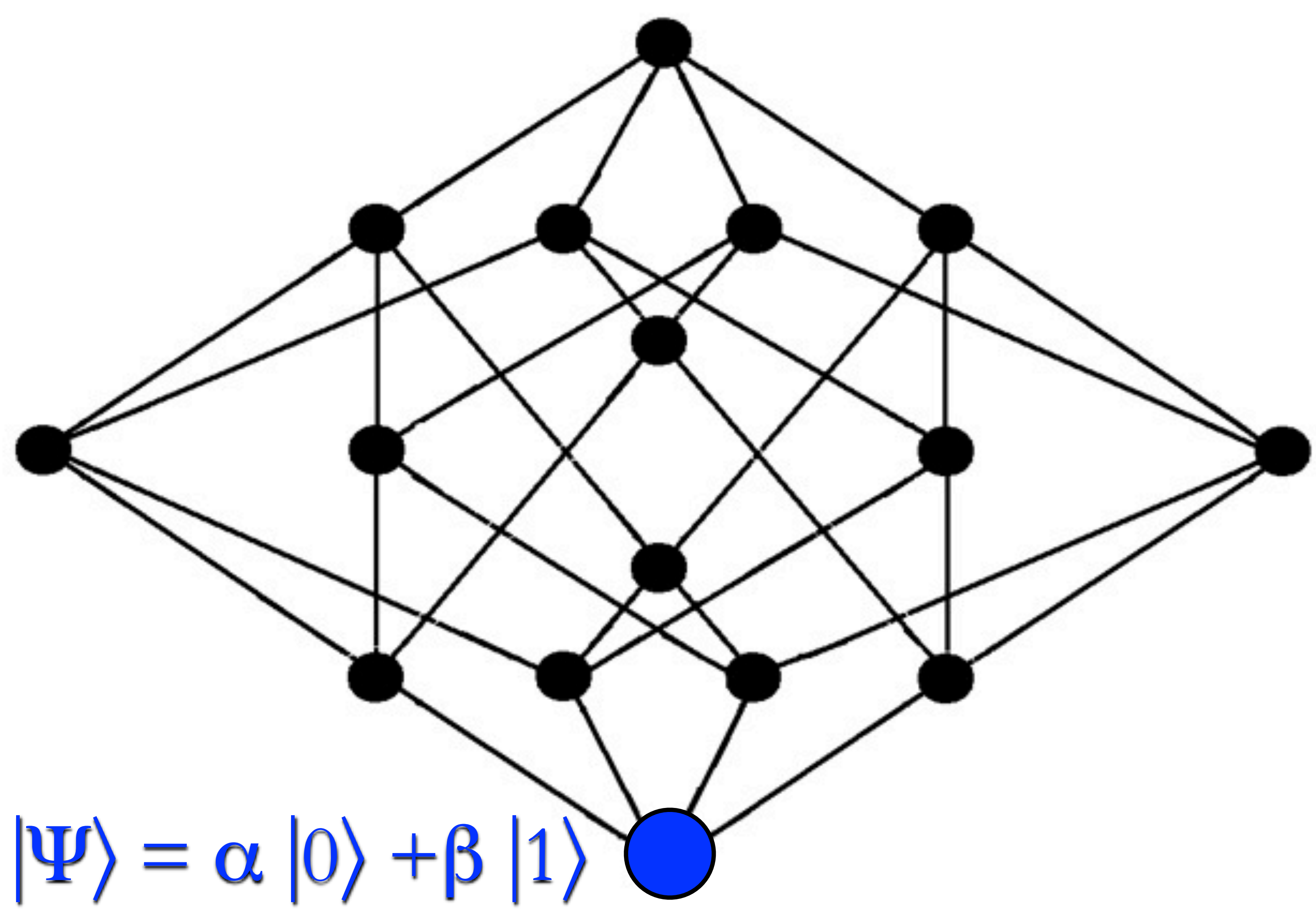

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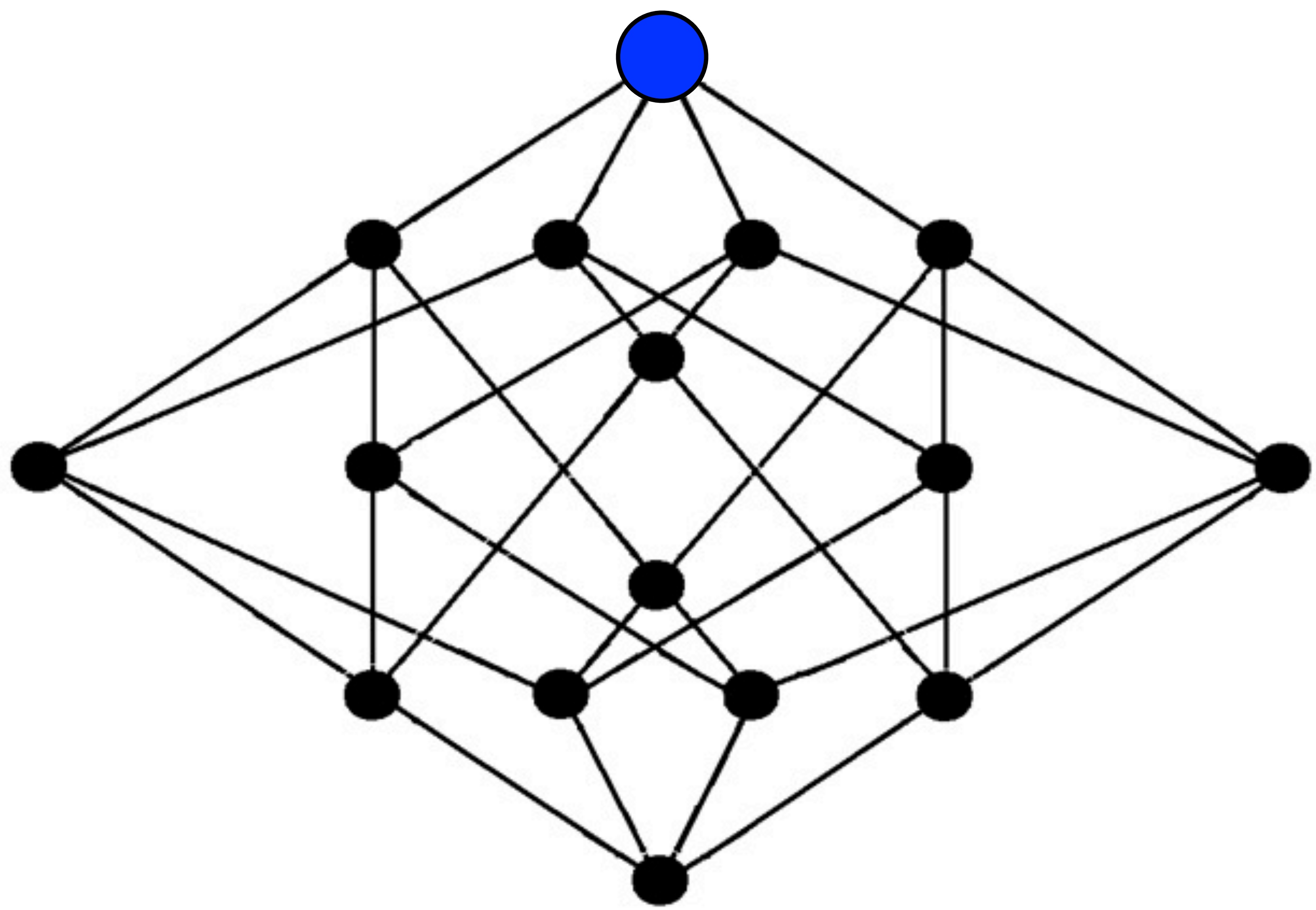

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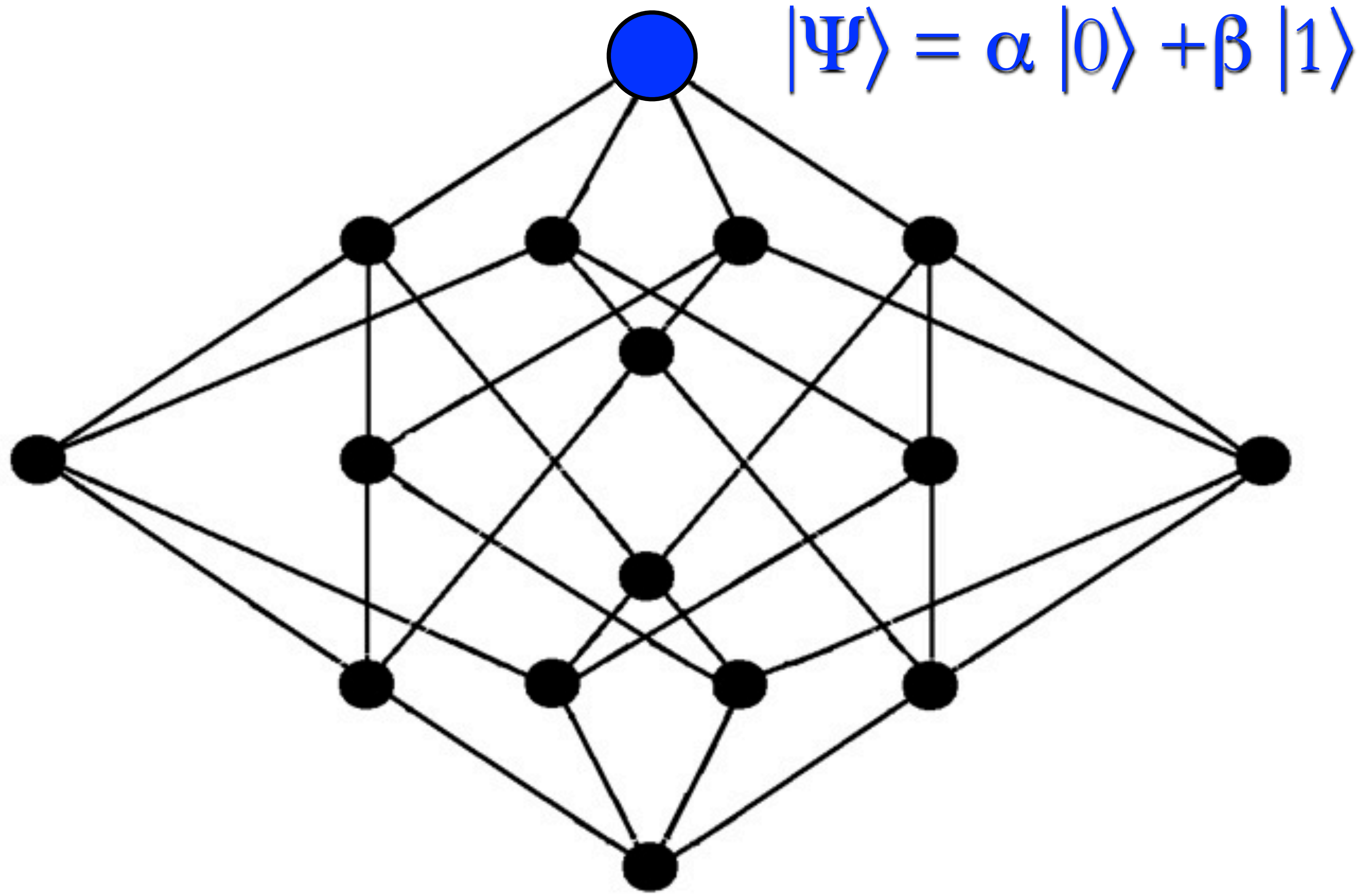
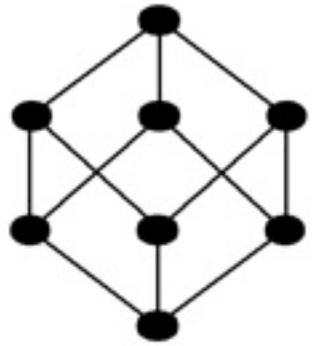
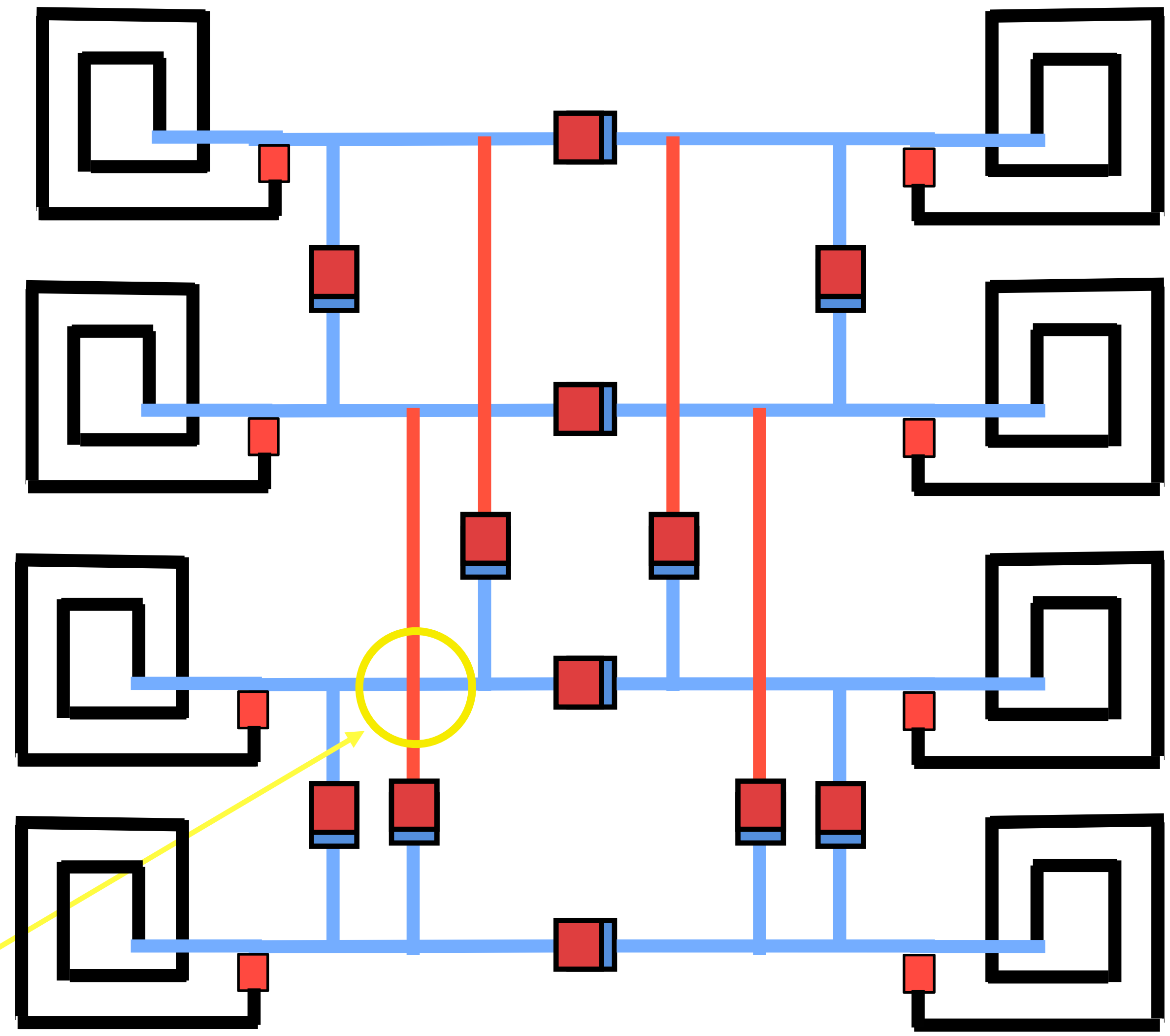


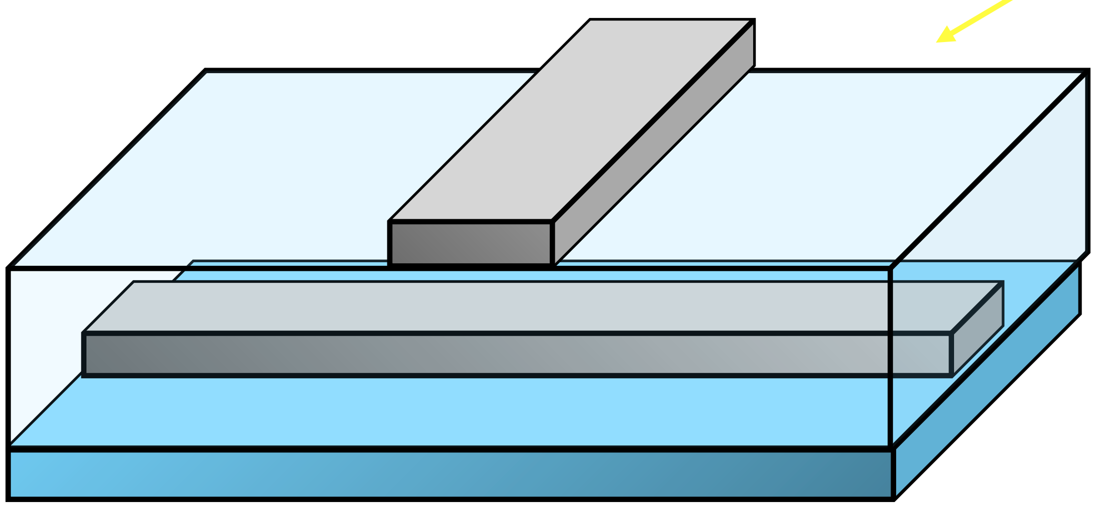
Phase Qubit Cube 1 2 3





Circuits do not need to be simple two-dimensional layouts.

Multi-layer interconnects allow many crossovers and complex couplings.



IEEE TRANSACTIONS ON APPLIED SUPERCONDUCTIVITY, VOL. 15, NO. 2, JUNE 2005

Fabrication Process of Planarized Multi-Layer Nb Integrated Circuits

Tetsuro Satoh, Kenji Hinode, Hiroyuki Akaike, Shuichi Nagasawa, Yoshihiro Kitagawa, and Mutsuo Hidaka

Abstract—To improve the operating speed and density of Nb single-flux-quantum integrated circuits, we developed an advanced fabrication process based on NEC's standard process. We fabricated planarized six-Nb-layer circuit structures using this advanced process. This new structure has four Nb wiring layers for greater design flexibility. To shield the magnetic field produced by the DC bias current, the DC bias power supply layer was placed under the groundplane. The critical current density of the Josephson junction was 10 kA/cm^2. We fabricated and tested more than 10 wafers and demonstrated that the six-layer circuits were successfully planarized. We also confirmed insulation between each Nb layer and the reliability of superconducting

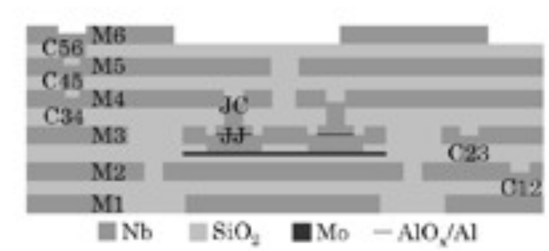
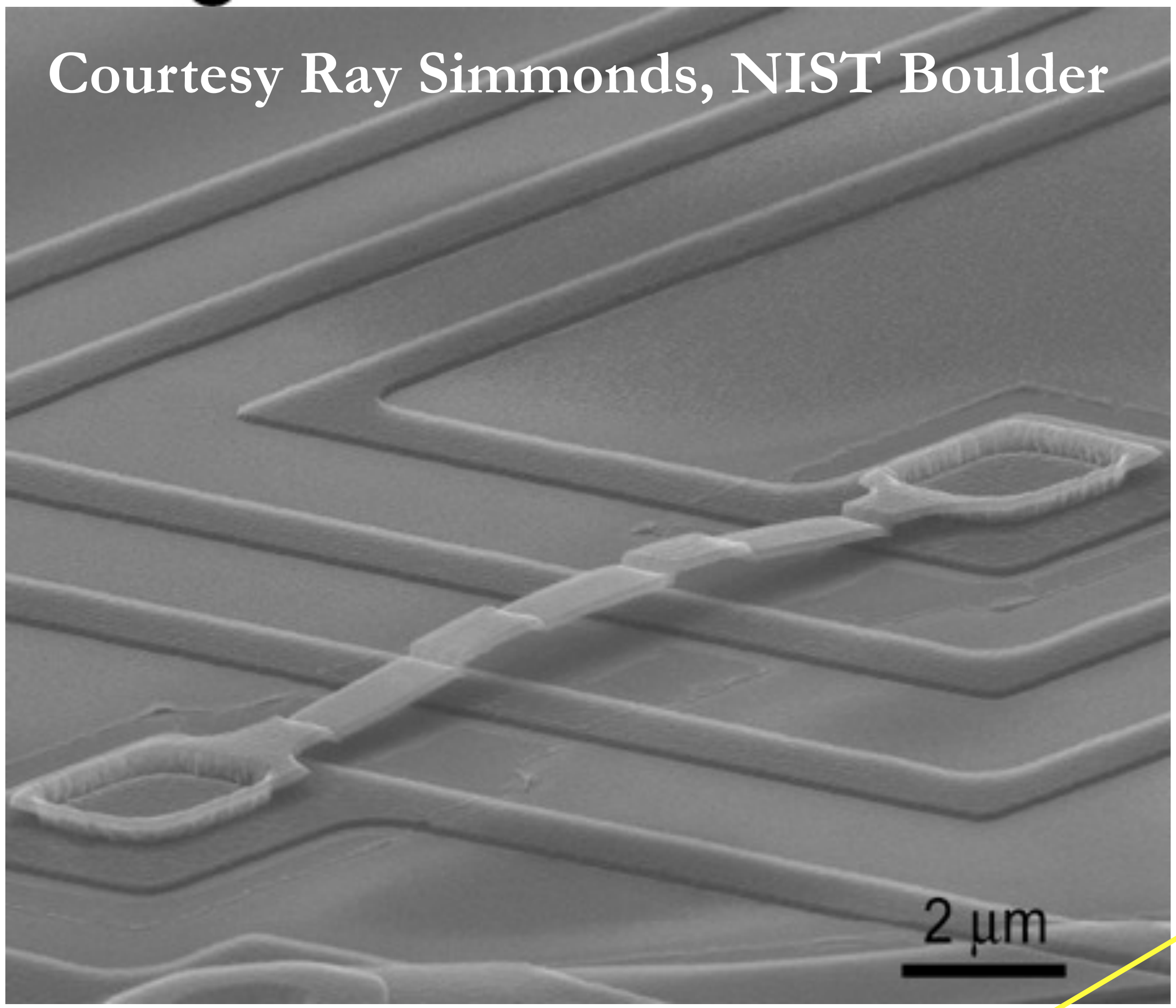
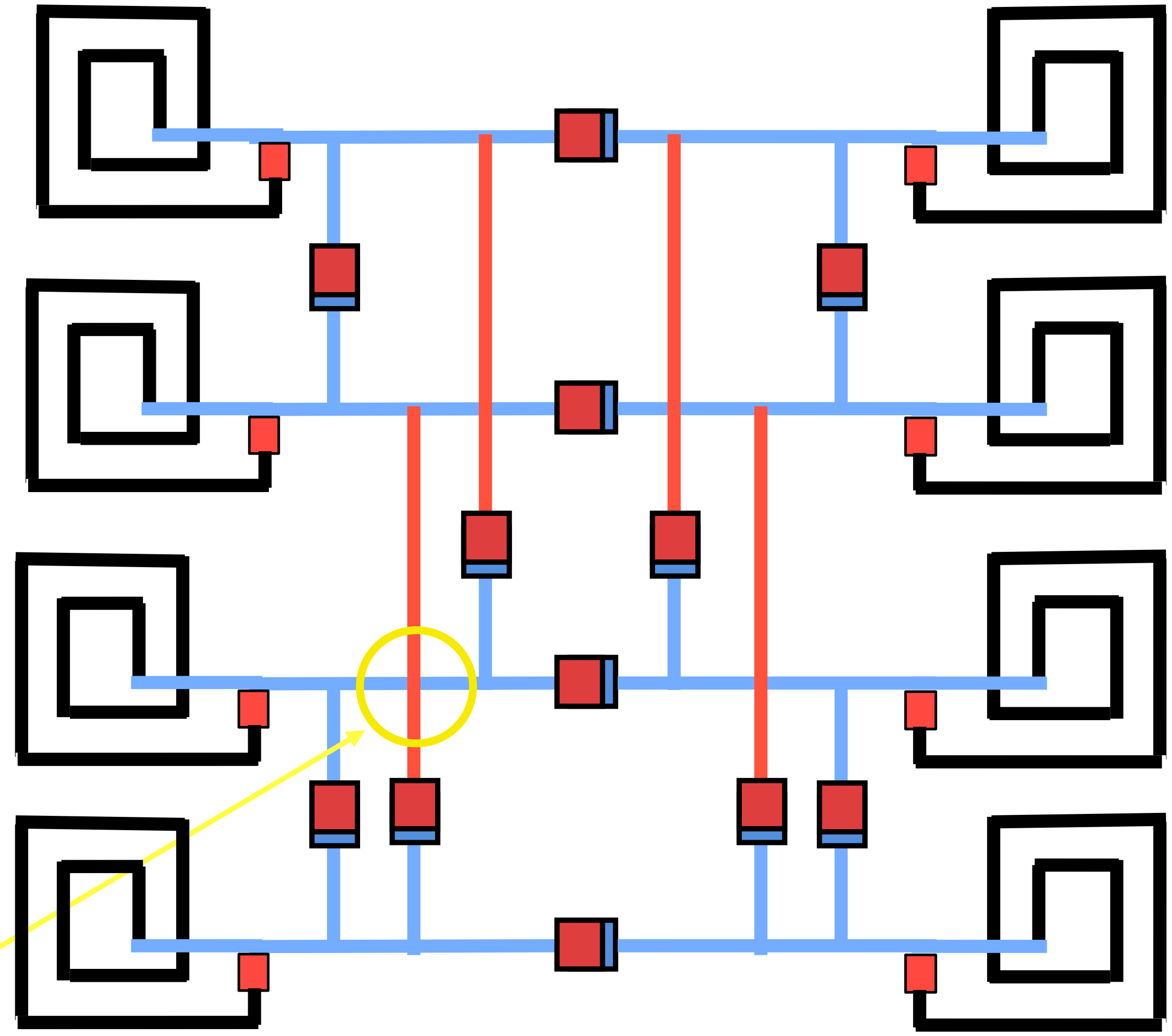
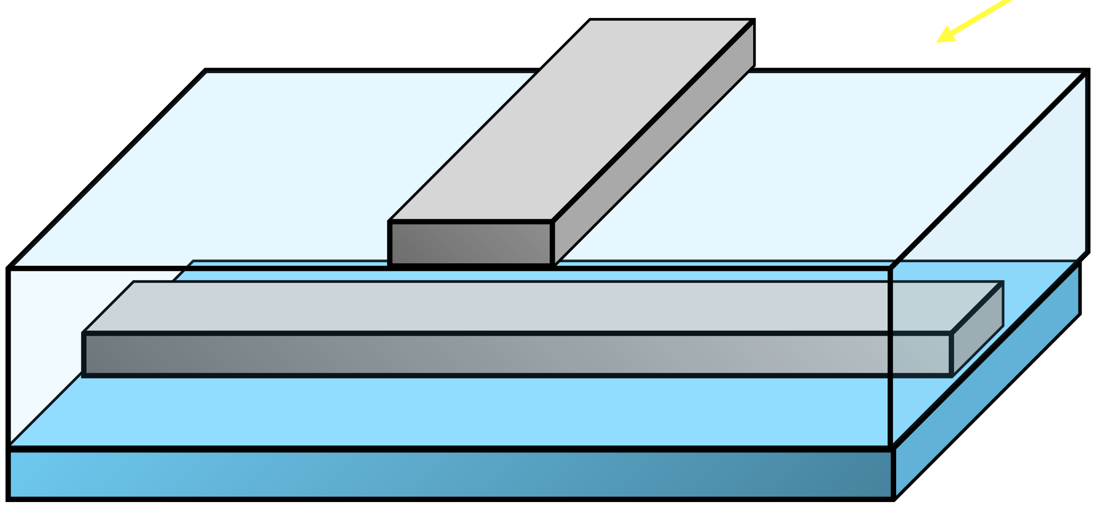


Fig. 1. Schematic illustration of a fabricated circuit structure.

Phase Qubit Cube 1 2 3







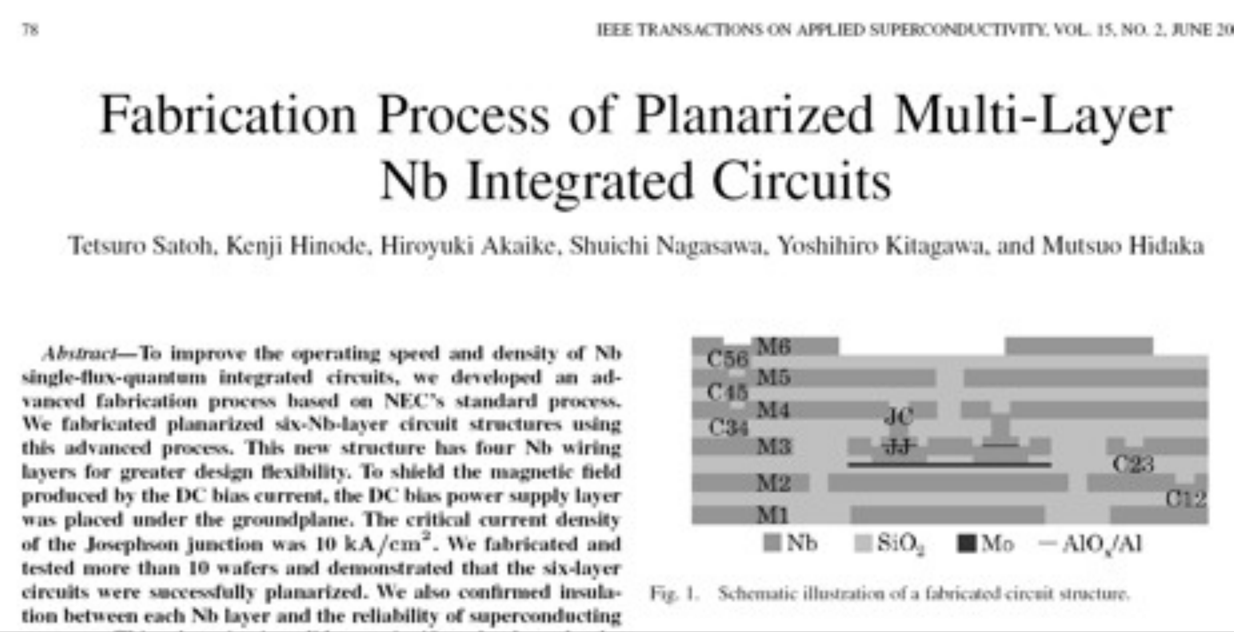


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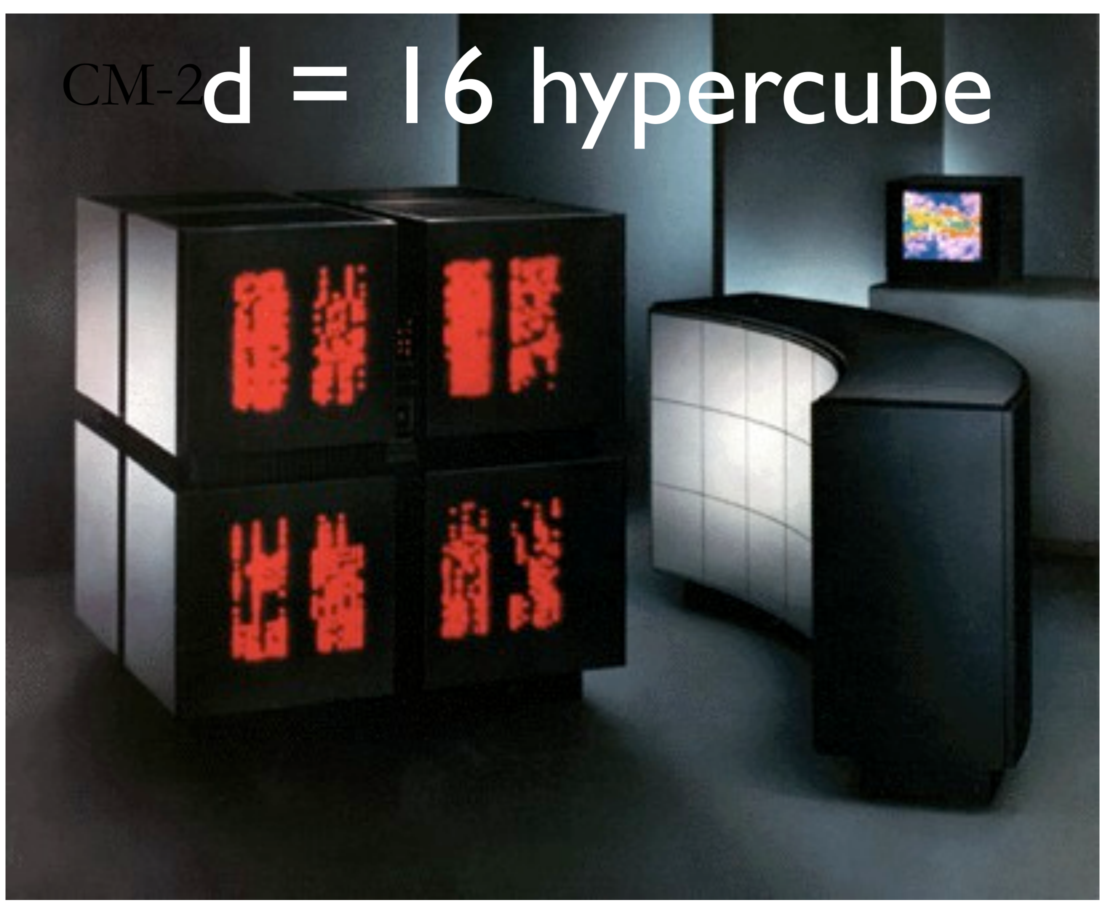
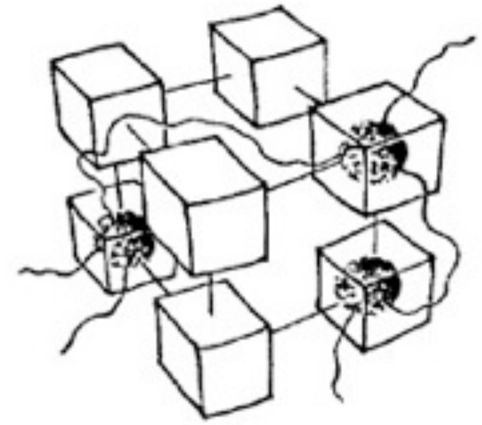
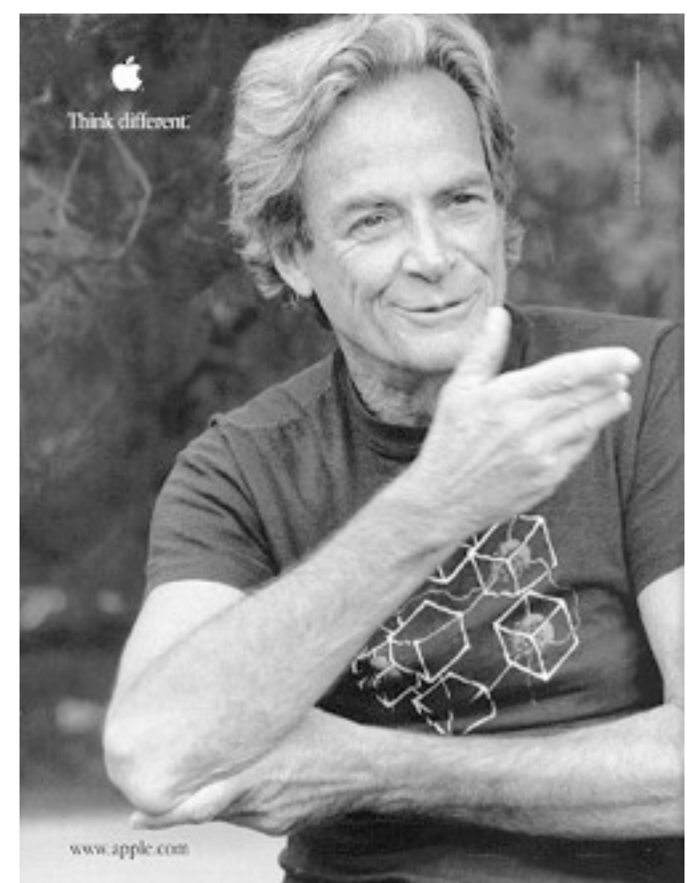
Fig. 1. Schematic illustration of a fabricated circuit structure.

Classical Hypercubes

- The hypercube design has previously been used for (classical) supercomputers such as the **Connection Machine** by **Thinking Machines**, co-founded by Daniel Hillis, for which **Feynman** was a consultant.





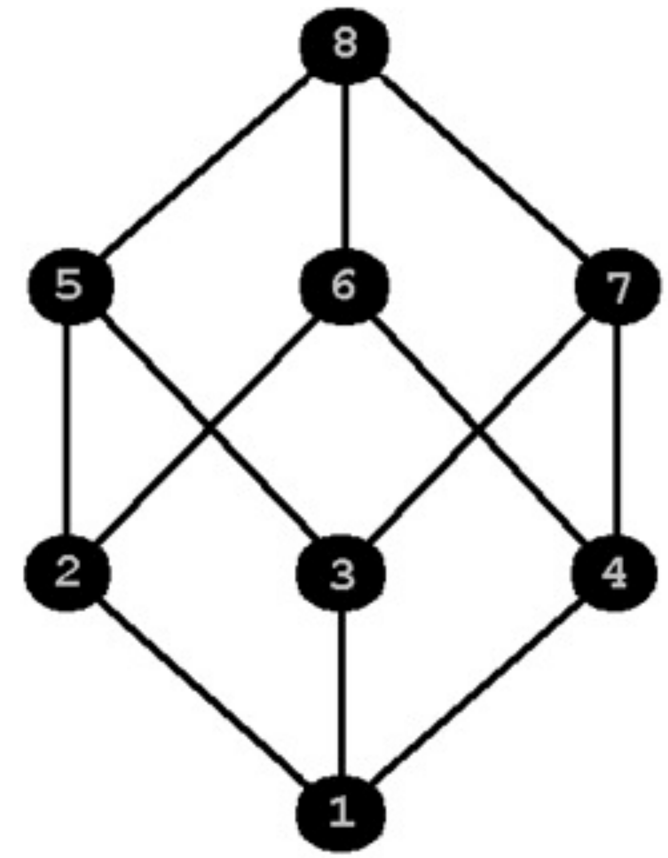


Phase Qubit Cube

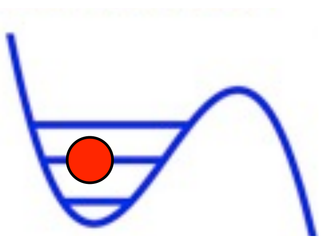
$$H = \frac{1}{2}\left(\Phi_0 / 2\pi\right)^{-2} \sum_{jk} p_j (C^{-1})_{jk} p_k - (\Phi_0 / 2\pi) \sum_j (I_{cj} \cos\gamma_j + I_j \gamma_j)$$

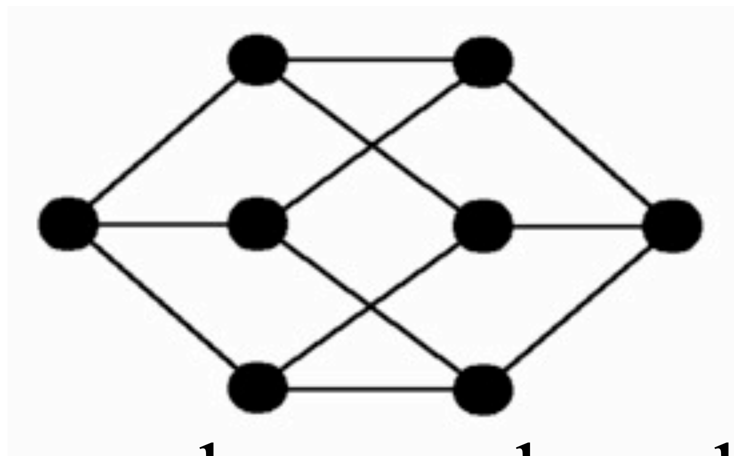
$$\zeta = \frac{C_c}{C_j + 3C_c}$$

$$C = (C_j + 3C_c)\begin{pmatrix} 1 & -\zeta & -\zeta & -\zeta & 0 & 0 & 0 & 0 \\ -\zeta & 1 & 0 & 0 & -\zeta & -\zeta & 0 & 0 \\ -\zeta & 0 & 1 & 0 & -\zeta & 0 & -\zeta & 0 \\ -\zeta & 0 & 0 & 1 & 0 & -\zeta & -\zeta & 0 \\ 0 & -\zeta & -\zeta & 0 & 1 & 0 & 0 & -\zeta \\ 0 & -\zeta & 0 & -\zeta & 0 & 1 & 0 & -\zeta \\ 0 & 0 & -\zeta & -\zeta & 0 & 0 & 1 & -\zeta \\ 0 & 0 & 0 & 0 & -\zeta & -\zeta & -\zeta & 1 \end{pmatrix}$$



Cube Dynamics



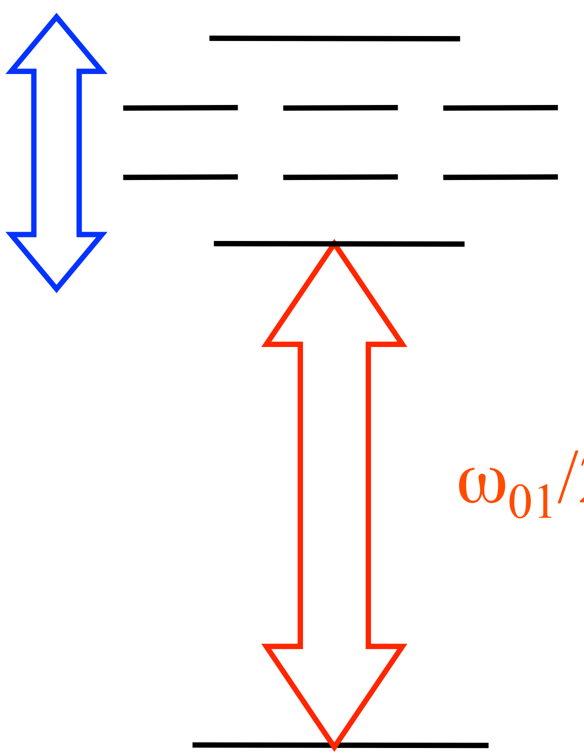


Eigenvalues are nearly evenly spaced ⇒ almost perfect oscillations!

Spectrum

$$\varepsilon(k) \approx \hbar\omega_0\zeta\, k$$

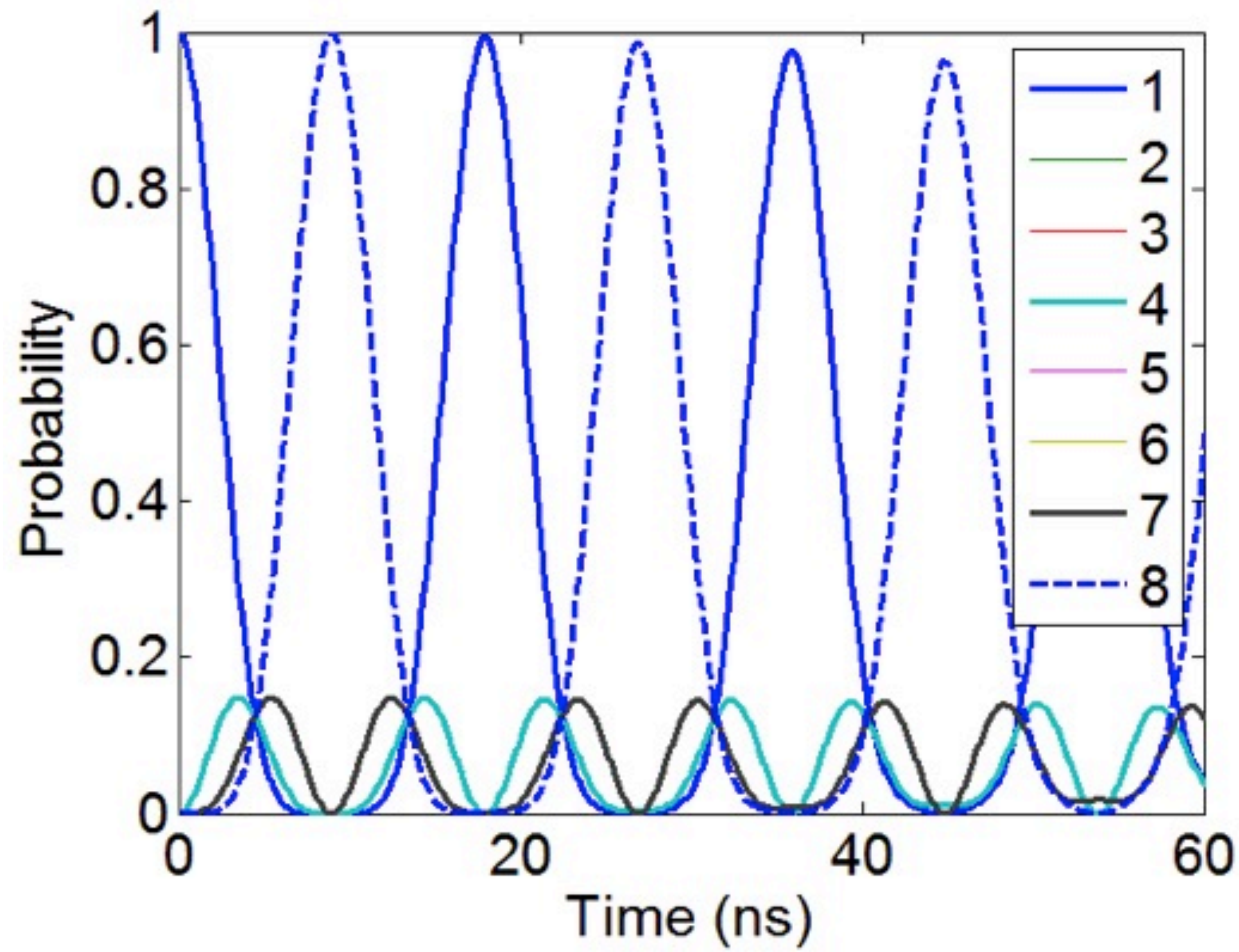
$3\,\zeta\,\omega_0/2\pi \approx 180$ MHz



$$\sum_j a_j |0\cdots 1_j \cdots 0\rangle$$

$\omega_{01}/2\pi \approx 5.76$ GHz

$|0¼0\rangle$



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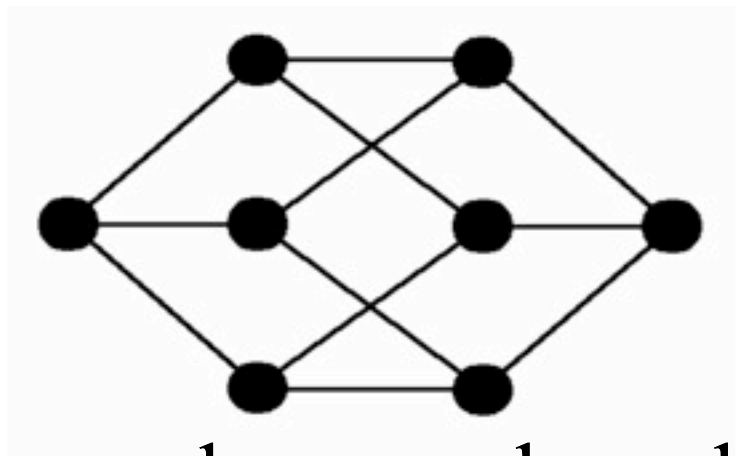

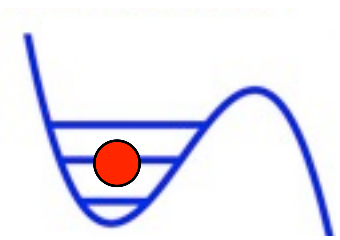

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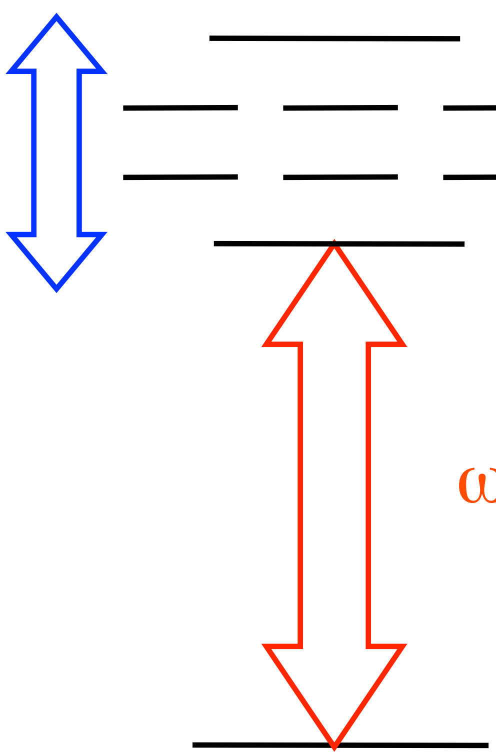

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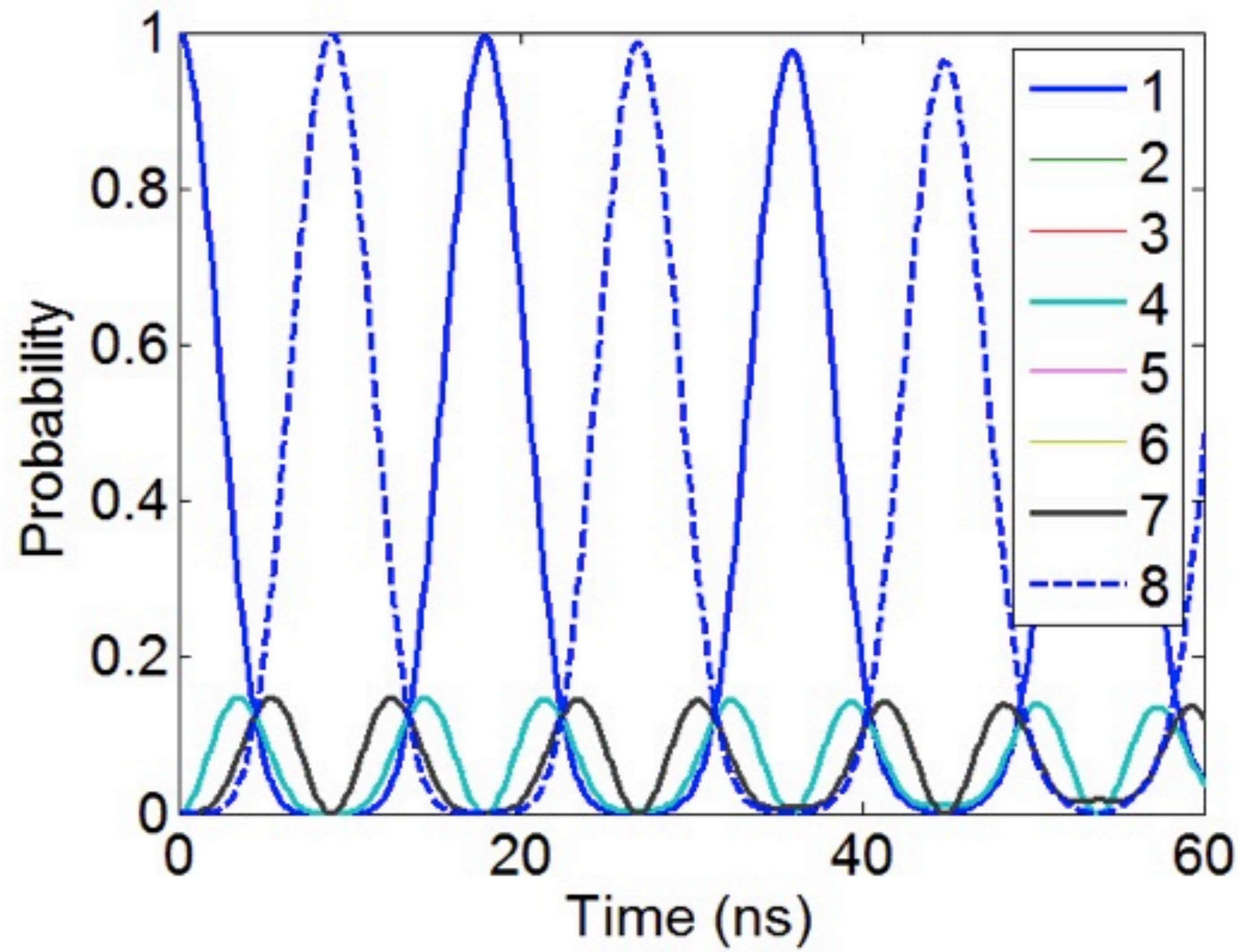

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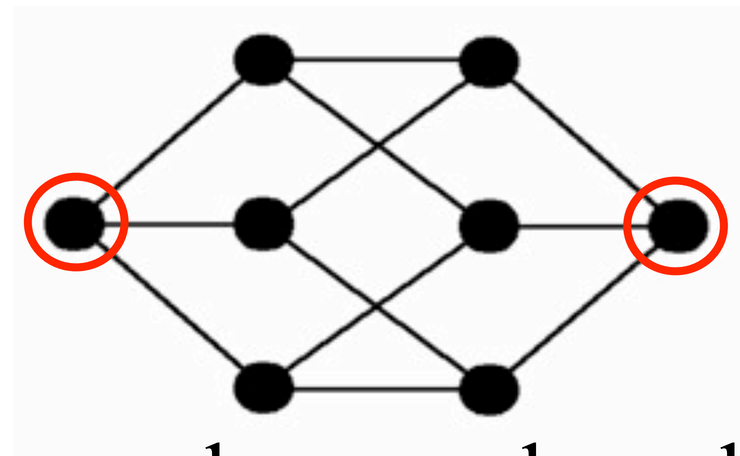

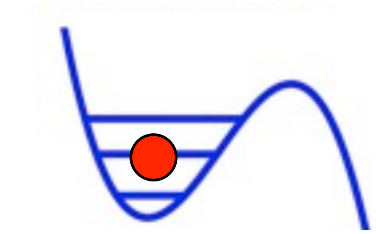

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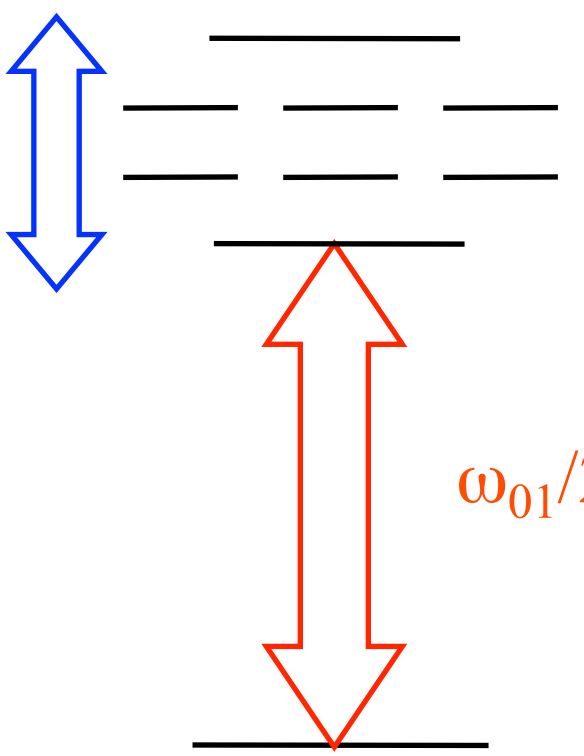

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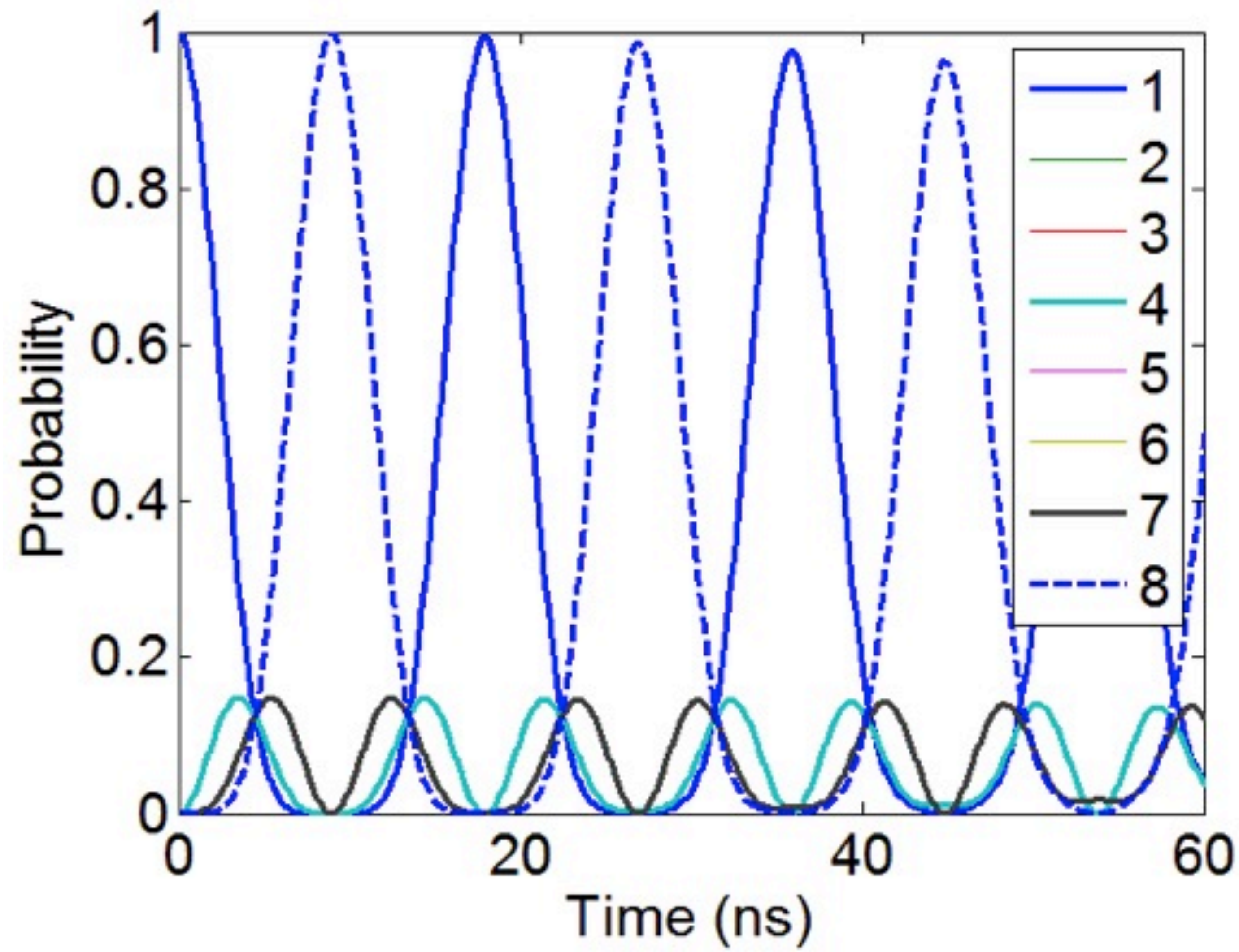
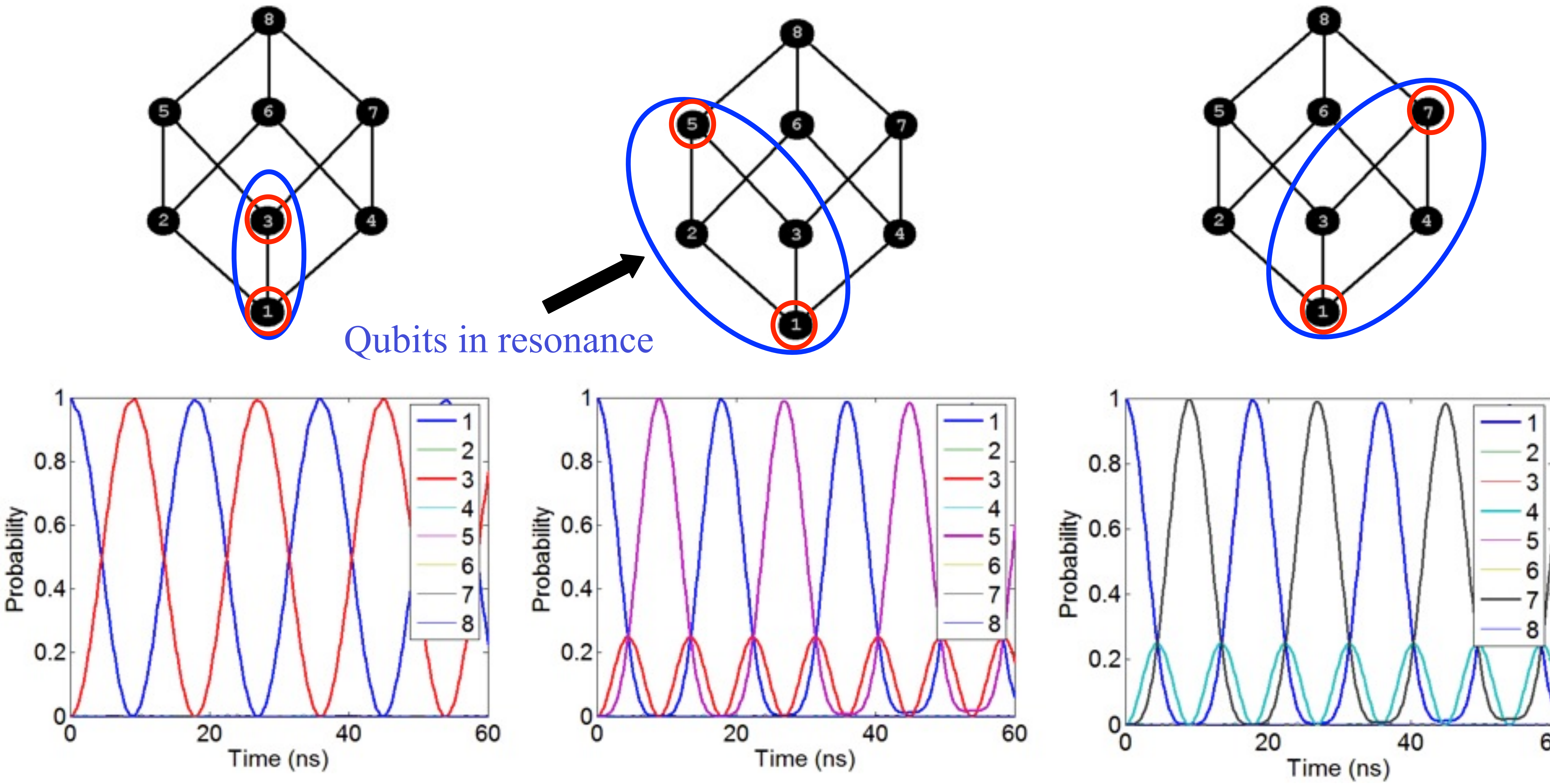


Tunable Cube Dynamics



Hypercube transport can be guided between any two sites by tuning qubit energies, all with the same propagation time!

Problem 1: Long-Range Coupling

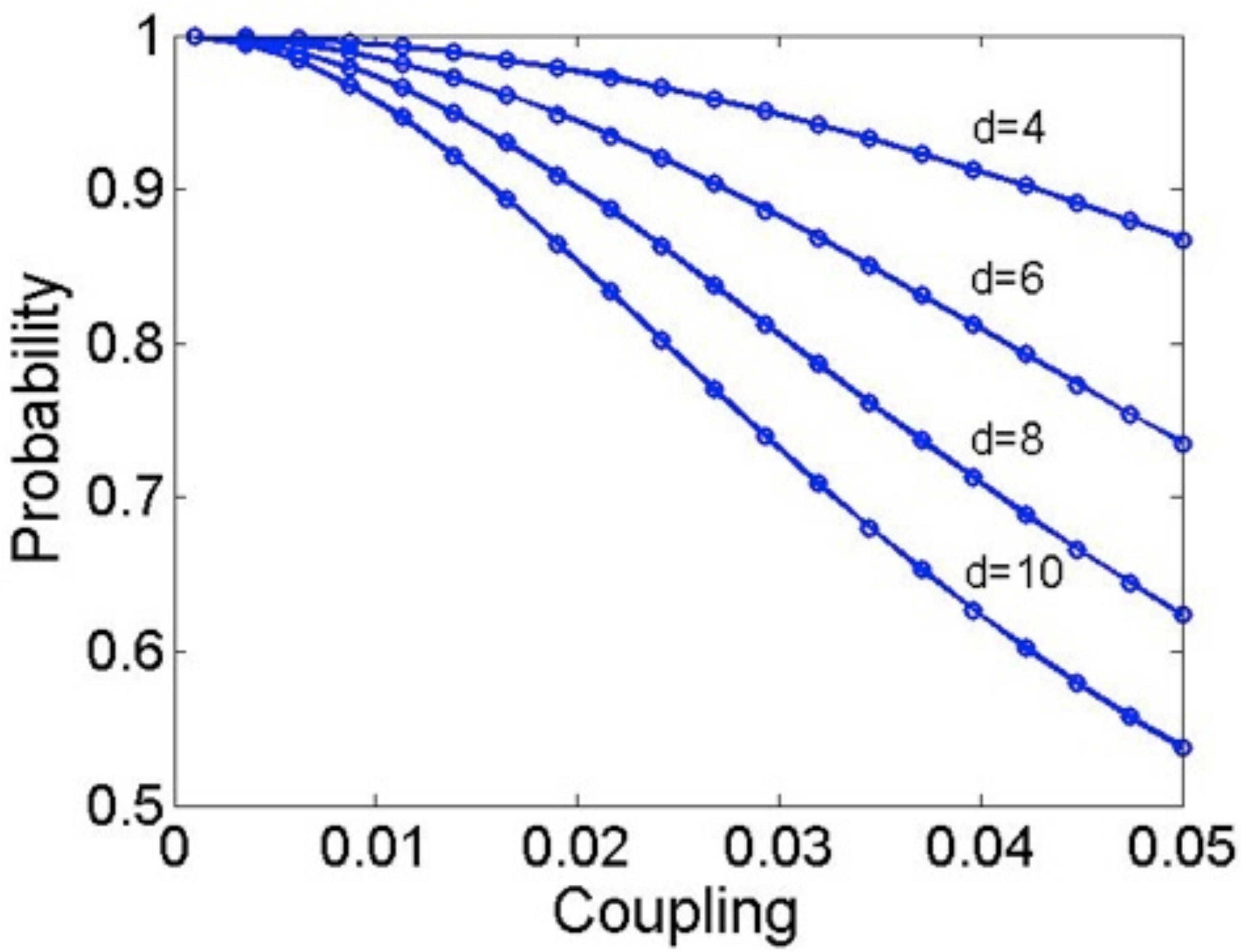
The coupling of junctions is through the *inverse* capacitance matrix.

$$H = \frac{1}{2}(\Phi_0/2\pi)^{-2}\sum_{jk} p_j (C^{-1})_{jk} p_k - (\Phi_0/2\pi)\sum_j (I_{cj}\cos\gamma_j + I_j\gamma_j)$$

$$C = (C_j + d\,C_c)(I - \zeta_d A_d), \quad \zeta_d = C_c/(C_j + d\,C_c)$$

$$C^{-1} = (C_j + d\,C_c)^{-1}(I + \zeta_d A_d + \zeta_d^2 A_d^2 + \cdots)$$

Long-range couplings



Long-range couplings distort the eigenvalue spectrum and degrade the state propagation. Effect gets worse with increasing hypercube dimension (d) and coupling (ζ).

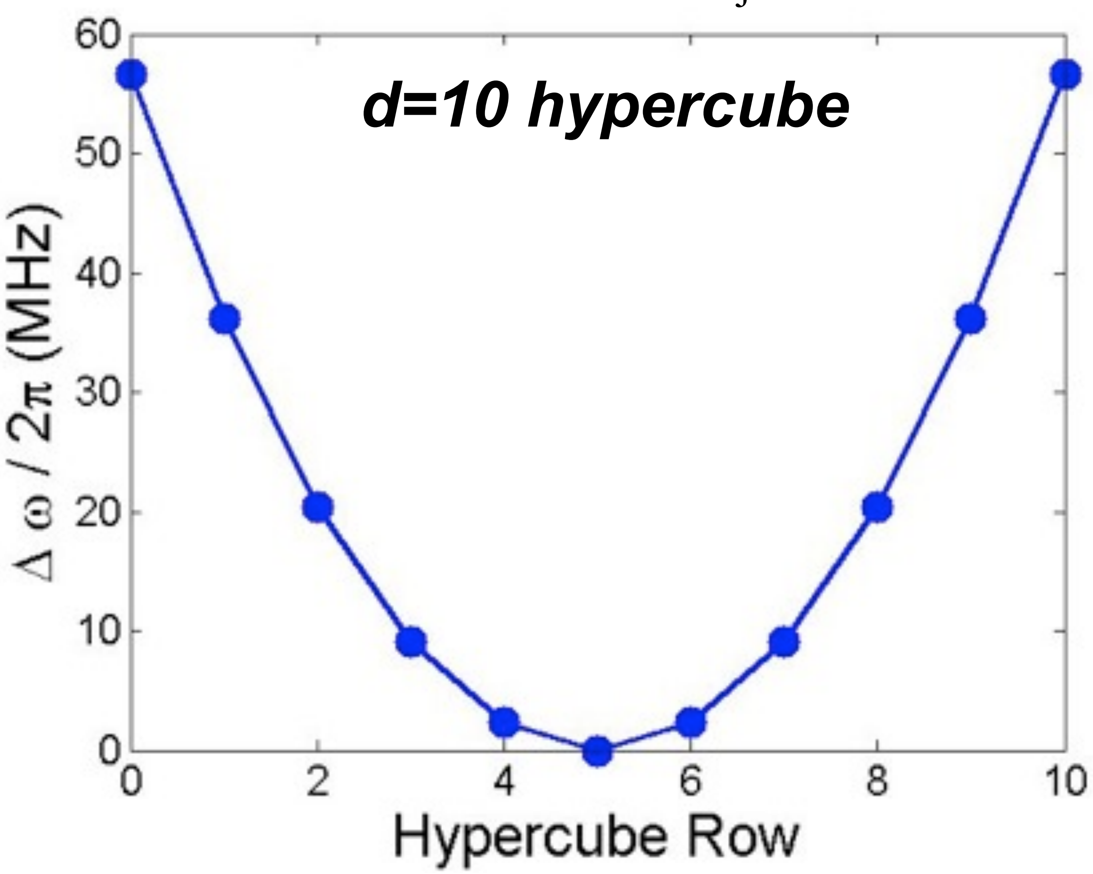
Tight-binding simulation

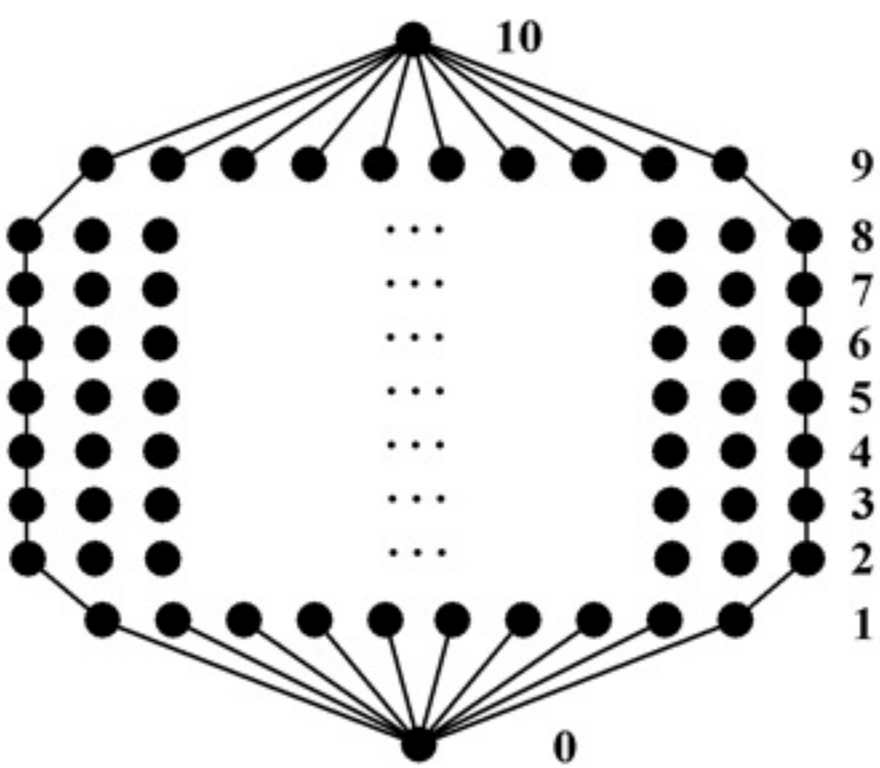
Correcting Long-Range Couplings

A simple fix is to vary the “on-site” energies (transition frequencies of each qubit) to compensate for the long-range couplings. By perturbation theory (using the angular momentum mapping) one can show that, at lowest order, the ***optimal choice of energies is quadratic!***

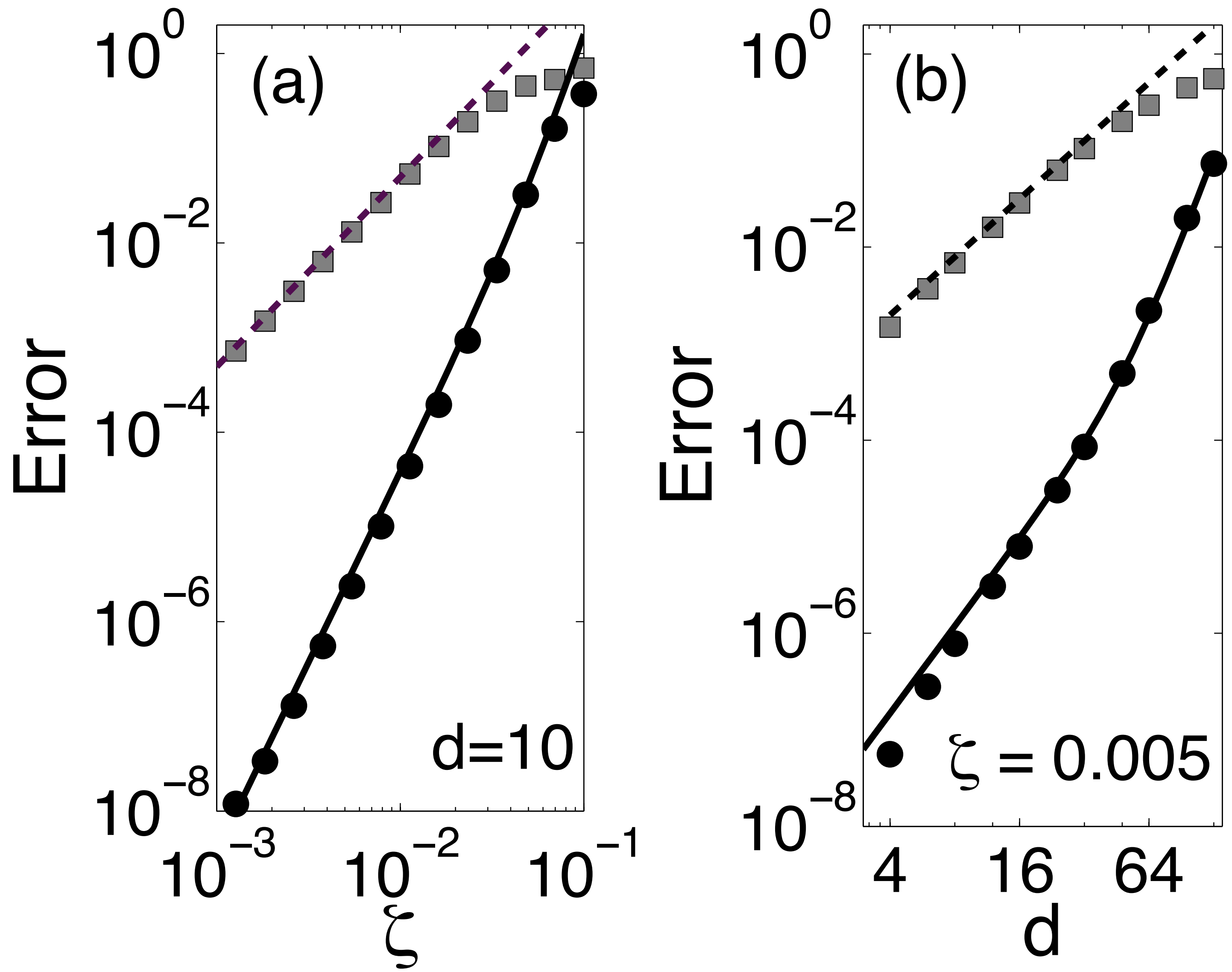
$$\Delta\omega_k = 4\omega_0\zeta^2\left(k - d/2\right)^2$$

$\omega_0/2\pi = 6$ GHz $\quad C_c/C_j = 0.01$





Corrected State Transfer



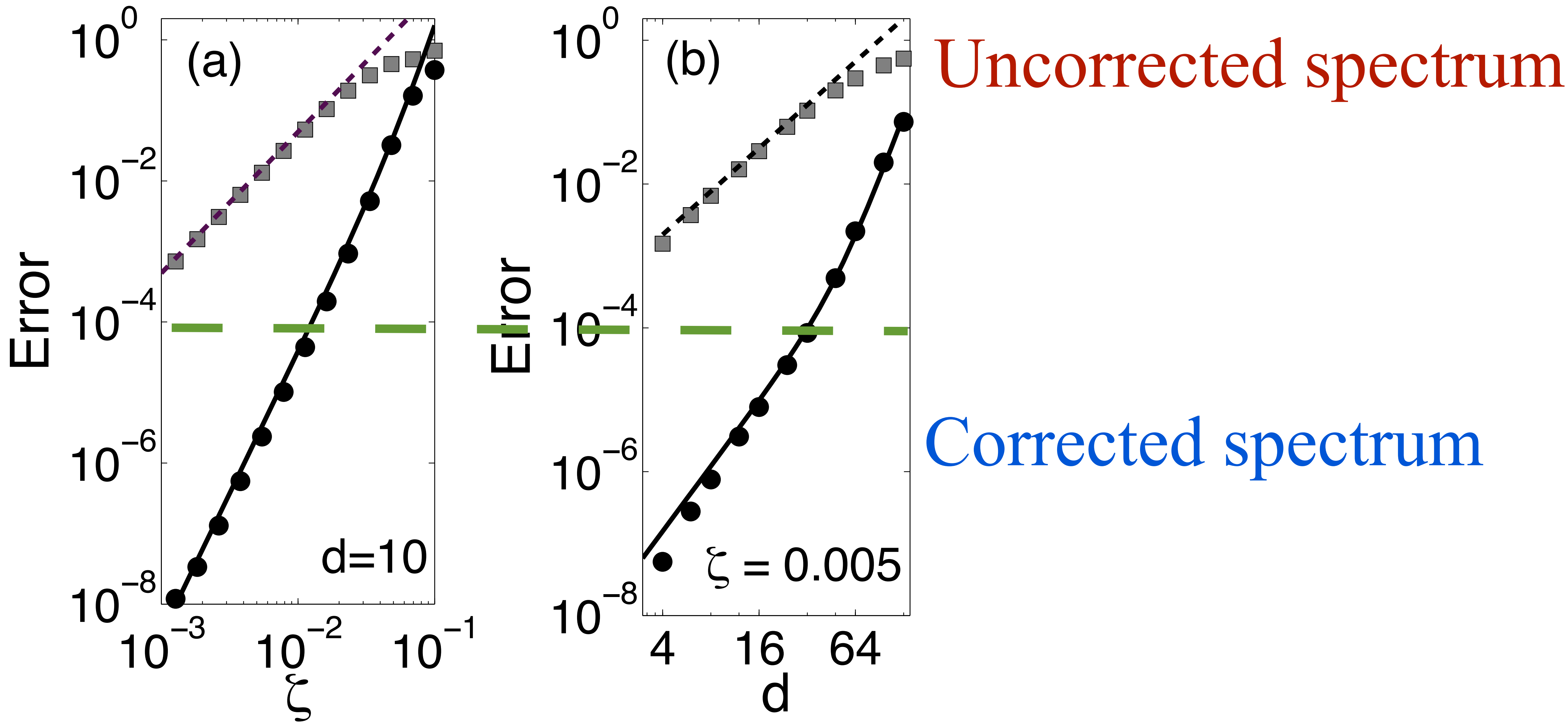
Uncorrected spectrum

Corrected spectrum

After correcting spectrum, fidelity of state transfer becomes applicable to quantum information processing for reasonable coupling strengths and for modestly large hypercube networks.

Tight-binding simulation

Corrected State Transfer



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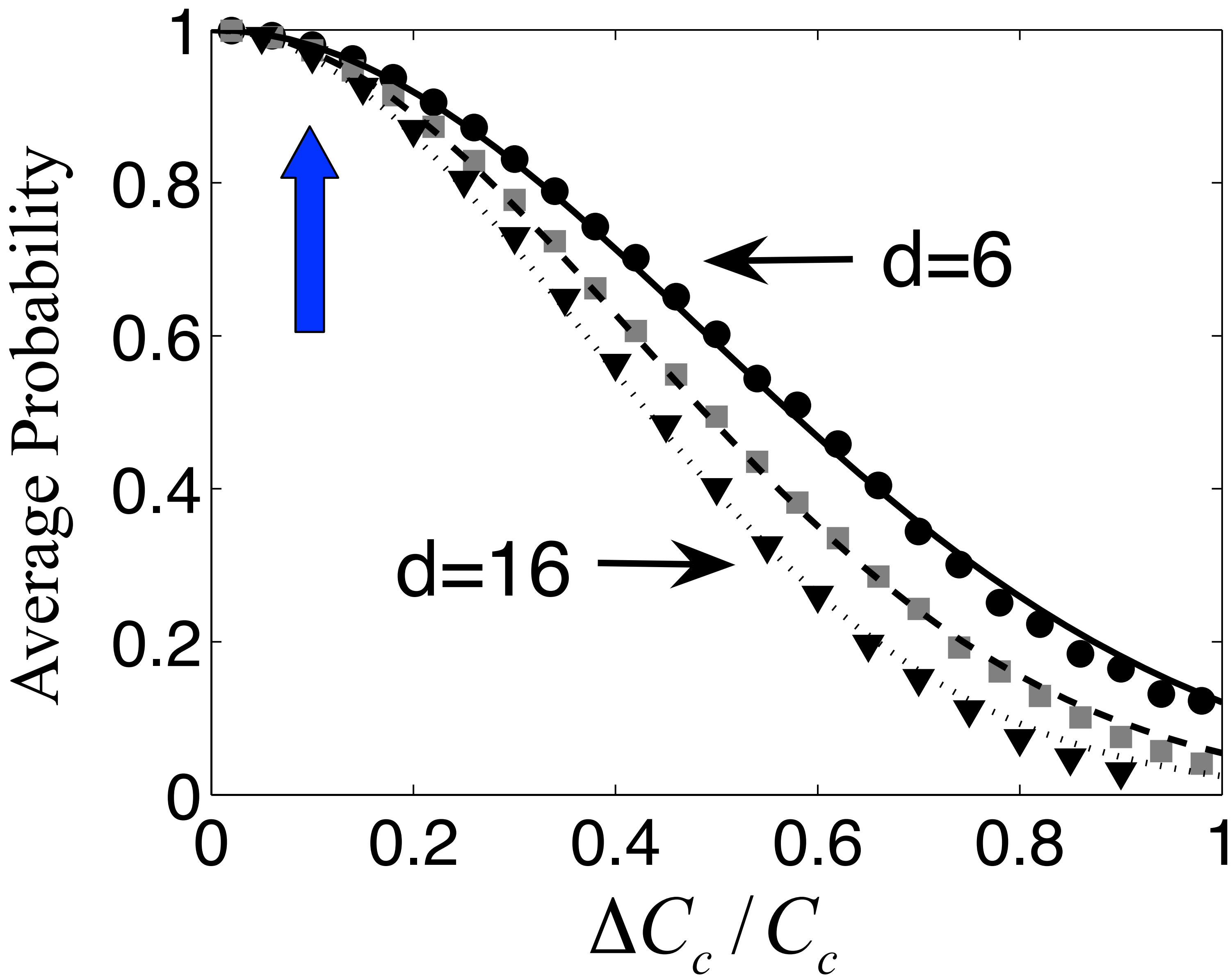
Tight-binding simulation

Problem 2: Disordered Couplings



Consistent with studies of **localization** (for disordered couplings):

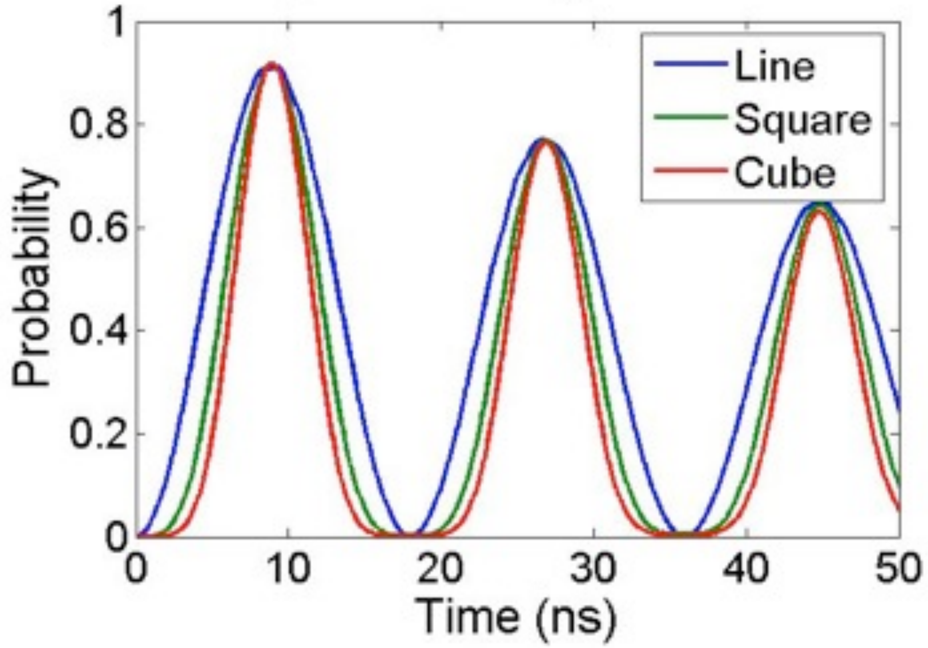
$$p_d \sim e^{-d/L},\, L \sim (\Delta C_c / C_c)^{-2}$$

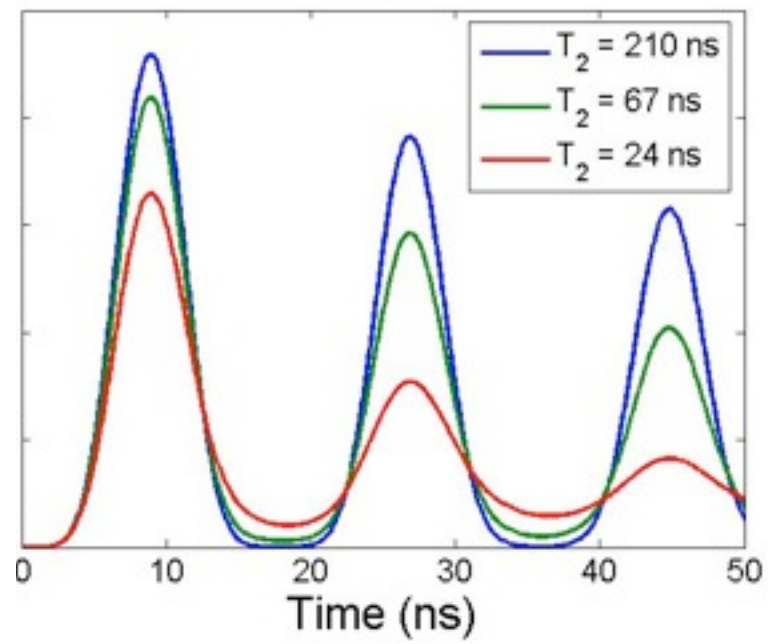
Even modest coupling disorder (~10%) has high transfer probability (>95%), for 2^{10} qubits!

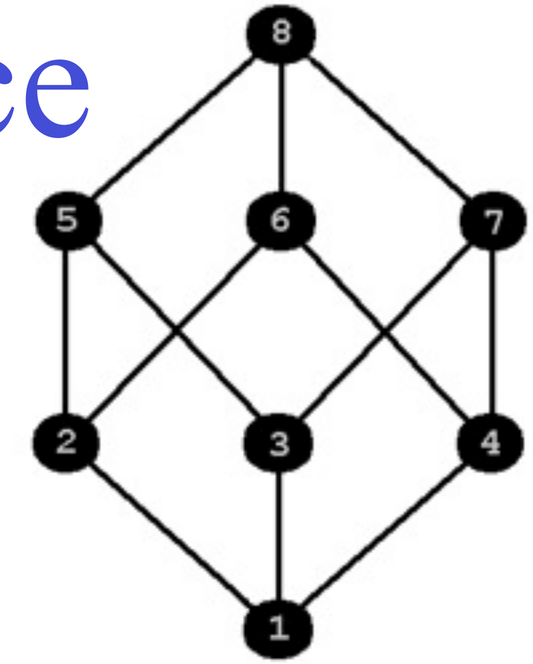
Tight-binding simulation

Problem 3: Decoherence

T_1 = 106 ns, T_2 = 210 ns $\omega_0 /2\pi = 6$ GHz Cube $C_c/C = 0.01$

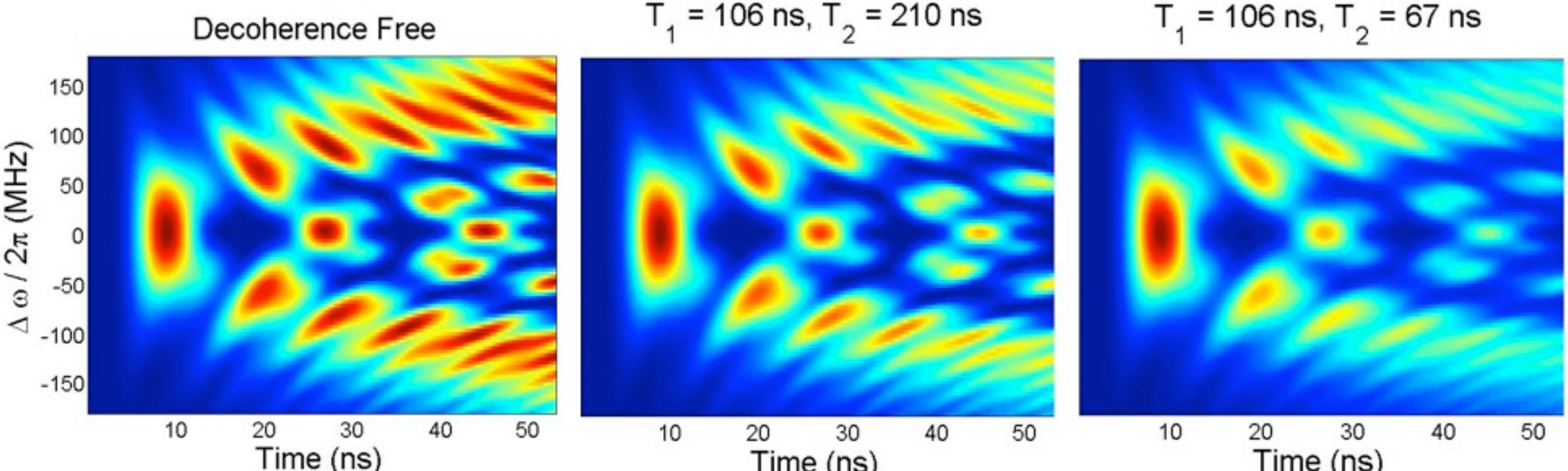






$\omega_2 = \ldots = \omega_6 = \omega_0$

$\omega_1 = \omega_8 = \omega_0 + \Delta\omega$



Density matrix simulation

For existing technology, transfer probabilities > 80% are possible.

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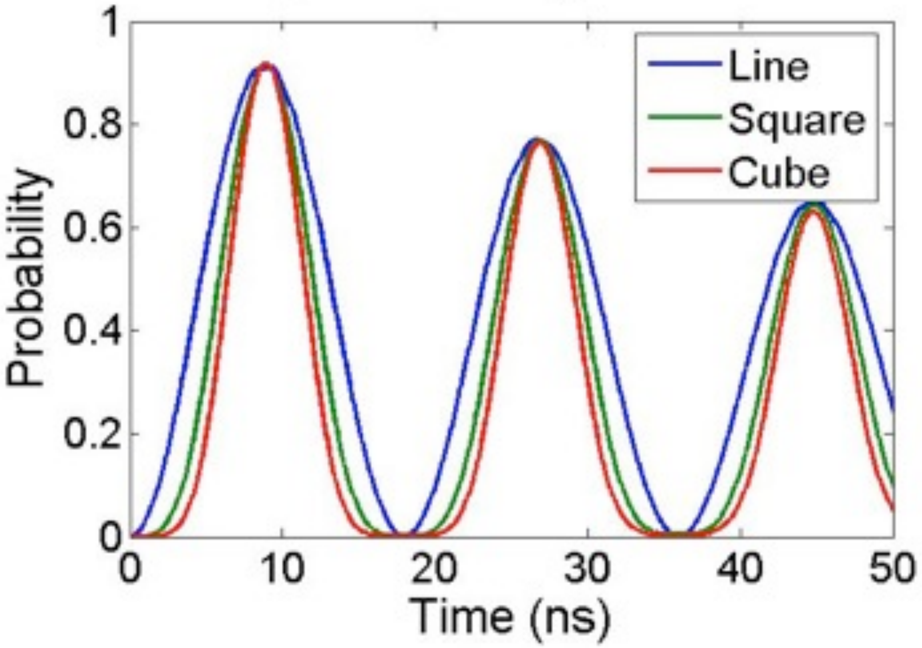

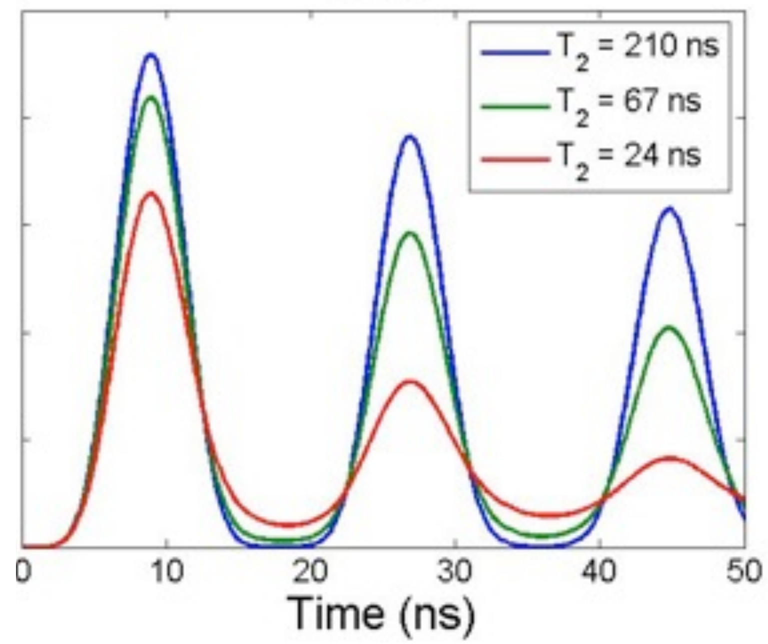

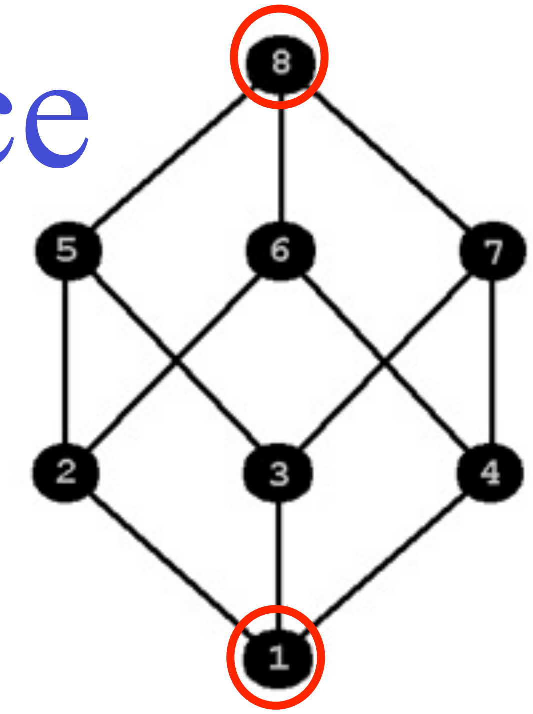

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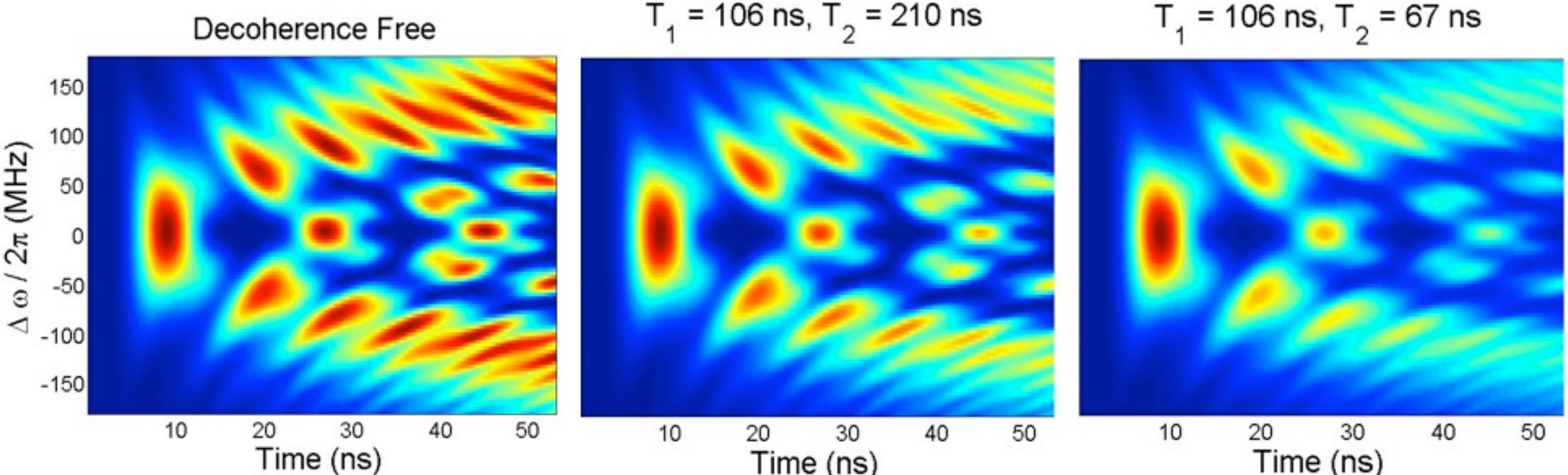

Density matrix simulation

For existing technology, transfer probabilities > 80% are possible.

Conclusions

- Nonadiabatic gates can be designed for phase qubits (and transmons?) in the regime of strong coupling (>1%), provided one accounts for the coupling of the qubit states with the auxiliary levels.
- Simulated gates for phase qubits (with tunneling, nonadiabatic couplings beyond qubit+auxiliary levels) have fidelities greater than 0.99.
- Qubit designs can be extended from artificial atoms and molecules to *artificial solids,* such as hypercubes, with novel transport properties that can be demonstrated using existing technology.