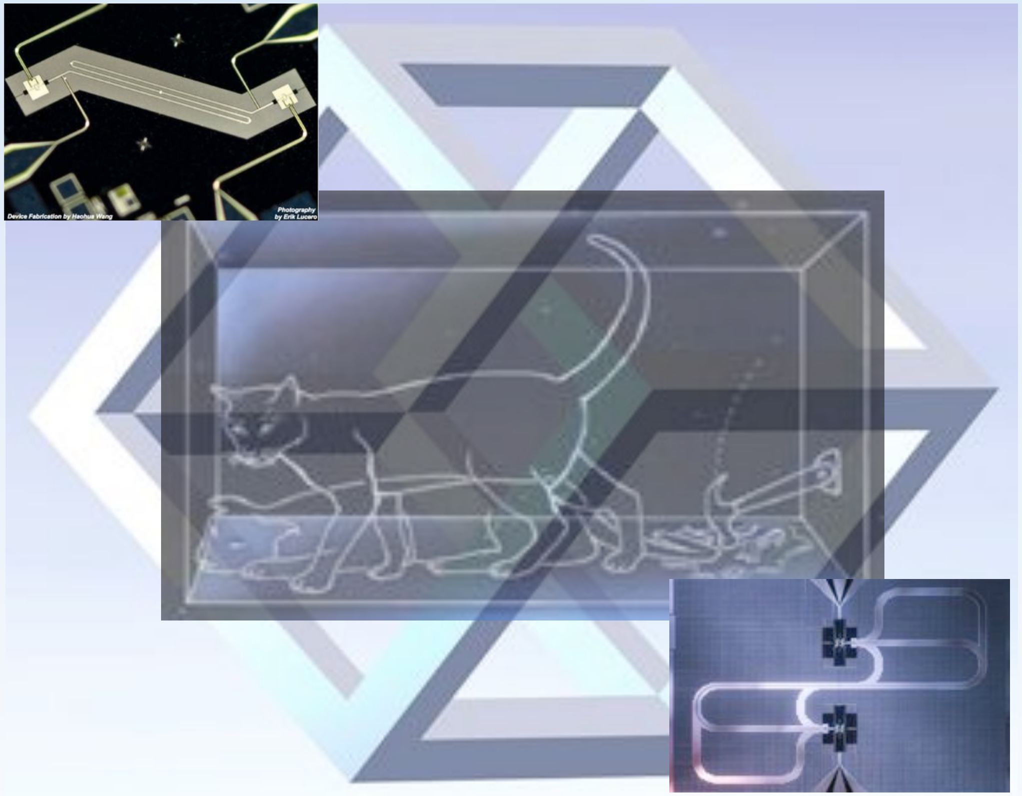
Entanglement in Superconducting Quantum Circuits



Frederick W. Strauch

Sigma Xi Talks November 18, 2011

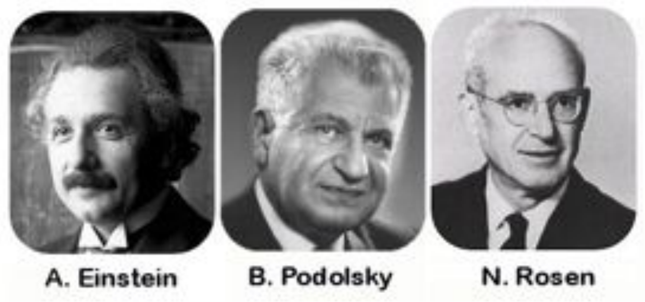
Timeline of QM + Entanglement

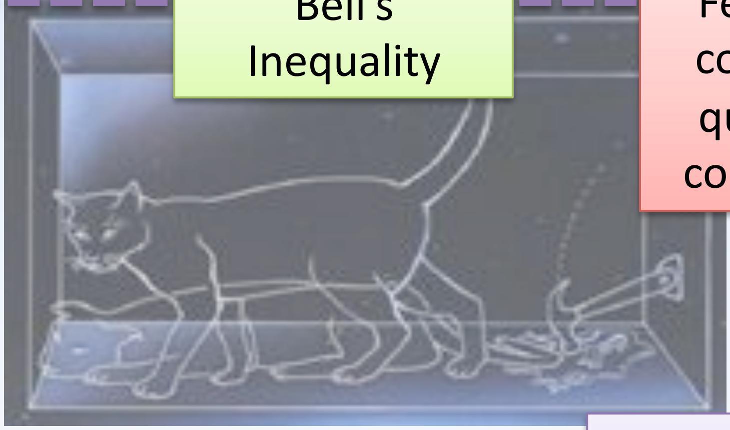
1905
Einstein postulates photon

1913
Bohr model of the atom

1920s
Heisenberg, Schrödinger, and Dirac formalize QM

1935
Einstein-Podolsky-Rosen "paradox", Entanglement, and Schrödinger's Cat


A. Einstein B. Podolsky N. Rosen

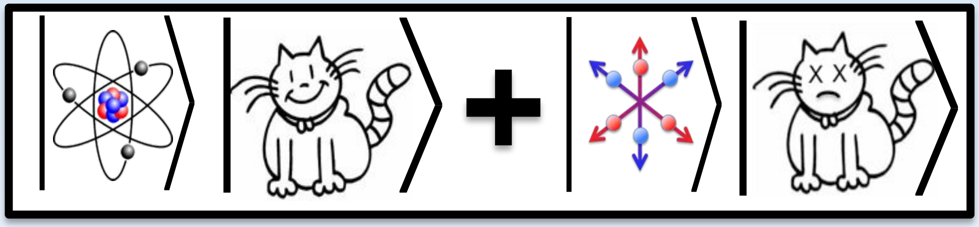


1964
Bell's Inequality

1982
Feynman considers quantum computers

Quantum Computation and Quantum Information

Entanglement



- "[The meaning of entanglement is]: the best possible knowledge of a whole does not necessarily include the best possible knowledge of all its parts, even though they may be entirely separate"

Quantum Computing

- Future devices that can harness entanglement to store and process information in parallel (*quantum parallelism*)

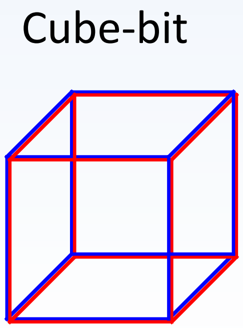
Q: $|input_1\rangle + |input_2\rangle + \cdots$

A: $|input_1\rangle|output_1\rangle + |input_2\rangle|output_2\rangle + \cdots$

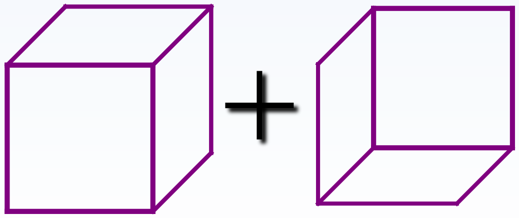
Quantum Bits

- Any system with two distinct states can represent a bit (0 or 1), the fundamental unit of information.
- Any quantum system with two distinct states (or any superposition thereof) can represent a quantum bit (**qubit**) of quantum information (e.g. spins:

$$|0\rangle = |\Uparrow\rangle, \quad |1\rangle = |\Downarrow\rangle$$



$$|\Psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$



Quantum Information Processing

- Quantum information allows superposition:
  - 3 classical bits: 000 or 001 or 010 or … 111

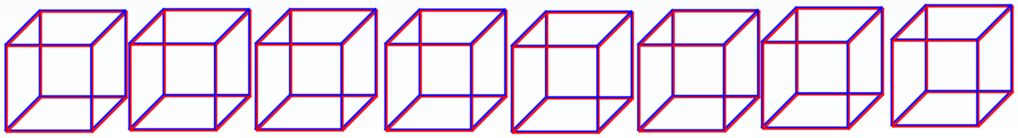
    (8 distinct possibilities)
  - 3 quantum bits:

    $|\Psi\rangle = (|000\rangle + |001\rangle + |010\rangle + \ldots |111\rangle)/2^{3/2}$

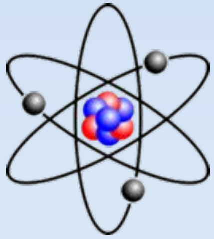
    (8 simultaneous possibilities)
- Quantum computers use superpositions to sample large number of simultaneous possibilities:

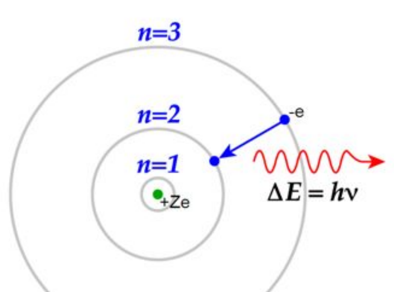
$$|\Psi\rangle = (\overbrace{|\underbrace{00\ldots00}_{\text{N qubits}}\rangle + \ldots + |11\ldots11\rangle}^{2^{\text{N}}\text{ possibilities}})/2^{\text{N}/2}$$

Cube-byte



Ψ Quantum Mechanics



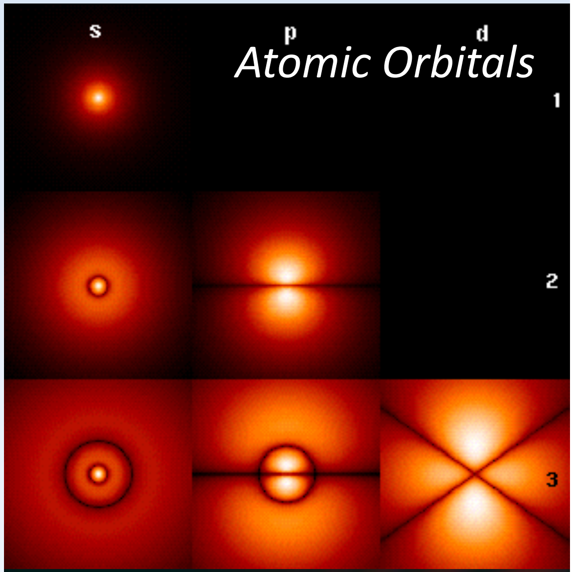


Bohr Atom

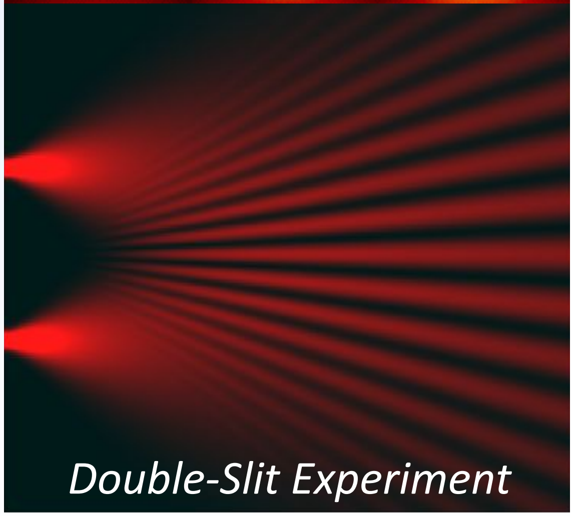
$$E = h\,f$$

Planck Law

Spectral Lines



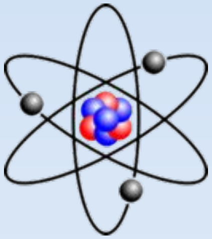
Atomic Orbitals

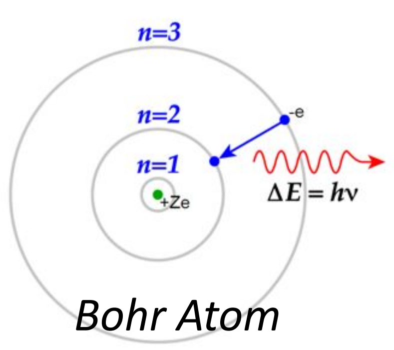


Double-Slit Experiment

- **Quantized Energy:** Energy comes in discrete packets (*quanta*), transferred by photons
- **Uncertainty principle**: position and momentum cannot be measured simultaneously (*complementary properties*)
- **Superposition**: Particles can be in many states or places at once (*wave-particle duality*)

Ψ Quantum Mechanics





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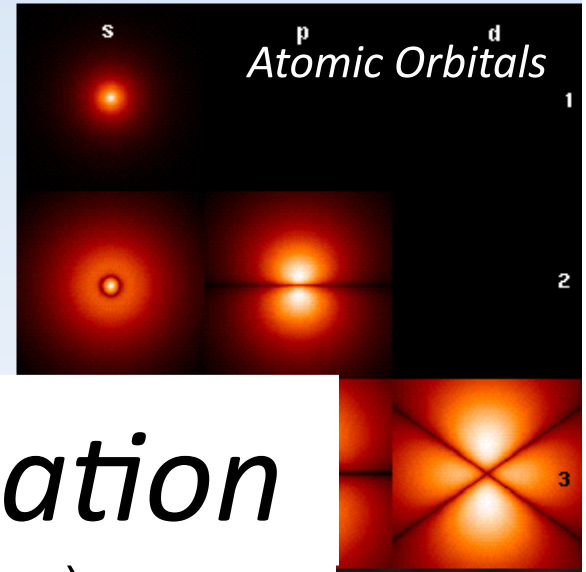


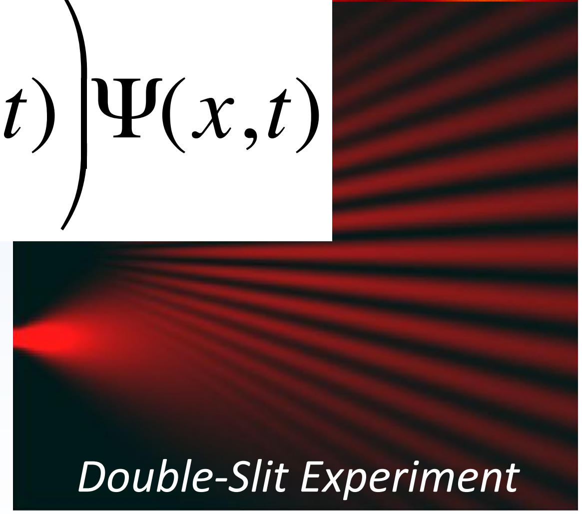
Atomic Orbitals

Schrödinger's Equation

$$i\hbar\partial_t\Psi(x,t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(x,t)\right)\Psi(x,t)$$

properties)

- **Superposition**: Particles can be in many states or places at once (*wave-particle duality*)



Double-Slit Experiment

Outline

- Superconducting Quantum Circuits
  - LC oscillators + Qubits
- Entangled Qubits
  - through Capacitors + Resonators
- Entangled Resonators
- Resonator Networks,
  Quantum Machines and Beyond

Timeline of Superconducting Quantum Circuits

1911
Kamerlingh Onnes
discovers
superconducity

1935
London brothers
propose
two-fluid model
using QM

1957
Bardeen, Cooper,
and Schrieffer
complete full QM
theory

1960s
Josephson and
Anderson study
coupled
superconductors

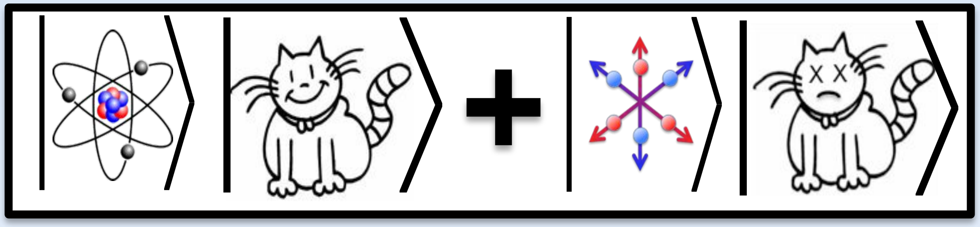
1980
Tony Leggett
proposes
macroscopic
quantum
effects

1982
Feynman
considers
quantum
computers

1985
David Deutsch
studies
quantum
algorithms
&
John Martinis
observes
quantized
energy levels
in Josephson
junction

Macroscopic Superpositions?

Tony Leggett

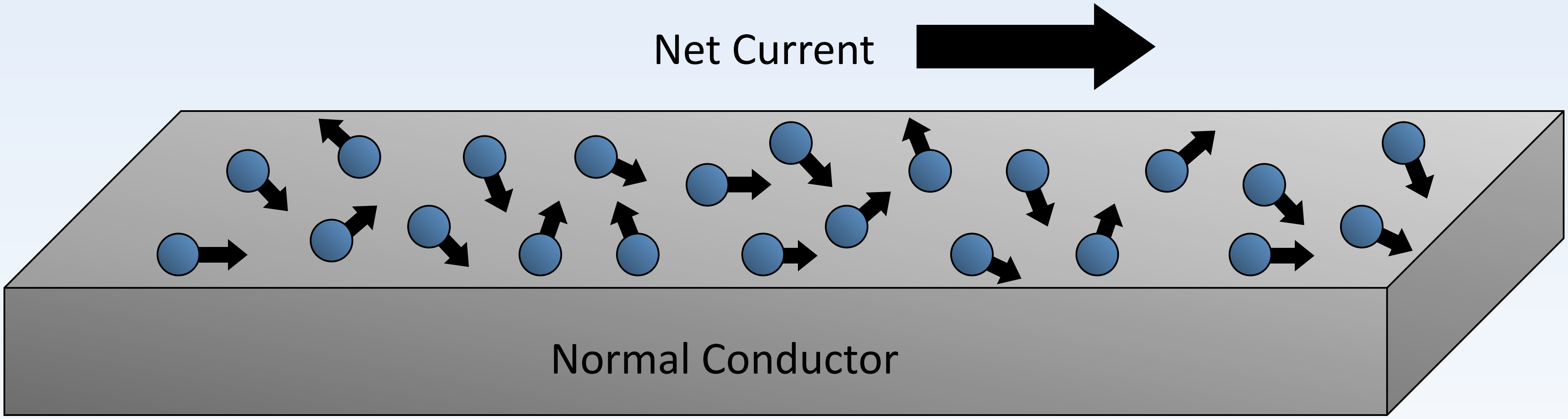


- "Is there actually any evidence that macroscopic systems can … be in quantum states which are linear superpositions of states with different macroscopic properties?"
- If the answer is **NO**, perhaps QM is simply not valid at the macroscopic level and some new law of nature prevents Schrödinger's Cat from ever getting out of the bag!
- He encouraged a study of certain quantum effects in superconductors

Prog. Theor. Phys. Suppl. **69** 80 (1980)
J. Phys.: Condens. Matter 14 R415 (2002)

Superconductivity

- In a normal conductor (at large temperatures), each electron moves independently.
- In a superconductor, the electrons form Cooper pairs; each center-of-mass moves with the same velocity.



Low energy excitations (dominant at low temperatures) involve this net motion of the Cooper pairs, all acting as one large artificial atom!

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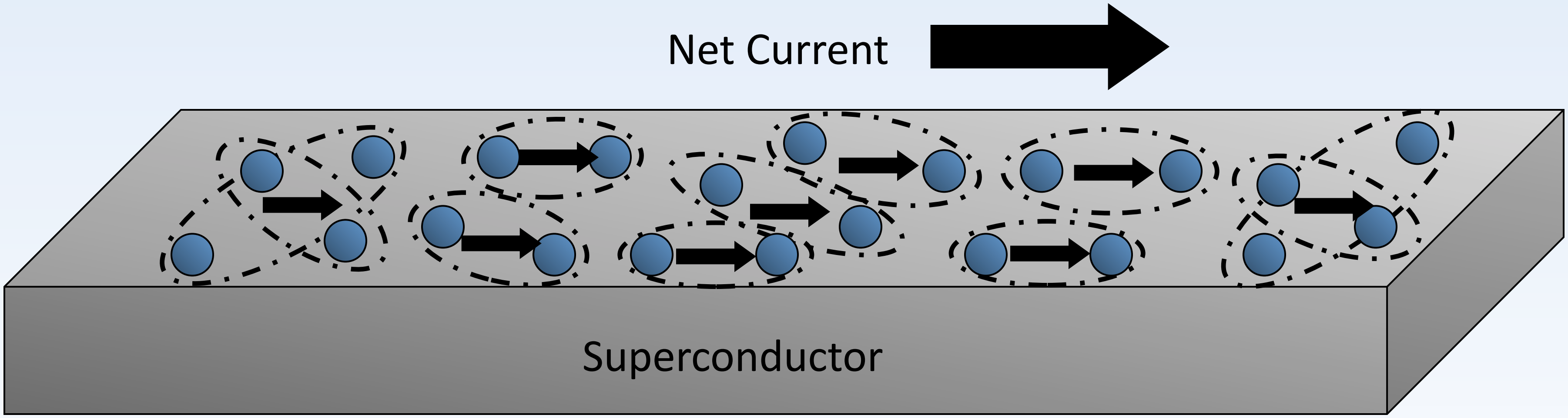


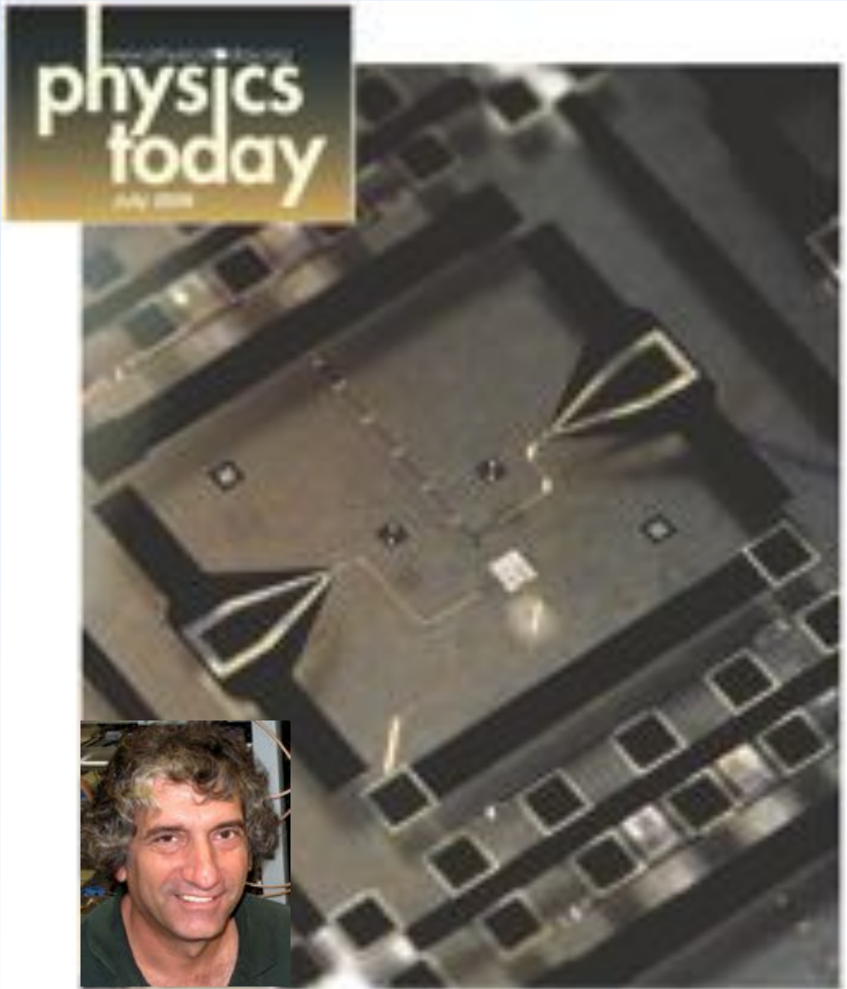
Low energy excitations (dominant at low temperatures) involve this net motion of the Cooper pairs, all acting as one large artificial atom!

Superconducting QCs

Potential for 10000s of quantum bits on a single microchip.

Ray Simmonds, NIST

Schoelkopf Lab

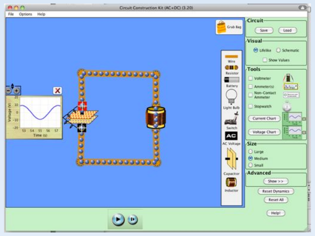


Martinis Group,
UC Santa Barbara

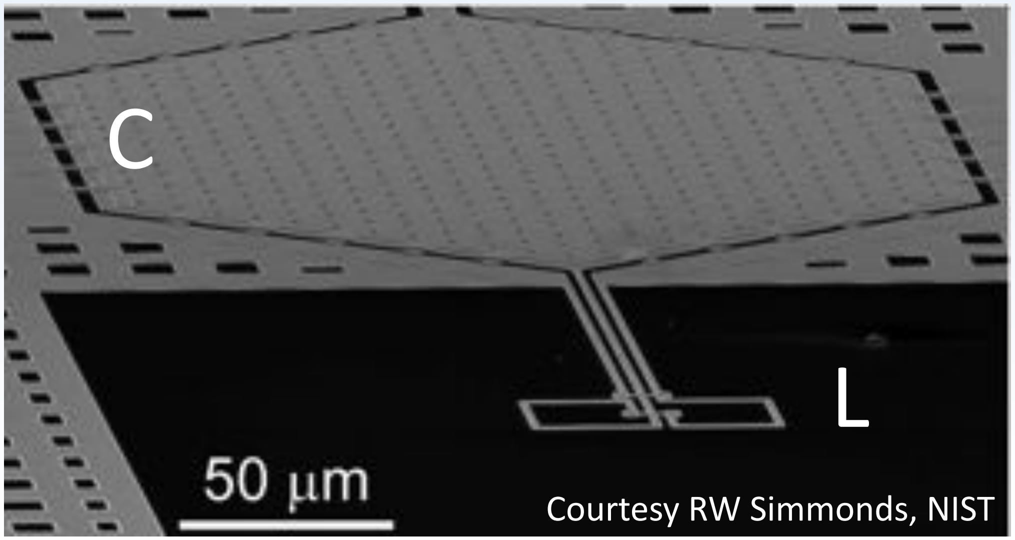
LC Oscillators

- Oscillating electric circuit
  - Moving charges ~ Kinetic energy
  - Stored electric fields ~ Potential energy
  - Energy oscillates at well-defined frequency (*simple harmonic oscillator*)

Simulation

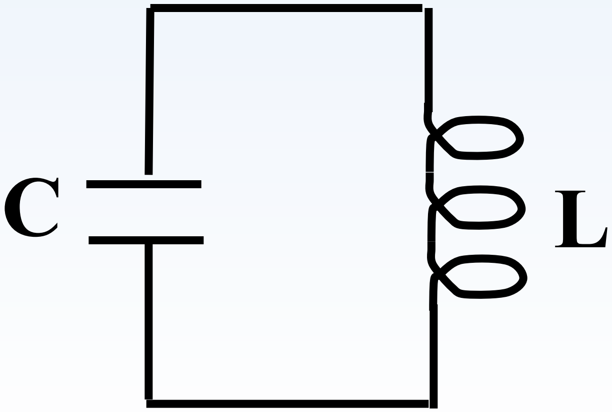


Superconducting LC

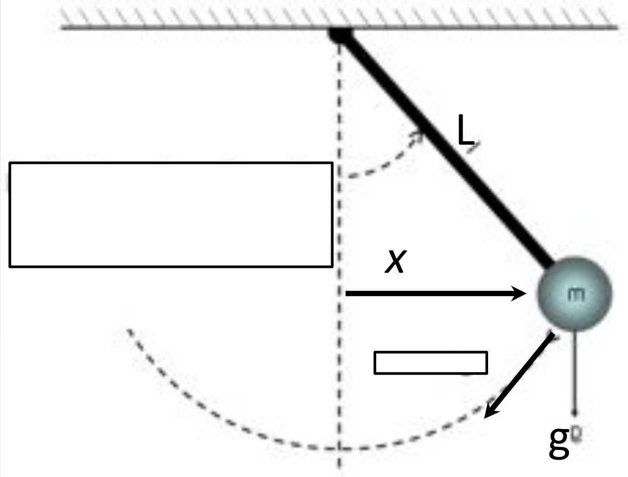


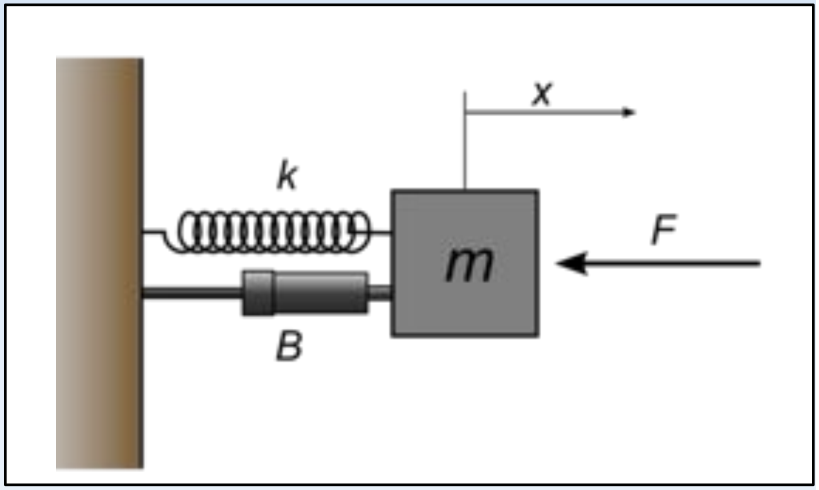
Courtesy RW Simmonds, NIST

Circuit Diagram



Other “Simple” Oscillators

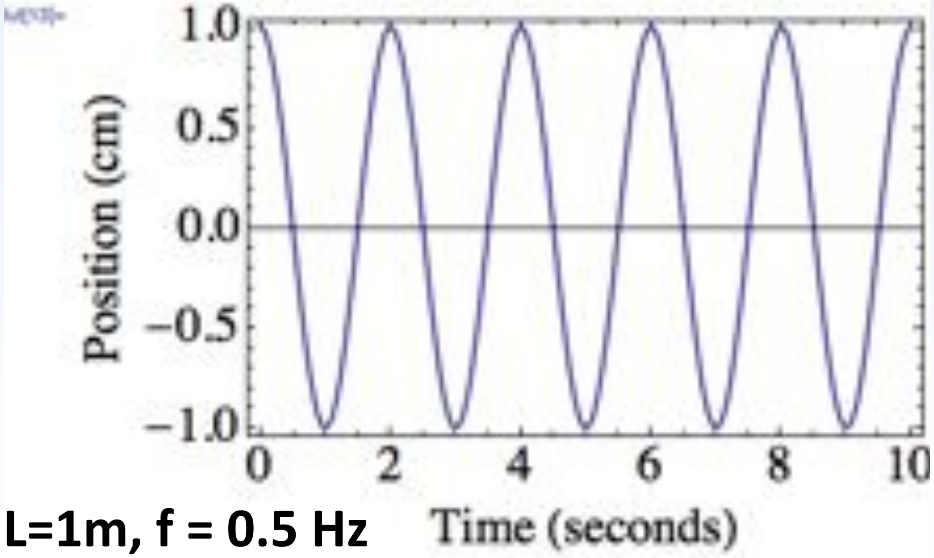




$x = A \cos(2 \pi f t)$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$



$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

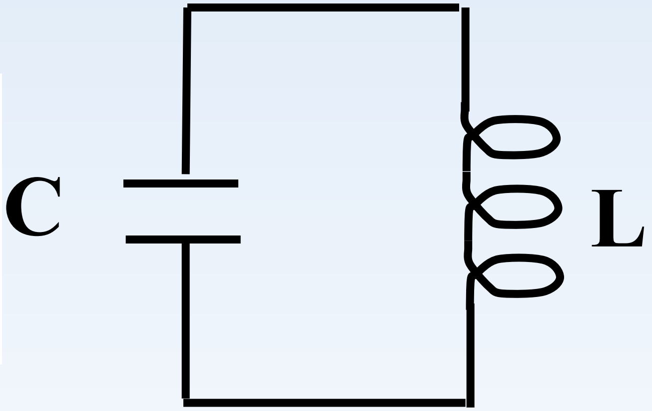
L=1m, f = 0.5 Hz

Ψ Quantum LC Oscillator

- Superconducting LC Oscillator described by a *wavefunction* for the total Cooper pair current $I=\Phi/L$: $\Psi(\Phi)$

$$E = \frac{1}{2C}Q^2 + \frac{1}{2}LI^2$$

$$\Leftrightarrow H = \frac{1}{2C}p_\Phi{}^2 + \frac{1}{2L}\Phi^2 \approx \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 x^2$$



- This has equally spaced quantized energy levels.

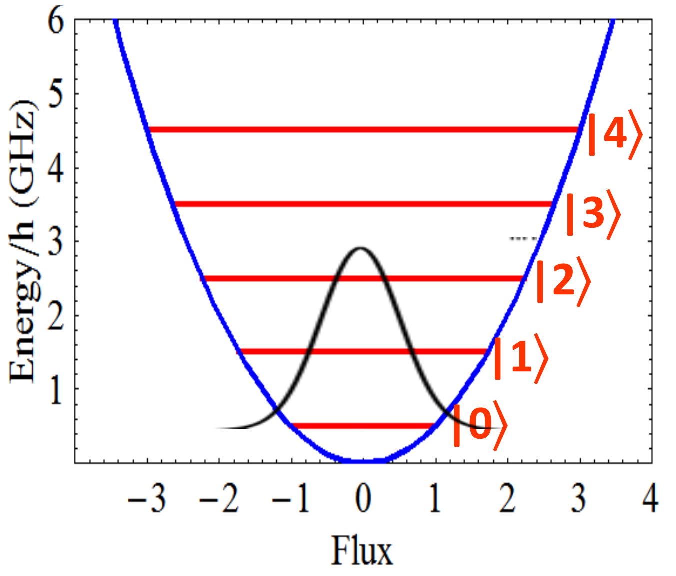
$$p \to -i\hbar\frac{d}{dx} \Rightarrow p_\Phi \to -i\hbar\frac{d}{d\Phi}$$

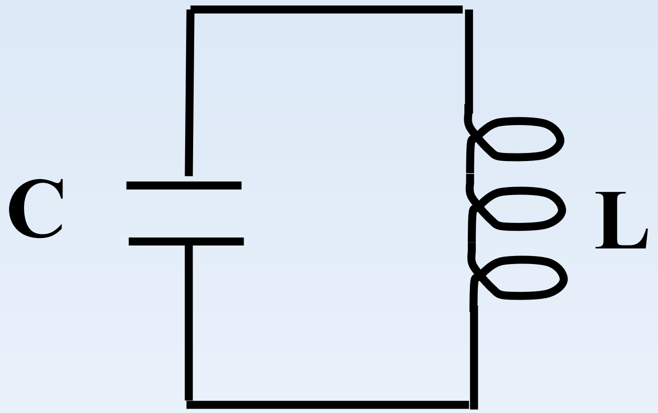
$$-\frac{\hbar^2}{2C}\frac{d^2\Psi}{d\Phi^2} + \frac{1}{2L}\Phi^2\Psi = E\Psi$$

Ψ Quantum LC Oscillator

$$-\frac{\hbar^2}{2C}\frac{d^2\Psi}{d\Phi^2}+\frac{1}{2L}\Phi^2\Psi=E\Psi$$

$$E_n=\hbar\omega_0\left(n+\tfrac{1}{2}\right),\ \omega_0=1/\sqrt{LC}$$





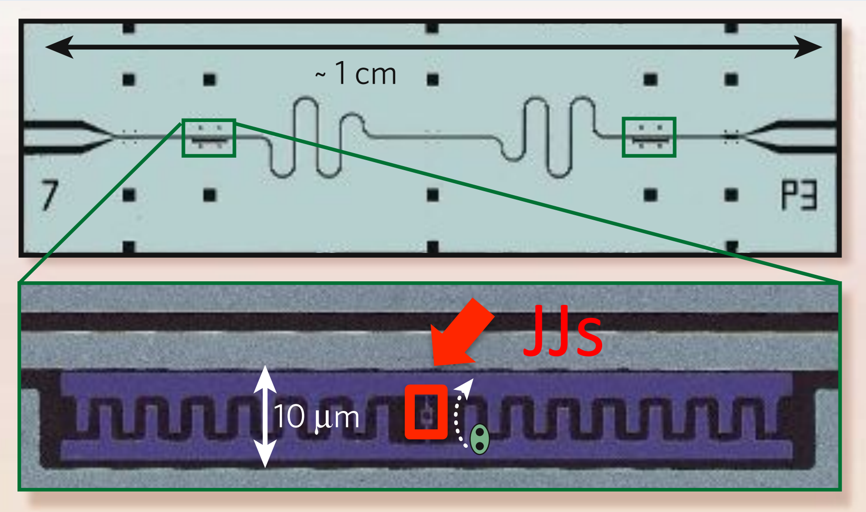
- This circuit allows the Cooper pair current to exhibit superposition!
- But there is a problem!
- ***Transitions*** between energy levels involve ***energy level differences***, which are ***constant*** for a harmonic oscillator.
- ***There is no way to excite individual energy levels*** (e.g. |0⟩ ↔ |1⟩) .

Superconducting Oscillators

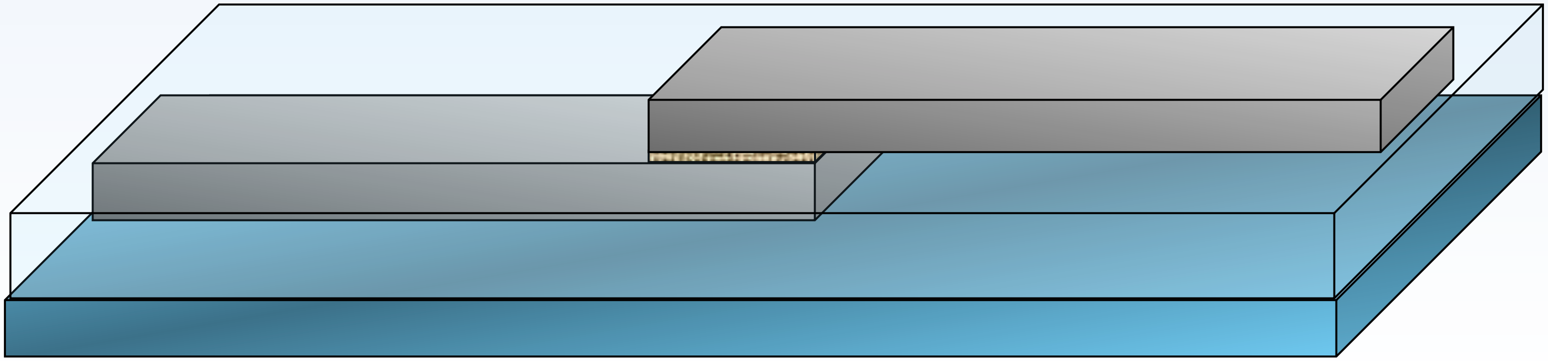
NIST Boulder

Josephson Junction

Yale



- Josephson Junctions change how electrons move (tunnel through barrier), giving a tunable inductor.



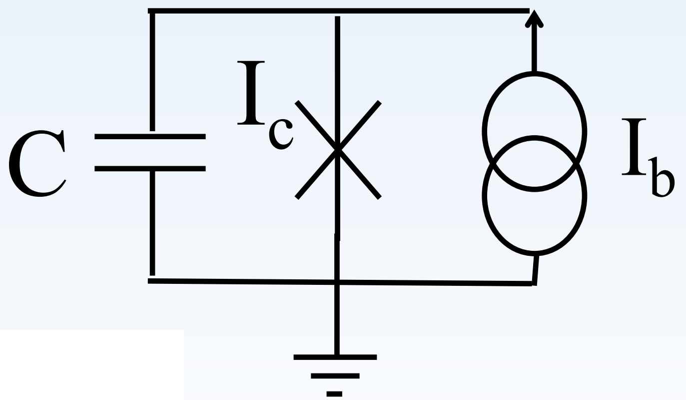
Josephson Junction

- Superconducting, tunable, anharmonic oscillator formed by an intrinsic capacitance, ***nonlinear inductor*** (energy stored through the ***phases*** of Cooper pairs).
- Circuit described by a wavefunction for the total Cooper pair current, given through the phase difference γ: $I = I_c \sin\gamma : \Psi(\gamma)$

$$E = \frac{1}{2C}Q^2 - \frac{\Phi_0}{2\pi}\left(I_c\cos\gamma + I_b\gamma\right), \quad \Phi_0 = \frac{h}{2e}$$

$$\Leftrightarrow H = \frac{1}{2C\left(\Phi_0/2\pi\right)^2}p_\gamma^2 - \frac{\Phi_0}{2\pi}\left(I_c\cos\gamma + I_b\gamma\right)$$

$$-\frac{\hbar^2}{2C\left(\Phi_0/2\pi\right)^2}\frac{d^2\Psi}{d\gamma^2} - \frac{\Phi_0}{2\pi}\boxed{\left(I_c\cos\gamma + I_b\right)}\Psi = E\Psi$$

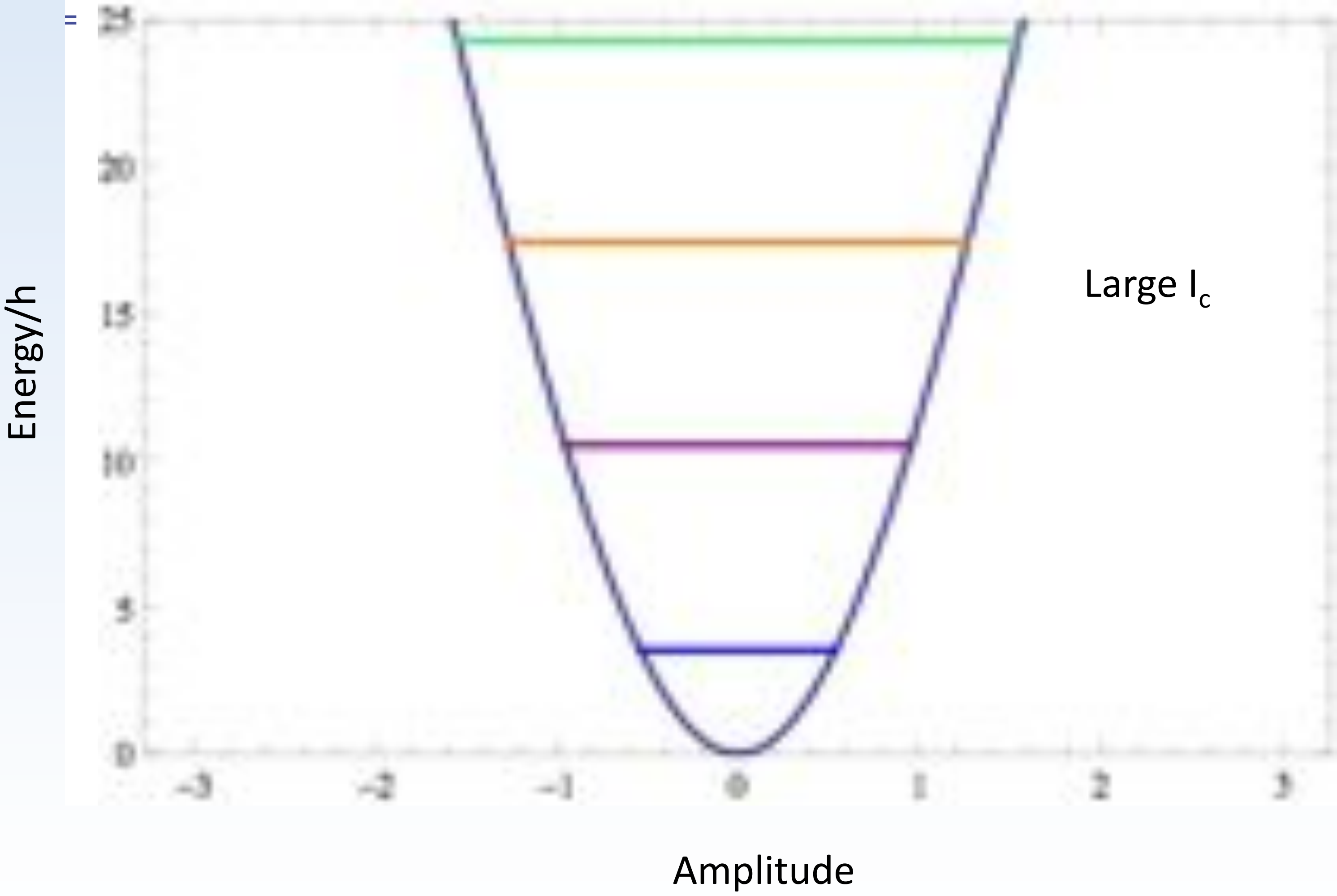


*Adjust **both I_c and I_b***

Tunable Quantum Oscillator

Adjust I_c* (high to low) *with I_b=0

$$-\frac{\hbar^2}{2C(\Phi_0/2\pi)^2}\frac{d^2\Psi}{d\gamma^2}-\frac{\Phi_0}{2\pi}(I_c\cos\gamma+I_b)\Psi=E\Psi$$

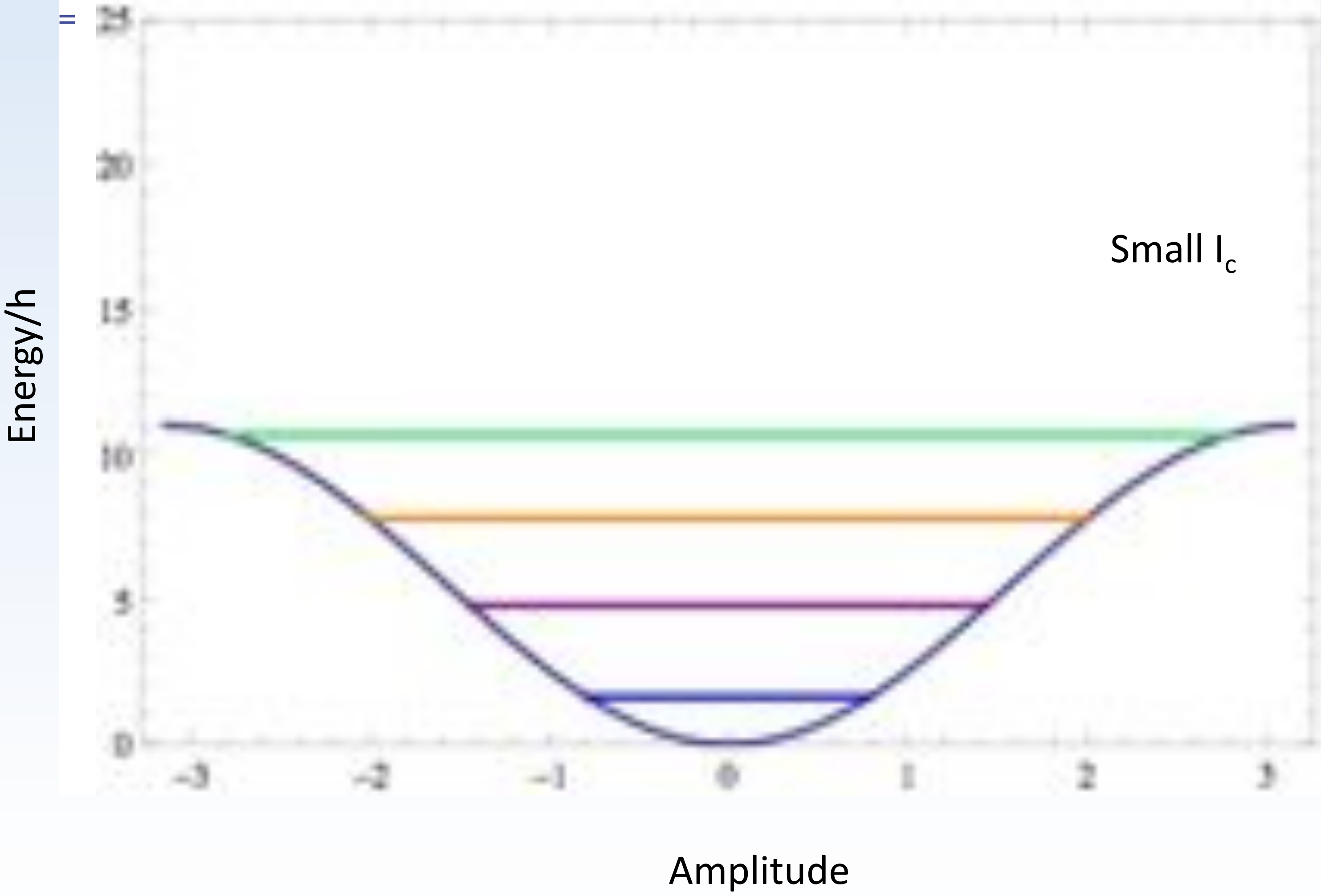


Large I_c

Tunable Quantum Oscillator

Adjust I_C (high to low) _with_ $I_b=0$

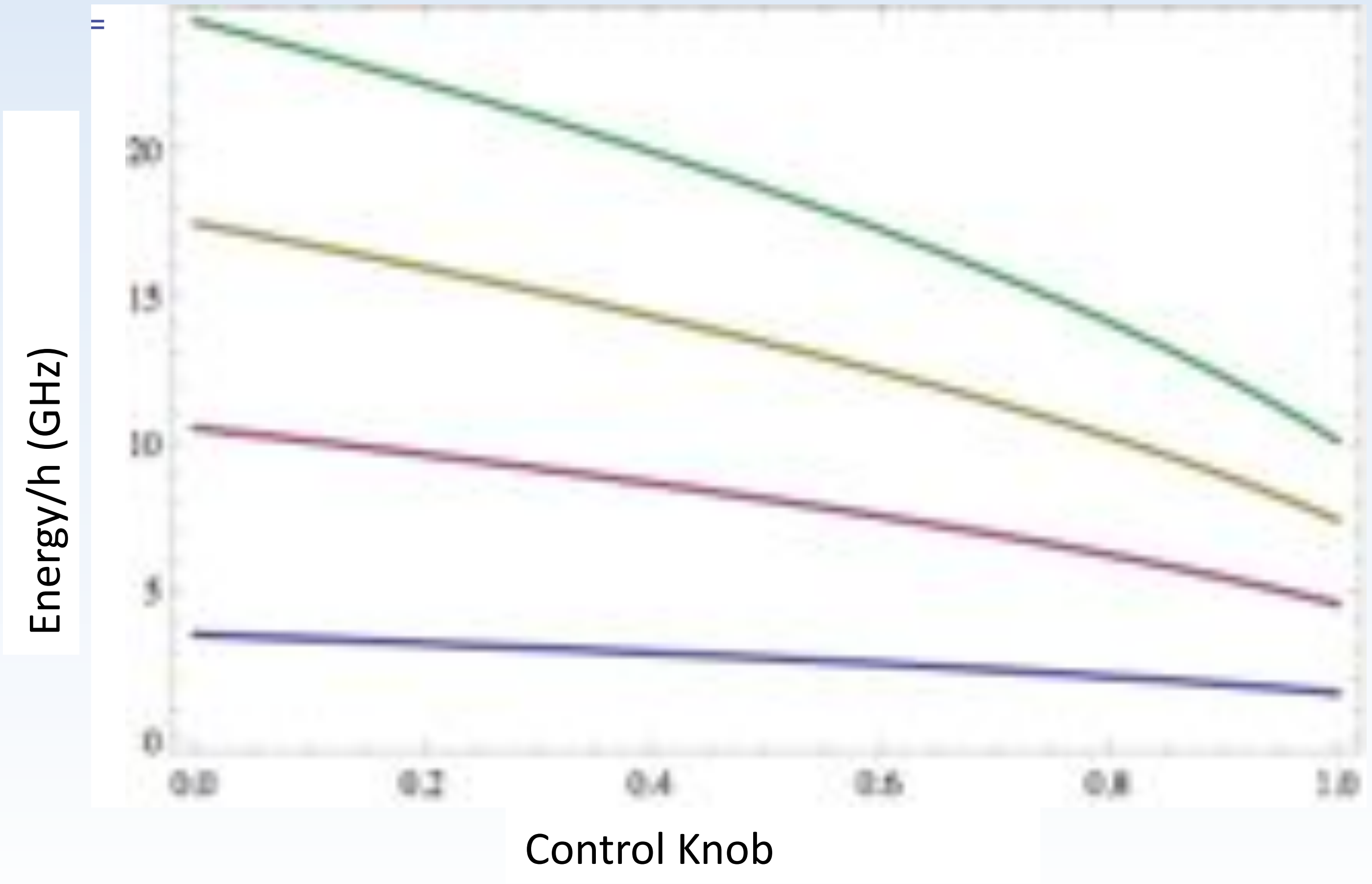
$$-\frac{\hbar^2}{2C(\Phi_0/2\pi)^2}\frac{d^2\Psi}{d\gamma^2}-\frac{\Phi_0}{2\pi}(I_c\cos\gamma+I_b)\Psi=E\Psi$$



Tunable Quantum Oscillator

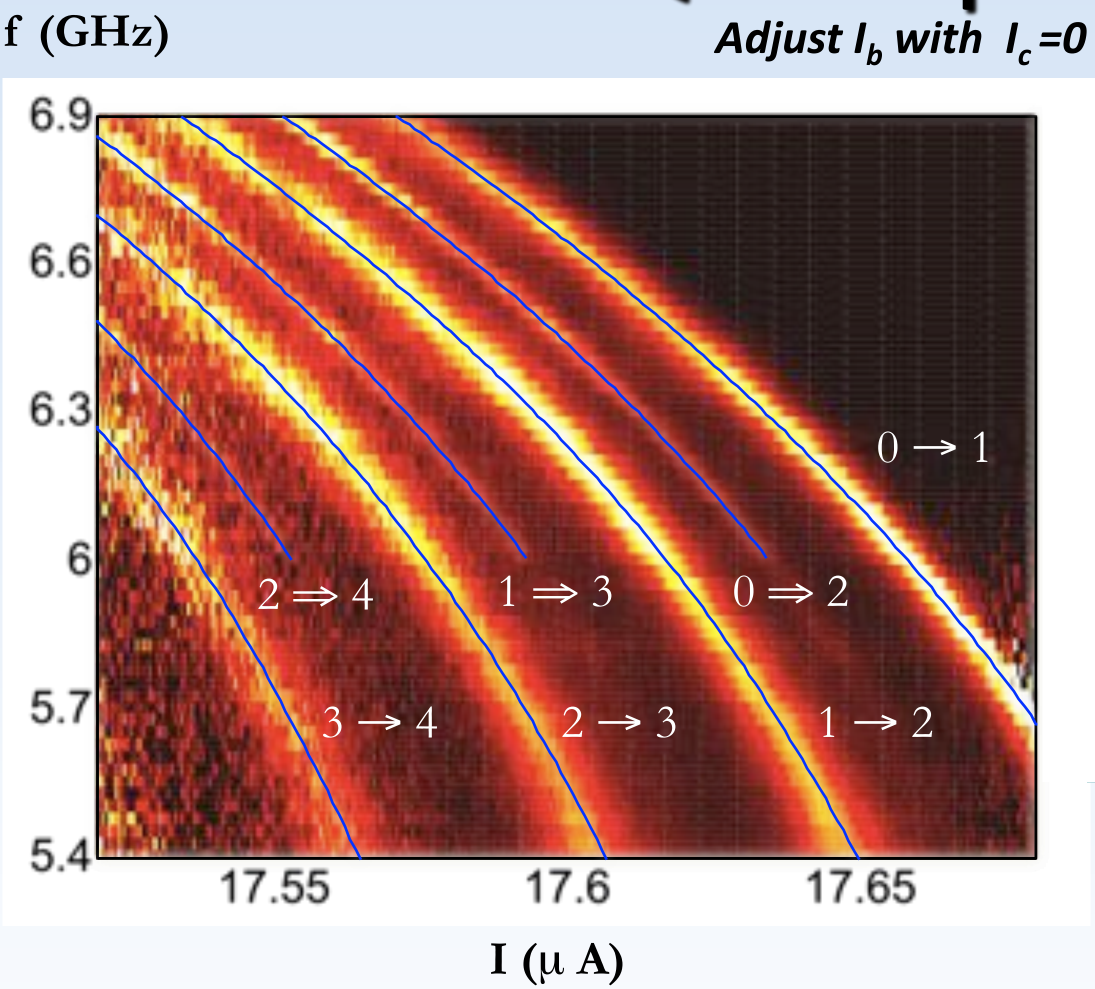
Adjust I_c **(high to low)** ***with $I_b=0$***

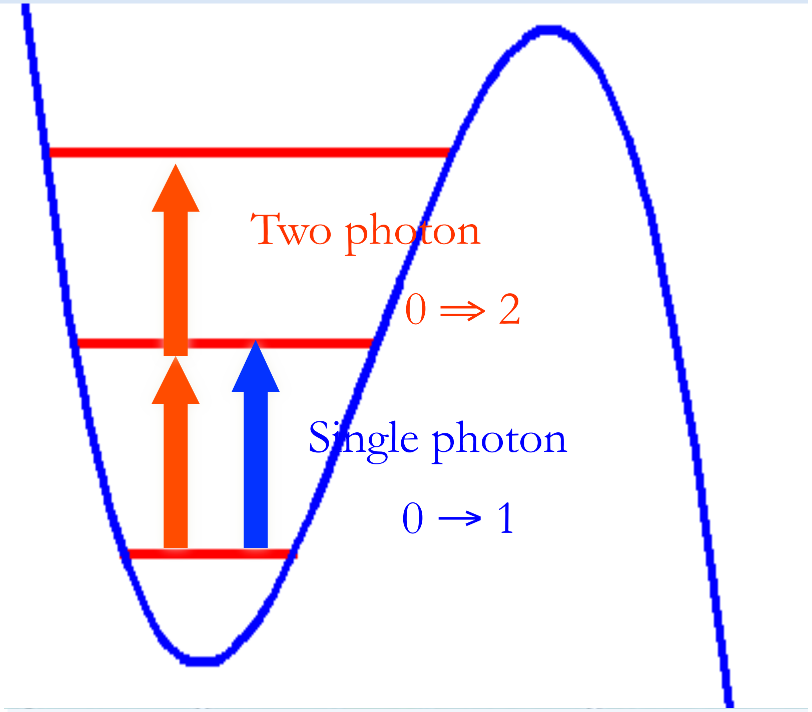
$$-\frac{\hbar^2}{2C\left(\Phi_0/2\pi\right)^2}\frac{d^2\Psi}{d\gamma^2}-\frac{\Phi_0}{2\pi}\left(I_c\cos\gamma+I_b\right)\Psi=E\Psi$$



Phase Qubit Spectroscopy

Adjust I_b with I_c=0





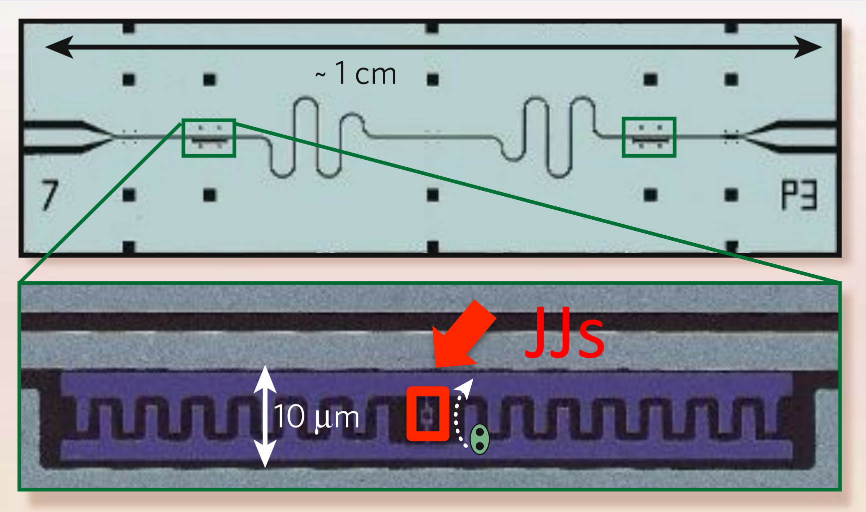
Sudeep Dutta et al. (Univ. Maryland)

Each microwave transition is an excitation of the junction with an increased tunneling rate. Bright indicates a large number of tunneling events, dark a small number of events.

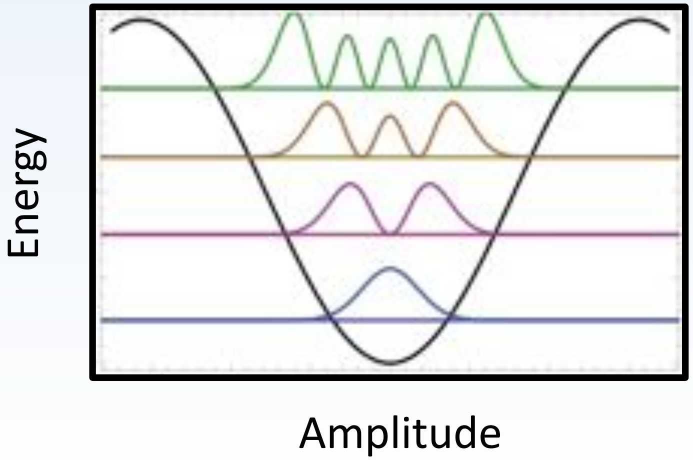
Superconducting Oscillators

NIST Boulder

Yale



- Key Properties:
  - – Oscillate like a pendulum
  - – Quantized Energy Levels:
  - – Tunable by external circuits!
  - – “Artificial Atoms”



Outline

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- **Entangled Qubits**
  - **through Capacitors + Resonators**
- Entangled Resonators
- Resonator Networks,
  Quantum Machines and Beyond

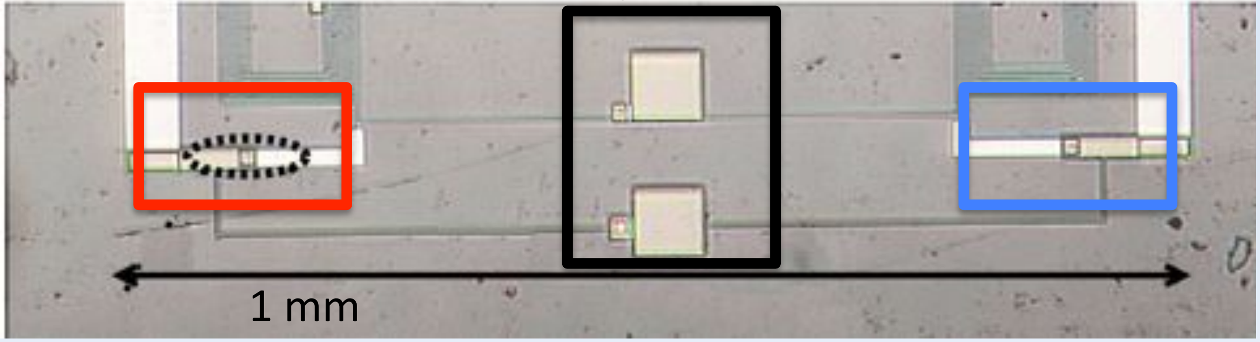
Coupled Phase Qubits

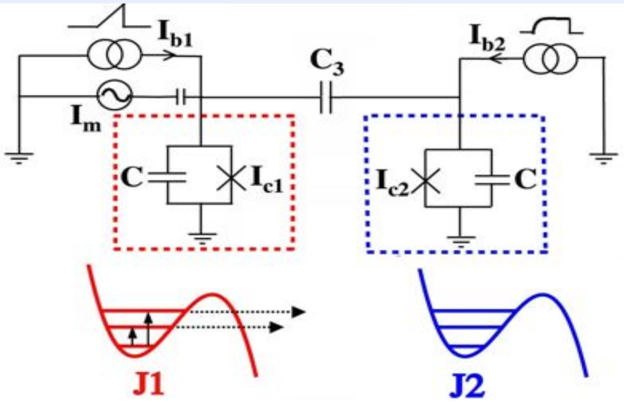
2001-2003

Junction 1

Coupling Capacitors

Junction 2





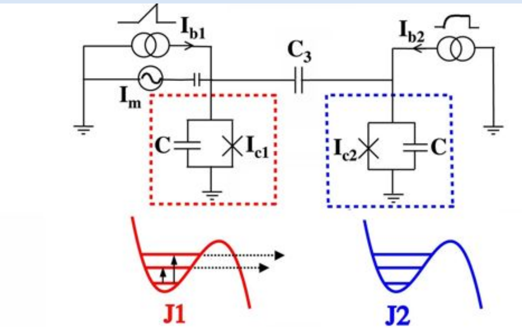
$$m = C\left(\Phi_0 / 2\pi\right)^2$$

$$\boxed{\xi = C_3/(C + C_3)}$$

$$H = \frac{1}{2m}(1+\xi)^{-1}(p_1{}^2 + p_2{}^2 + 2\xi p_1 p_2) - \frac{\Phi_0}{2\pi}(I_{c1}\cos\gamma_1 + I_1\gamma_1 + I_{c2}\cos\gamma_2 + I_2\gamma_2)$$

Coupled Phase Qubits: Theory

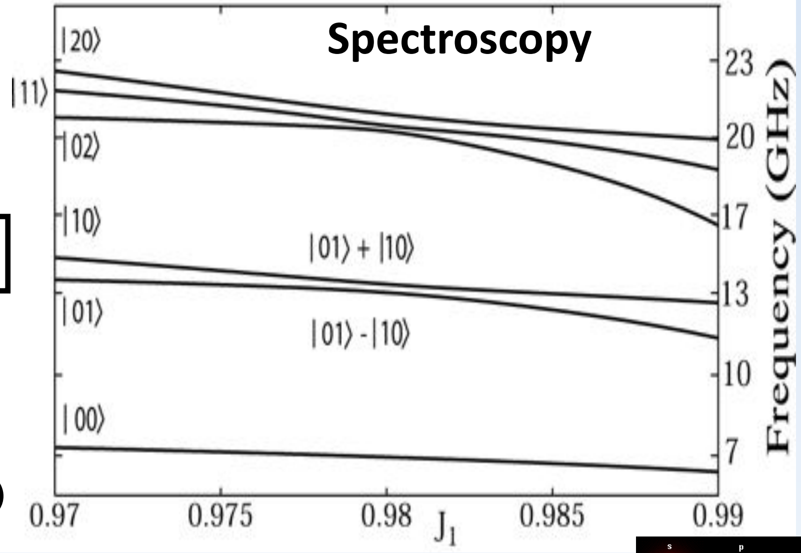
P. R. Johnson, F. W. Strauch *et al., Physical Review B* **67,** 020502(R) (2003).

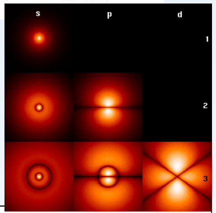


$$m = C(\Phi_0/2\pi)^2$$

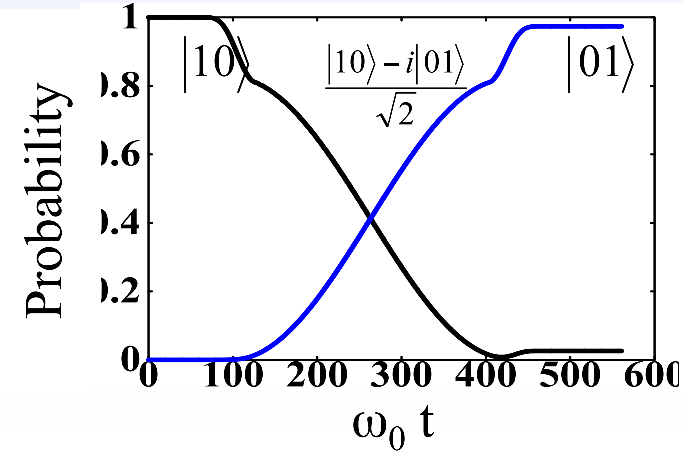
$$\boxed{\zeta = C_3/(C+C_3)}$$

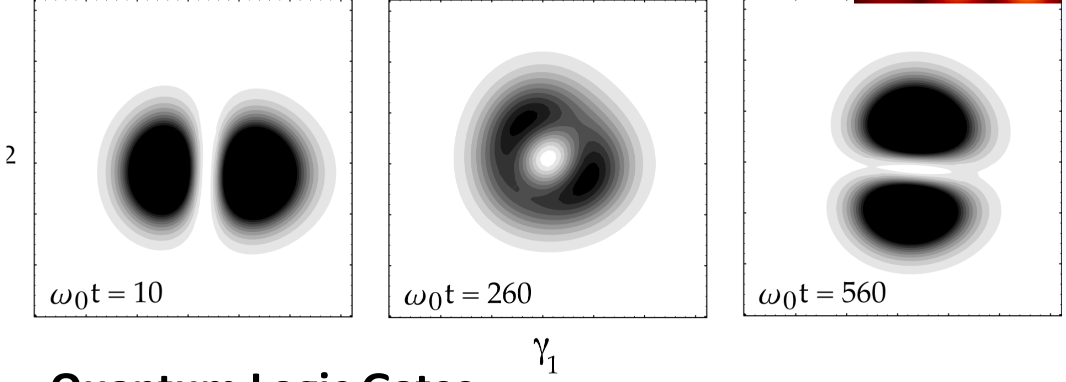
$$H = \frac{1}{2m}(1+\zeta)^{-1}(p_1^2 + p_2^2 + 2\zeta p_1 p_2) - \frac{\Phi_0}{2\pi}(I_{c1}\cos\gamma_1 + I_1\gamma_1 + I_{c2}\cos\gamma_2 + I_2\gamma_2)$$





F. W. Strauch *et al., Physical Review Letters* **91,** 167005 (2003).

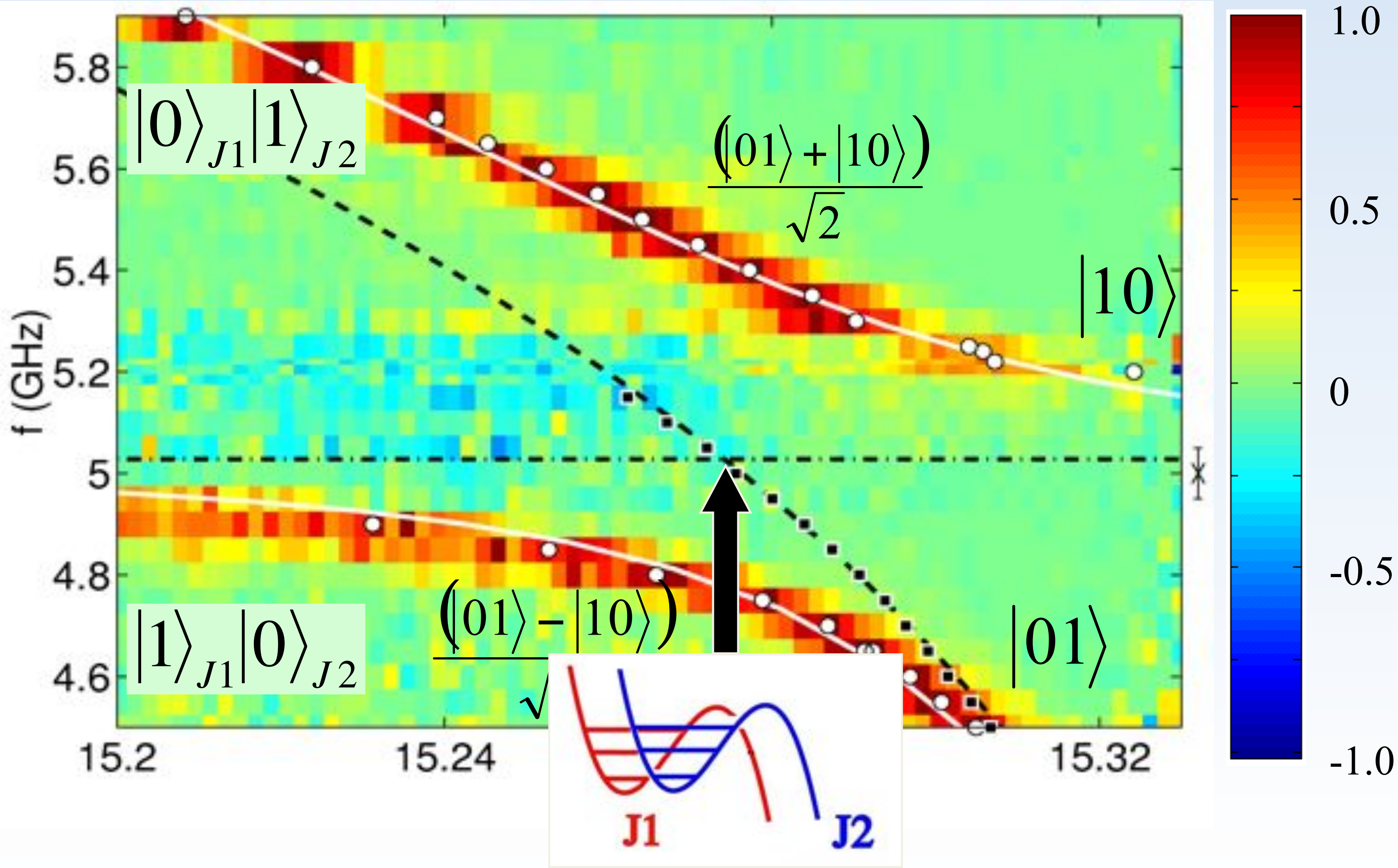




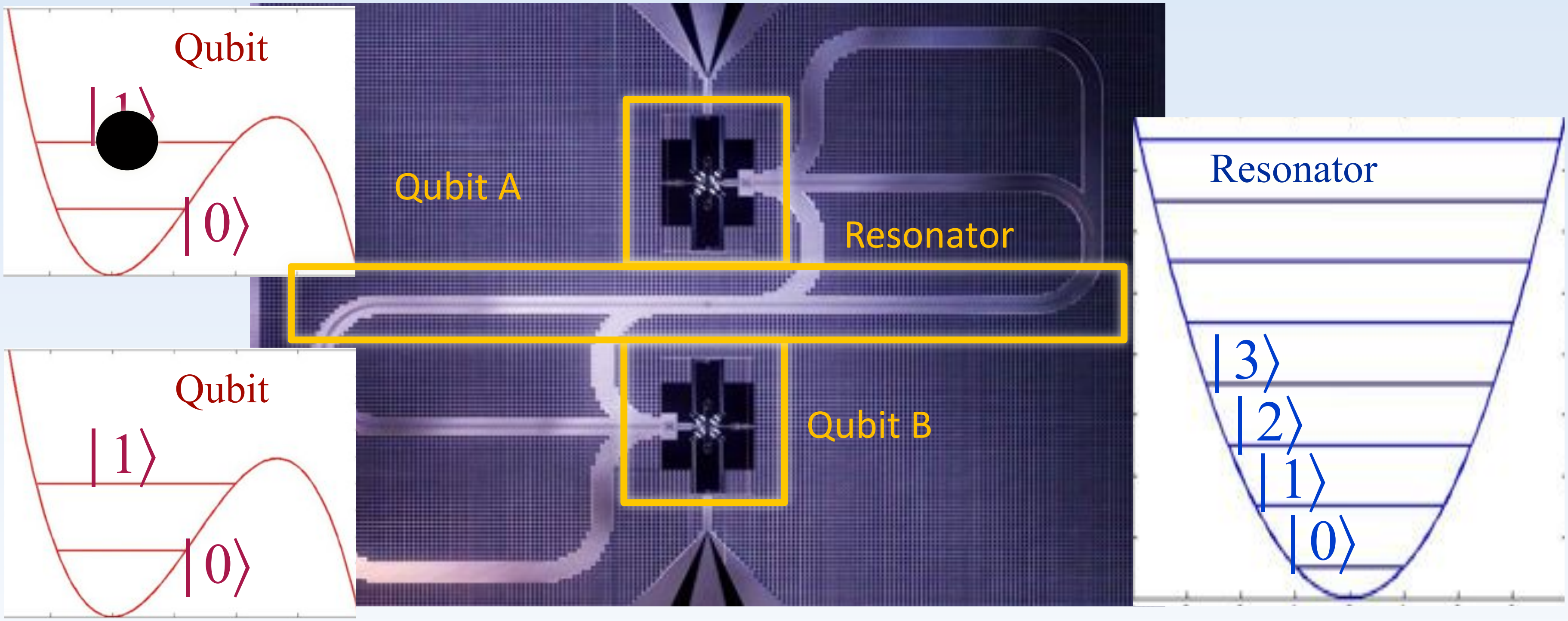
Quantum Logic Gates

Coupled Phase Qubits: Experiment

A. J. Berkley, H. Xu, R. C. Ramos, M. A. Gubrud, F. W. Strauch et al., *Science*, **300**, 1548 (2003).

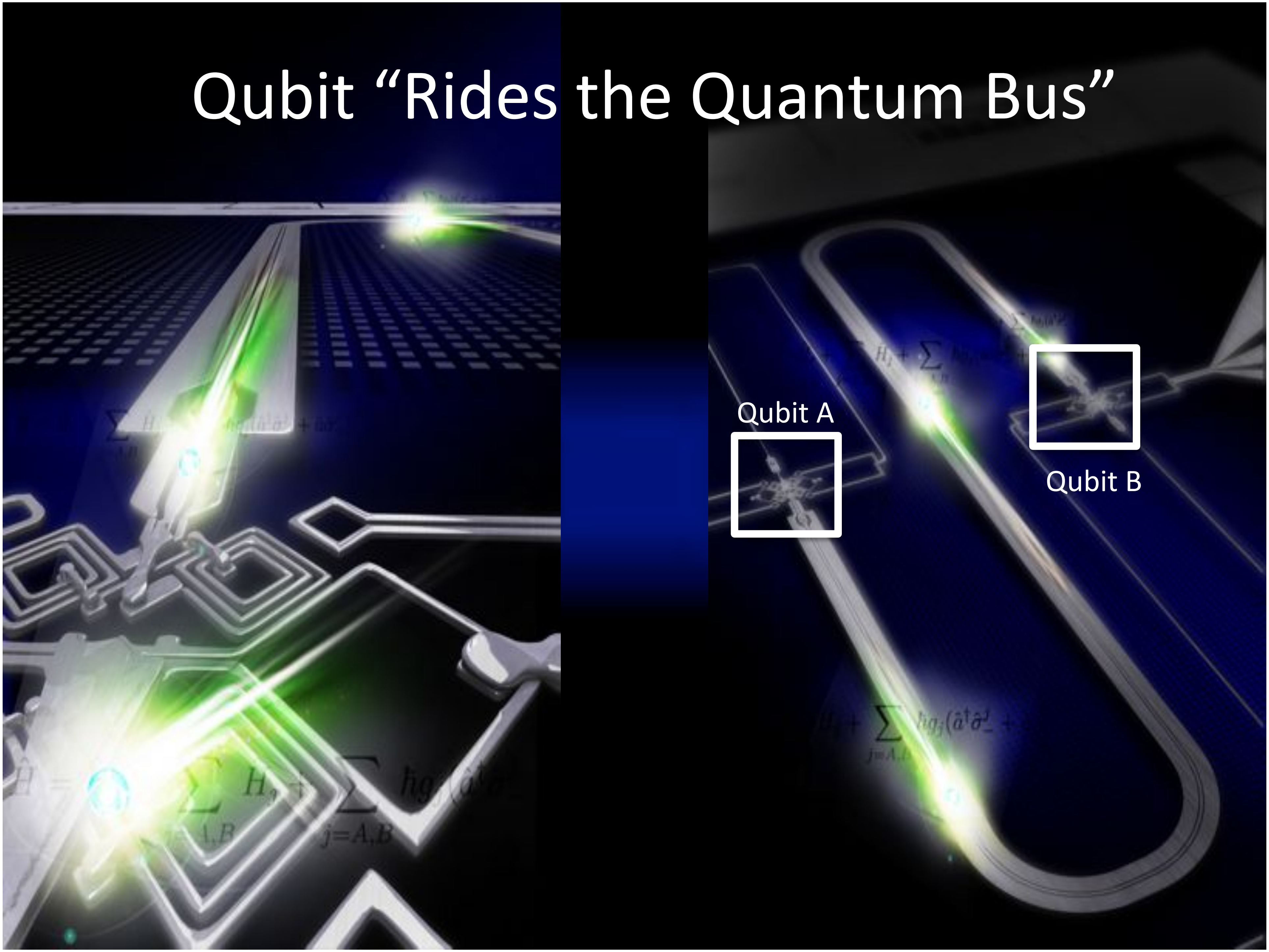


Coupling Qubits by Resonant Cavities

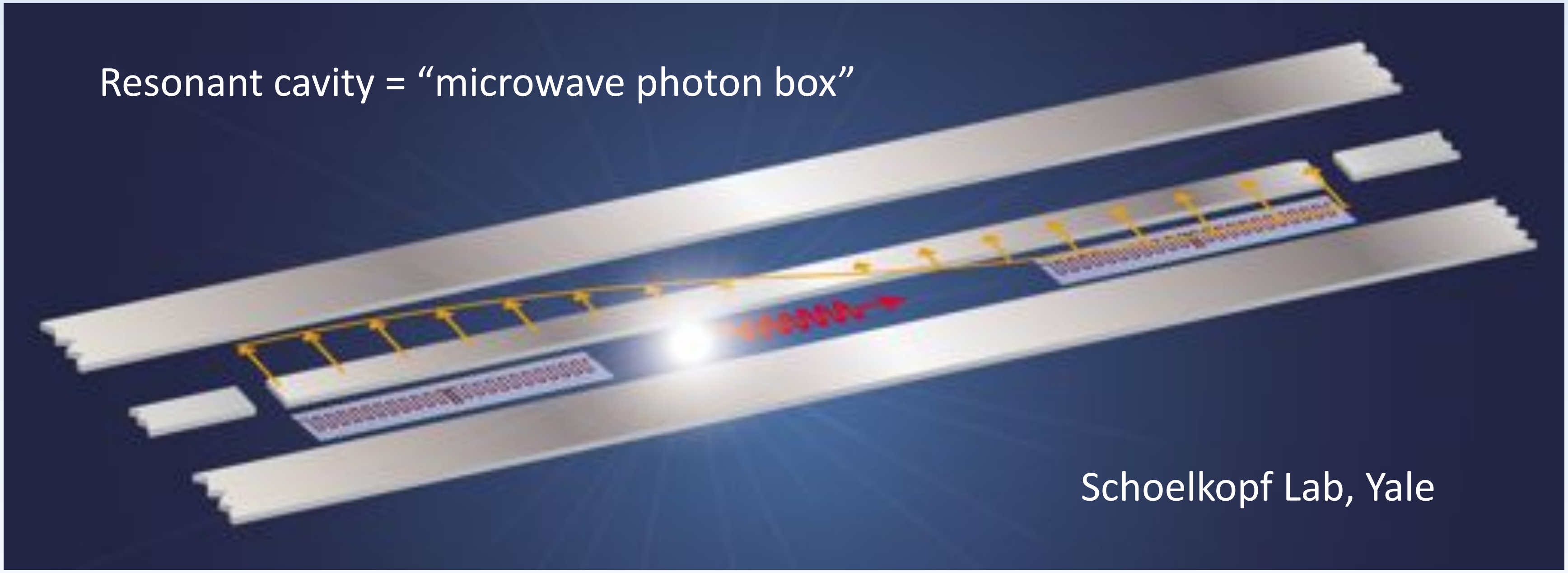


“Coherent quantum state storage and transfer between two phase qubits via a resonant cavity”, M. Sillanpaa, J. I. Park, and R. W. Simmonds, *Nature* **449**, 438 (2007)

Qubit “Rides the Quantum Bus”



Superconducting Resonator



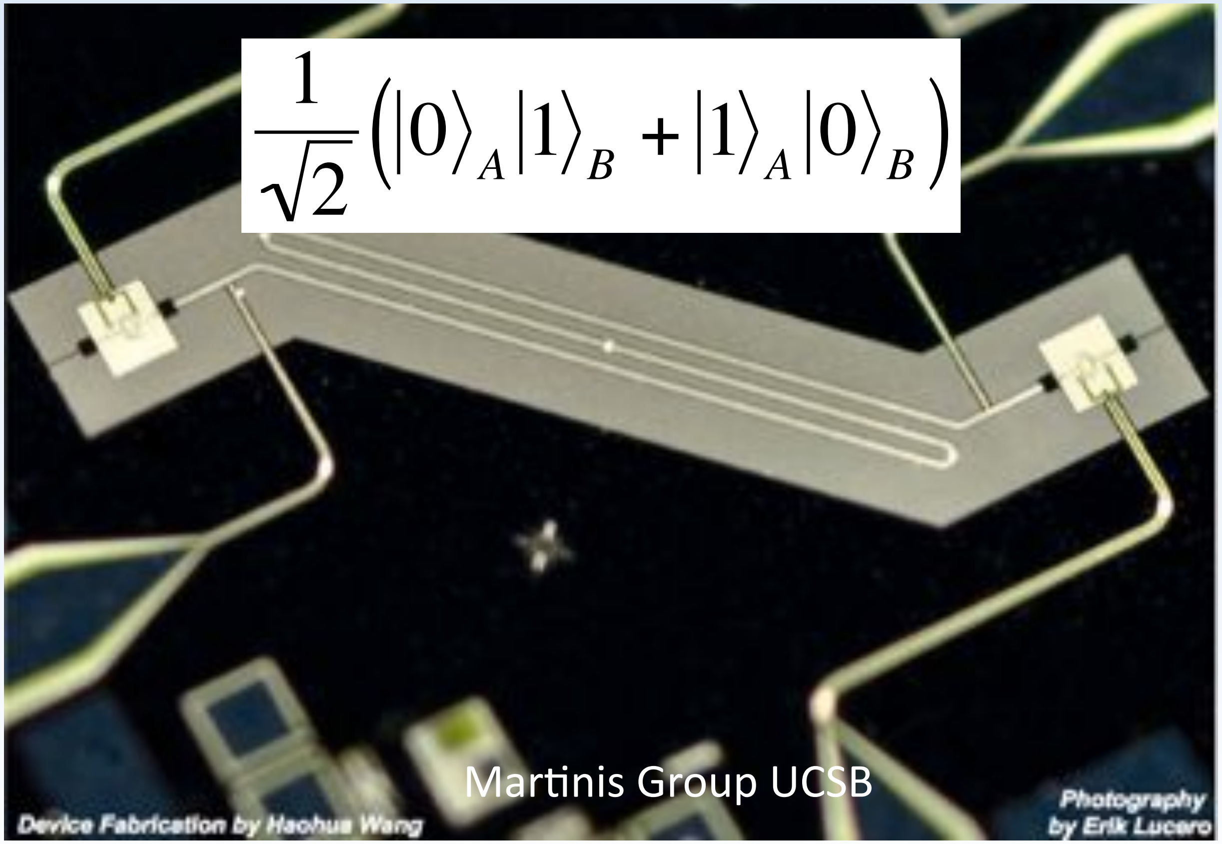
Resonant cavity = “microwave photon box”

Schoelkopf Lab, Yale

Bell Inequality Experiment

Create entangled qubits, measure *all* of their properties---violate Bell's inequality!

M. Ansmann *et al.,* "Violation of Bell's inequality in Josephson phase qubits", *Nature* **461**, 504 (2009)

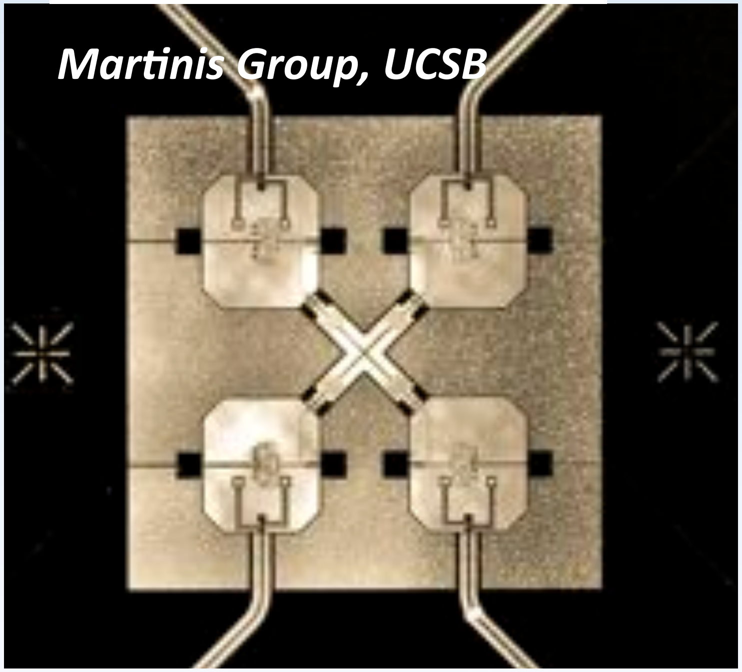


$$\frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B+|1\rangle_A|0\rangle_B\right)$$

Martinis Group UCSB

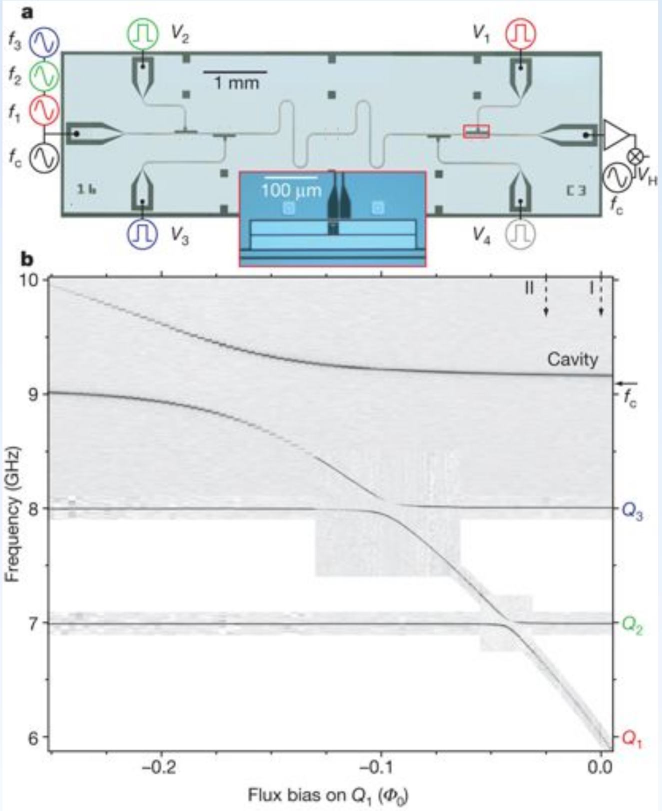
Three-Qubit Entanglement

$$\frac{1}{\sqrt{2}}\left(|000\rangle+|111\rangle\right)$$ GHZ state



Matthew Neeley *et al.*, "Generation of three-qubit entangled states using superconducting phase qubits", *Nature* **467**, 570 (2010)

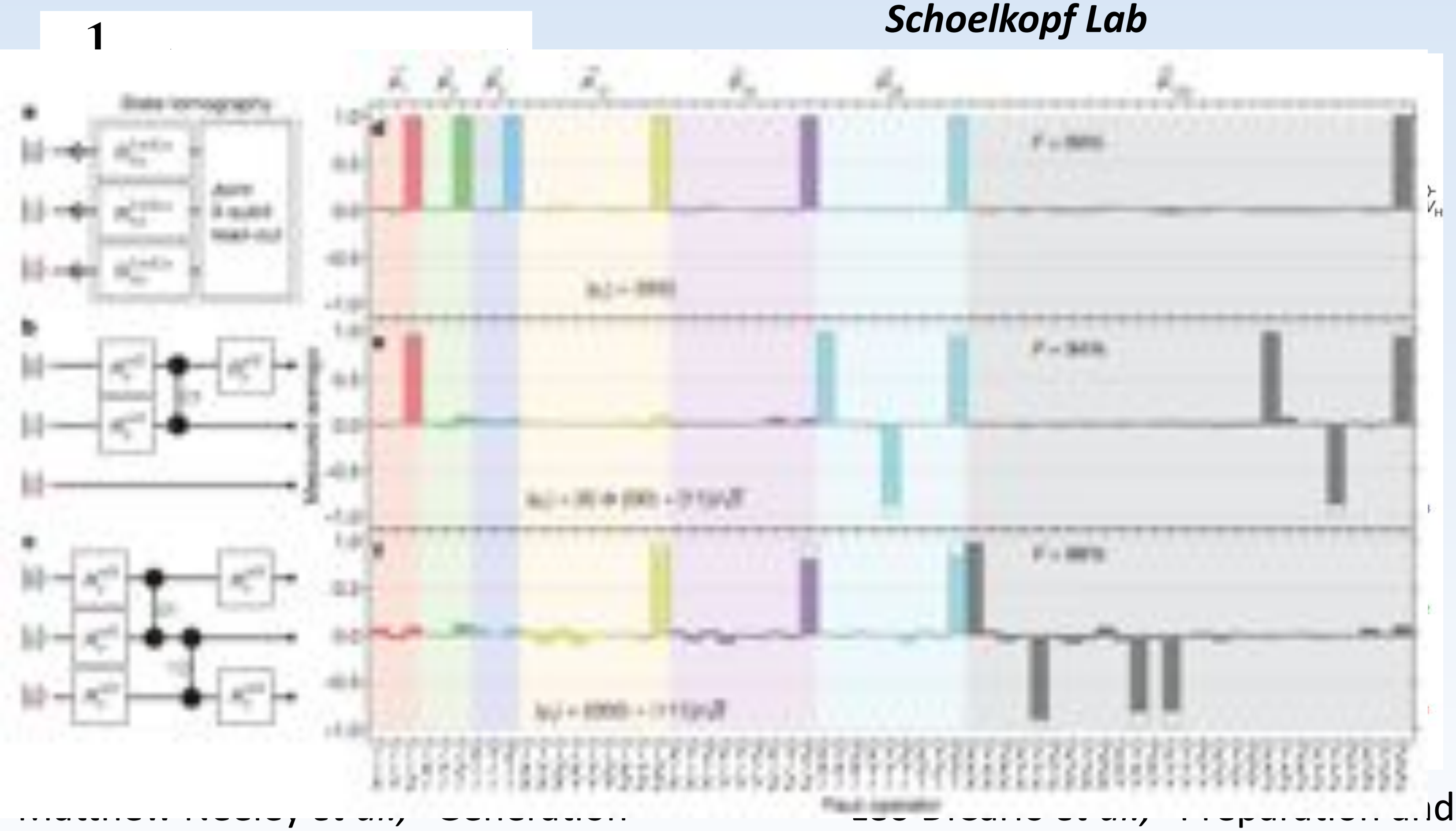
Schoelkopf Lab



Leo DiCarlo *et al.*, "Preparation and measurement of three-qubit entanglement in a superconducting circuit", *Nature* **467**, 574 (2010)

Three-Qubit Entanglement

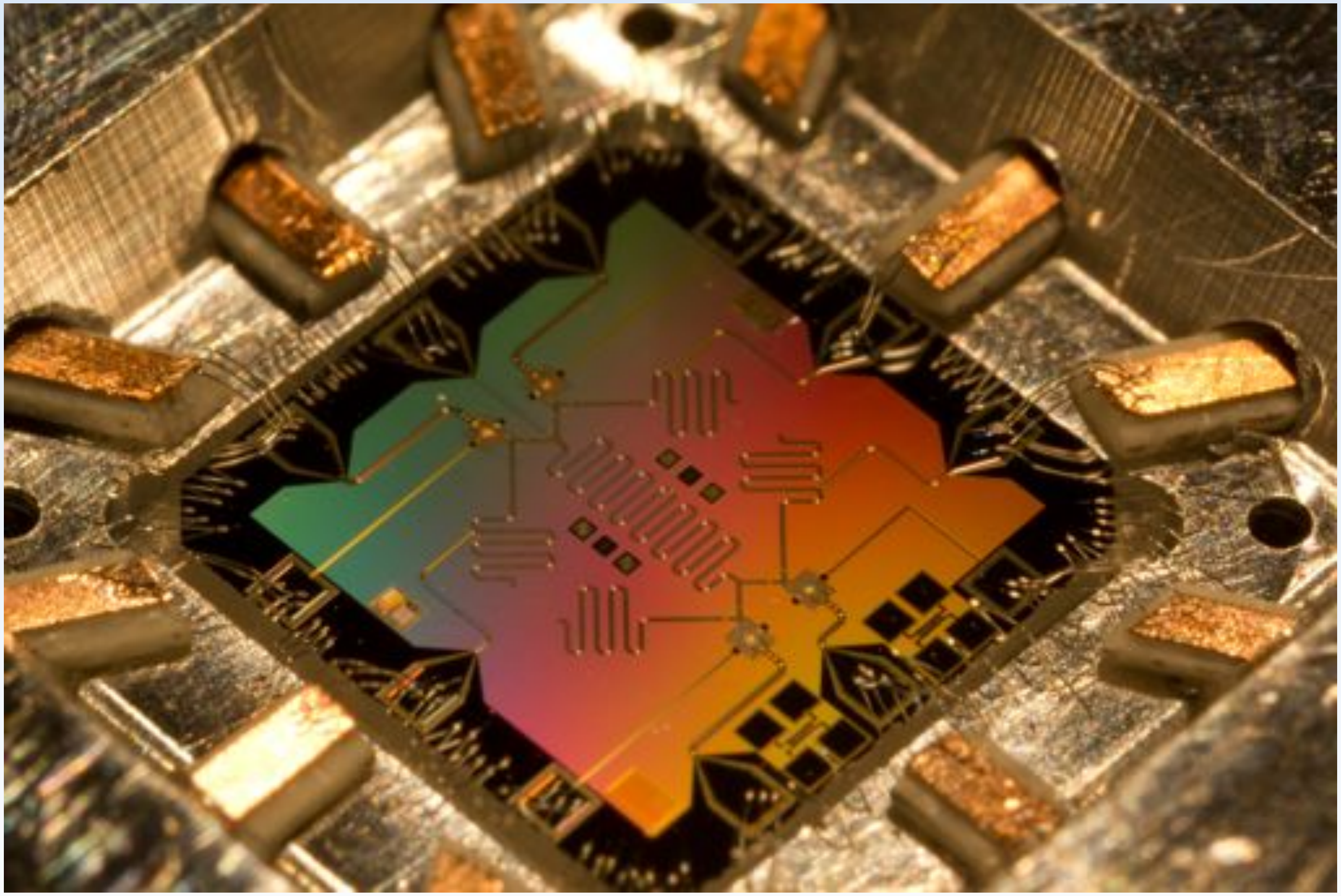
Schoelkopf Lab



of three-qubit entangled states using superconducting phase qubits", *Nature* **467**, 570 (2010)

measurement of three-qubit entanglement in a superconducting circuit", *Nature* **467**, 574 (2010)

Four Qubits + Five Resonators



Erik Lucero @ Martinis Group UCSB

Outline

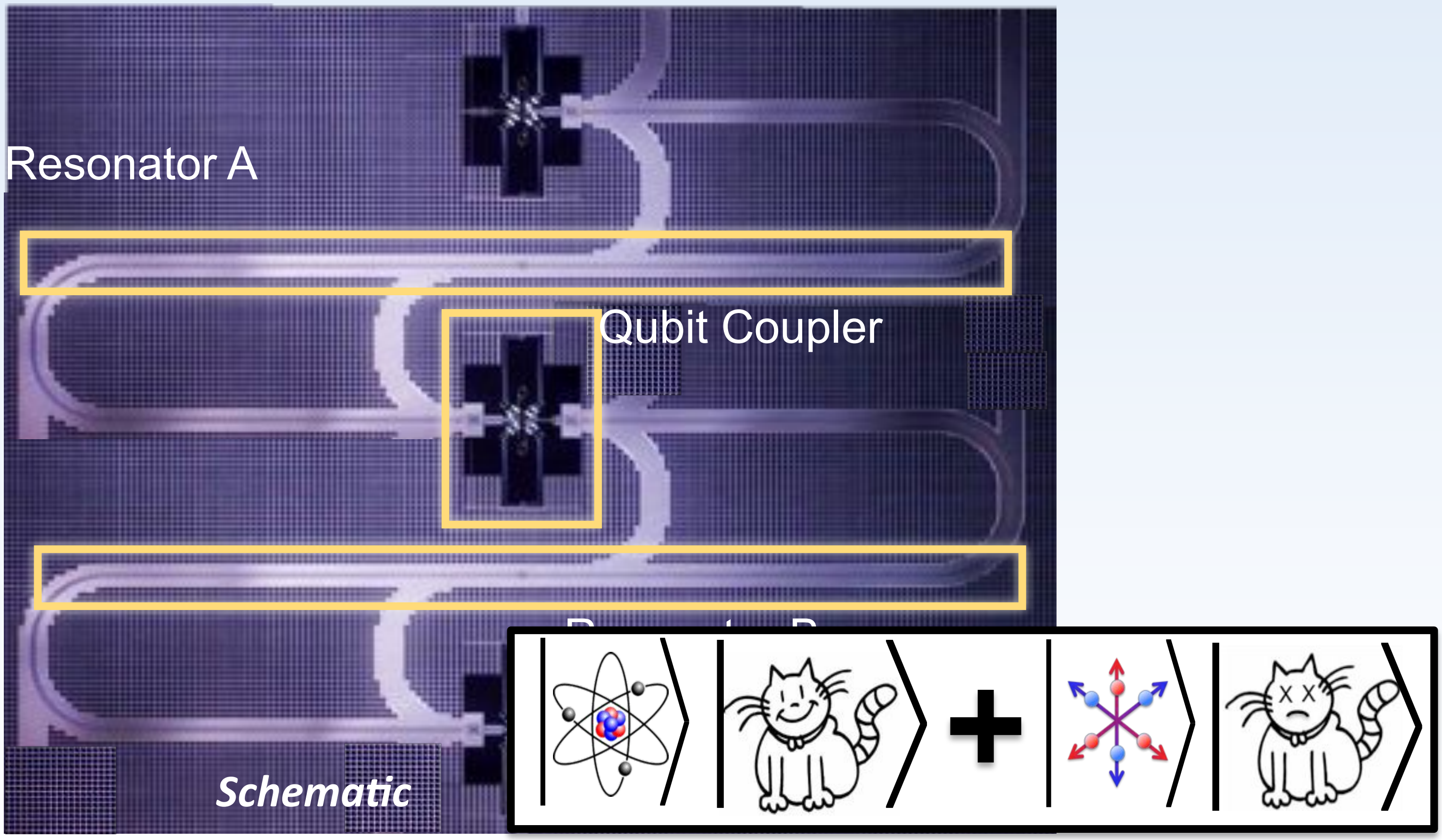
- Superconducting Quantum Circuits
  - LC oscillators + Qubits
- Entangled Qubits
  - through Capacitors + Resonators
- **Entangled Resonators**
- Resonator Networks,
  Quantum Machines and Beyond

Entangling Resonators

FWS, K Jacobs, and RW Simmonds,
Phys. Rev. Lett. **105**, 050501 (2010)

Algorithm to generate interesting entangled states, e.g. “NOON” states:

$$\frac{1}{\sqrt{2}}\left(|N\rangle_A|0\rangle_B+|0\rangle_A|N\rangle_B\right)$$



Arbitrary Control of a Superconducting Resonator

$$|0\rangle + i|3\rangle + |6\rangle$$

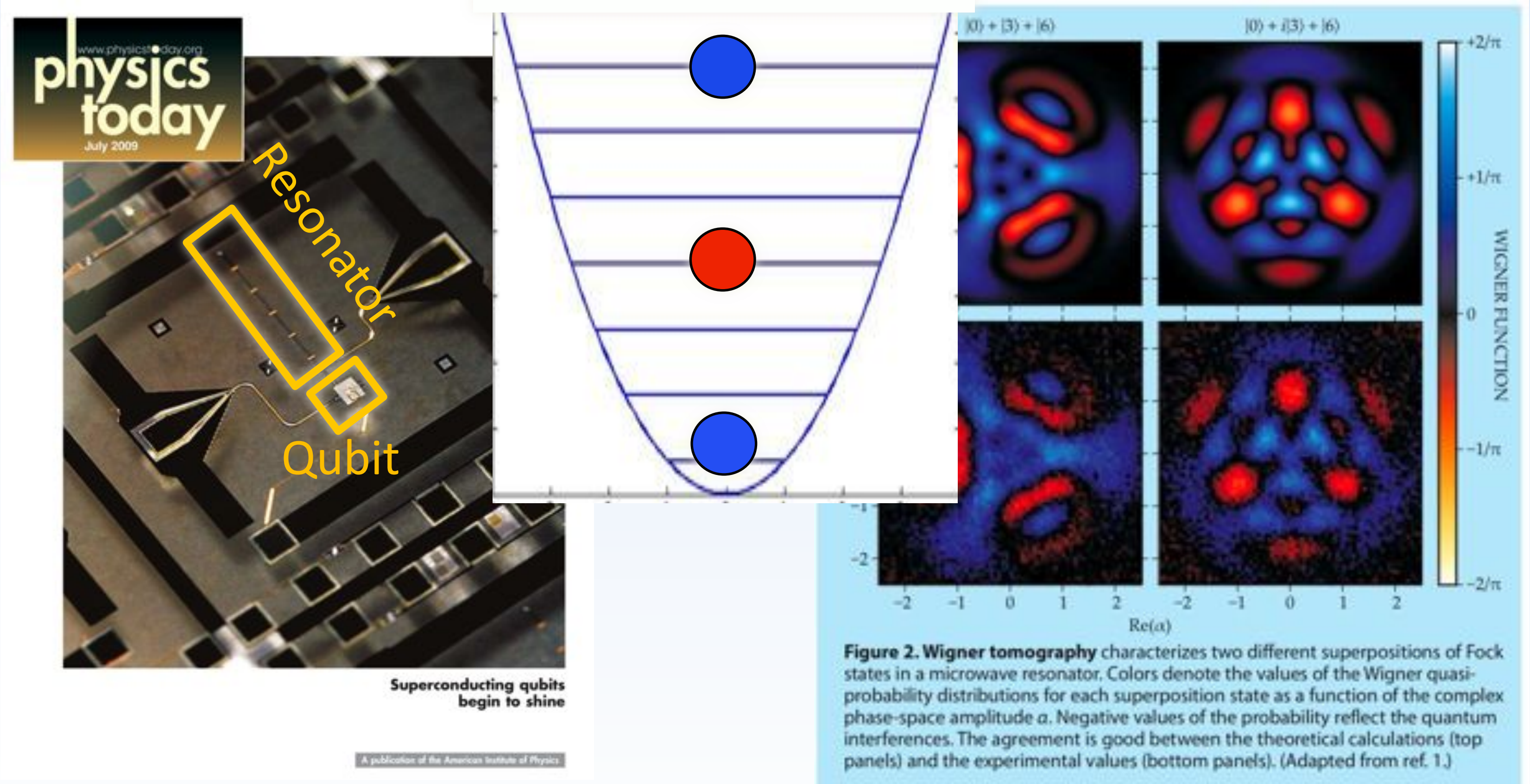


Figure 2. Wigner tomography characterizes two different superpositions of Fock states in a microwave resonator. Colors denote the values of the Wigner quasi-probability distributions for each superposition state as a function of the complex phase-space amplitude *α*. Negative values of the probability reflect the quantum interferences. The agreement is good between the theoretical calculations (top panels) and the experimental values (bottom panels). (Adapted from ref. 1.)

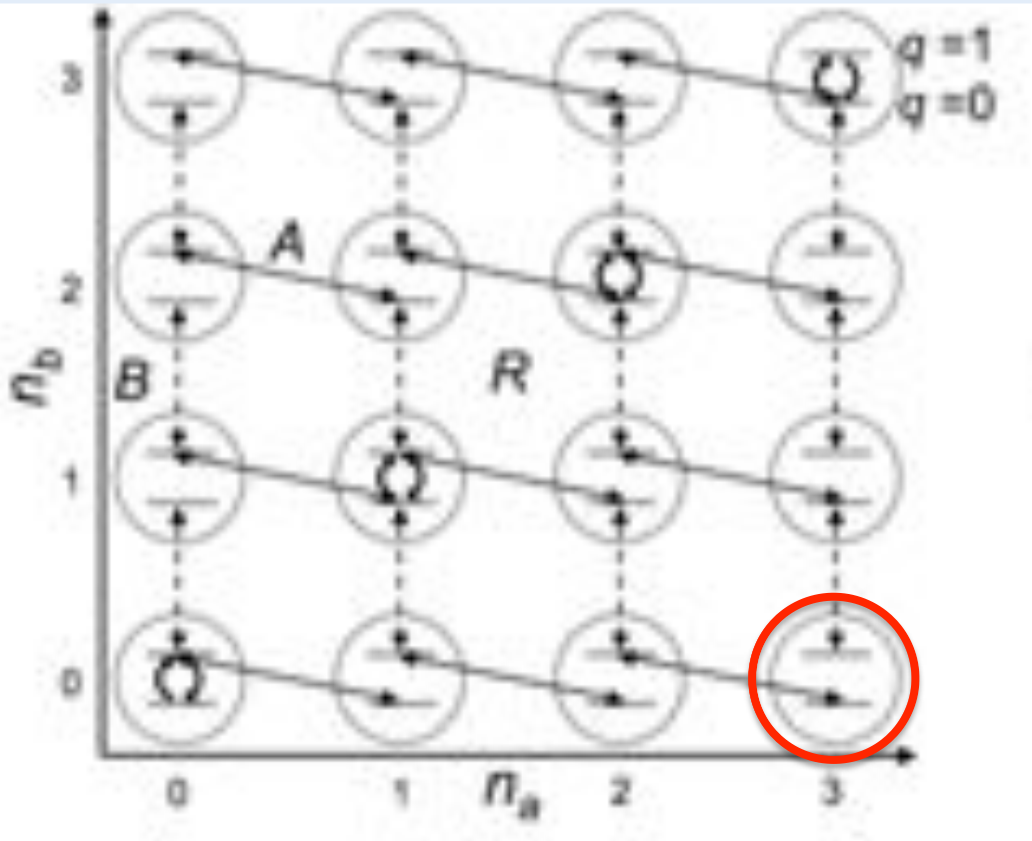
- Martinis Group, UC Santa Barbara (2008)

Entangled Resonator Theory

$$H/\hbar = \omega_q(t)|1\rangle\langle 1| + \omega_a \hat{n}_a + \omega_b \hat{n}_b + \overset{\mathbf{R}}{\boxed{\tfrac{1}{2}\left(\Omega(t)|1\rangle\langle 0| + \Omega^*(t)|0\rangle\langle 1|\right)}}$$

$$\mathbf{A}\ \boxed{+ g_a(\hat{\sigma}_+\hat{a}_- + \hat{\sigma}_-\hat{a}_+)}\boxed{+ g_b(\hat{\sigma}_+\hat{b}_- + \hat{\sigma}_-\hat{b}_+)}\ \mathbf{B}$$

$$\omega_a < \omega_q < \omega_b$$



n_a = # photons in resonator A
n_b = # photons in resonator B

- Rabi pulses (**R**) drive qubit transitions (q=0 → 1)
- Shift pulses (**A + B**) transfer quanta between qubit and resonators
- ***Program of A, B, R, can generate any entangled state we want!***

$$|\psi\rangle_{qubit}|3\rangle_A|0\rangle_B$$

NOON State Example

High NOON state:

J.P. Dowling, Contemp. Phys. **49,** 125 (2008)

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle_A|0\rangle_B + |0\rangle_A|3\rangle_B\right)$$

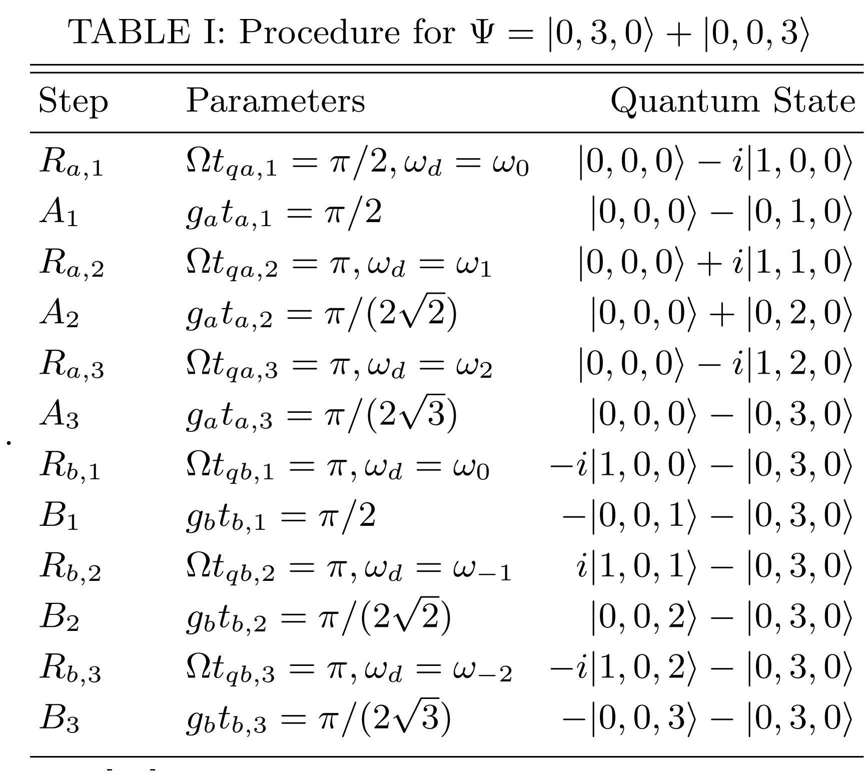
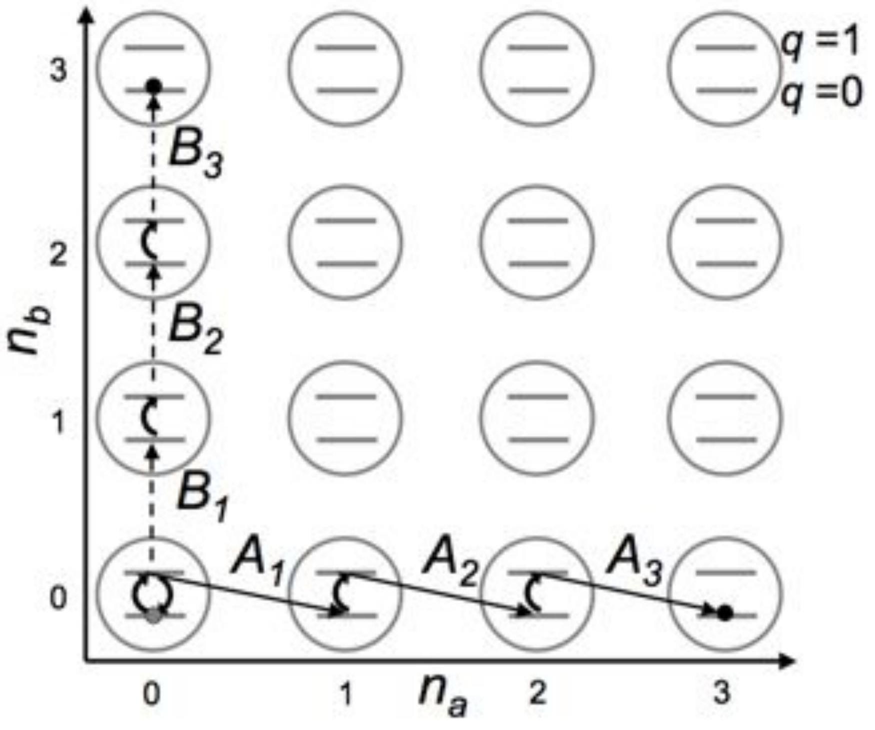


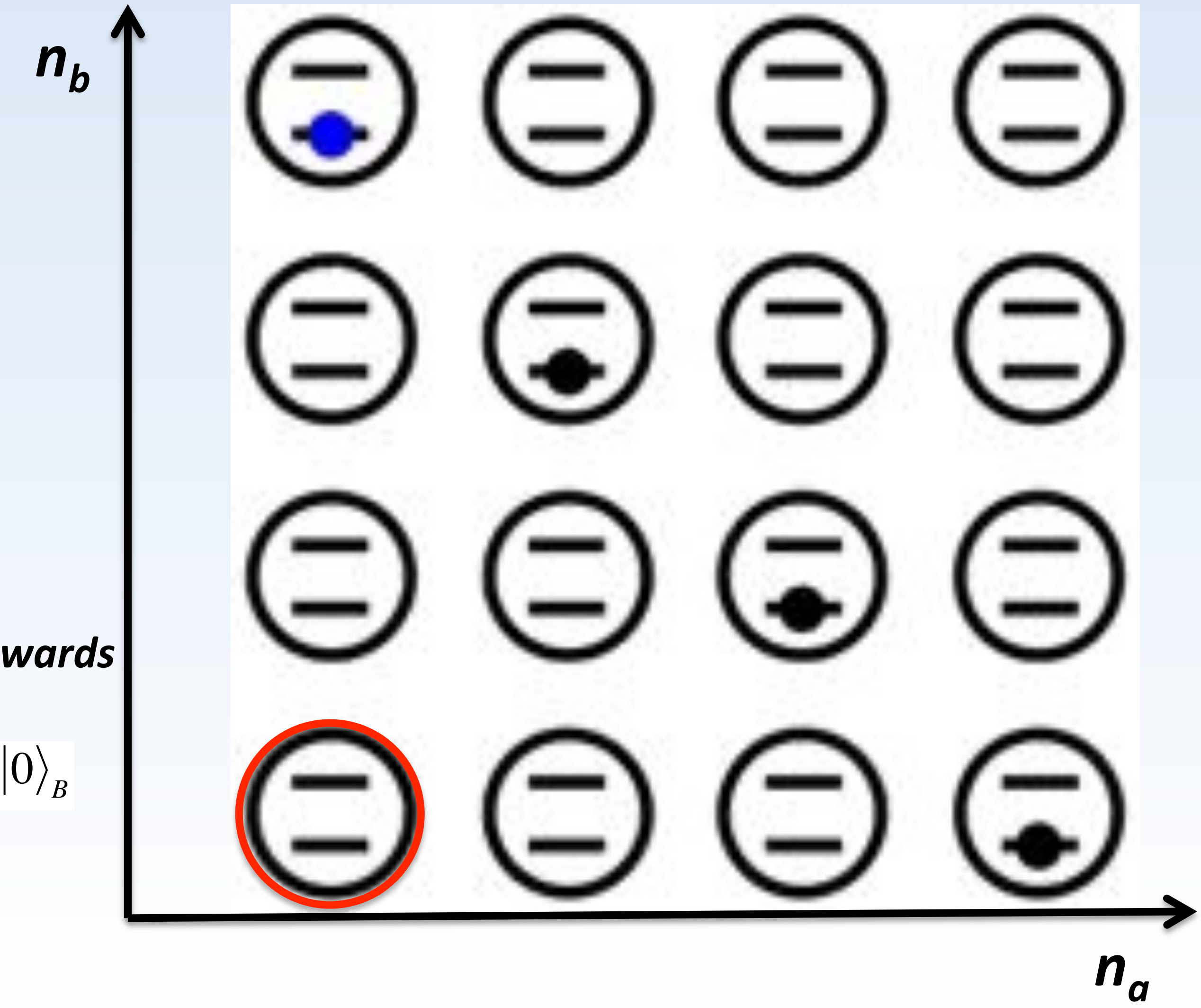
TABLE I: Procedure for $\Psi = |0,3,0\rangle + |0,0,3\rangle$

Step	Parameters	Quantum State
$R_{a,1}$	$\Omega t_{qa,1} = \pi/2, \omega_d = \omega_0$	$\|0,0,0\rangle - i\|1,0,0\rangle$
A_1	$g_a t_{a,1} = \pi/2$	$\|0,0,0\rangle - \|0,1,0\rangle$
$R_{a,2}$	$\Omega t_{qa,2} = \pi, \omega_d = \omega_1$	$\|0,0,0\rangle + i\|1,1,0\rangle$
A_2	$g_a t_{a,2} = \pi/(2\sqrt{2})$	$\|0,0,0\rangle + \|0,2,0\rangle$
$R_{a,3}$	$\Omega t_{qa,3} = \pi, \omega_d = \omega_2$	$\|0,0,0\rangle - i\|1,2,0\rangle$
A_3	$g_a t_{a,3} = \pi/(2\sqrt{3})$	$\|0,0,0\rangle - \|0,3,0\rangle$
$R_{b,1}$	$\Omega t_{qb,1} = \pi, \omega_d = \omega_0$	$-i\|1,0,0\rangle - \|0,3,0\rangle$
B_1	$g_b t_{b,1} = \pi/2$	$-\|0,0,1\rangle - \|0,3,0\rangle$
$R_{b,2}$	$\Omega t_{qb,2} = \pi, \omega_d = \omega_{-1}$	$i\|1,0,1\rangle - \|0,3,0\rangle$
B_2	$g_b t_{b,2} = \pi/(2\sqrt{2})$	$\|0,0,2\rangle - \|0,3,0\rangle$
$R_{b,3}$	$\Omega t_{qb,3} = \pi, \omega_d = \omega_{-2}$	$-i\|1,0,2\rangle - \|0,3,0\rangle$
B_3	$g_b t_{b,3} = \pi/(2\sqrt{3})$	$-\|0,0,3\rangle - \|0,3,0\rangle$



Programming Entanglement

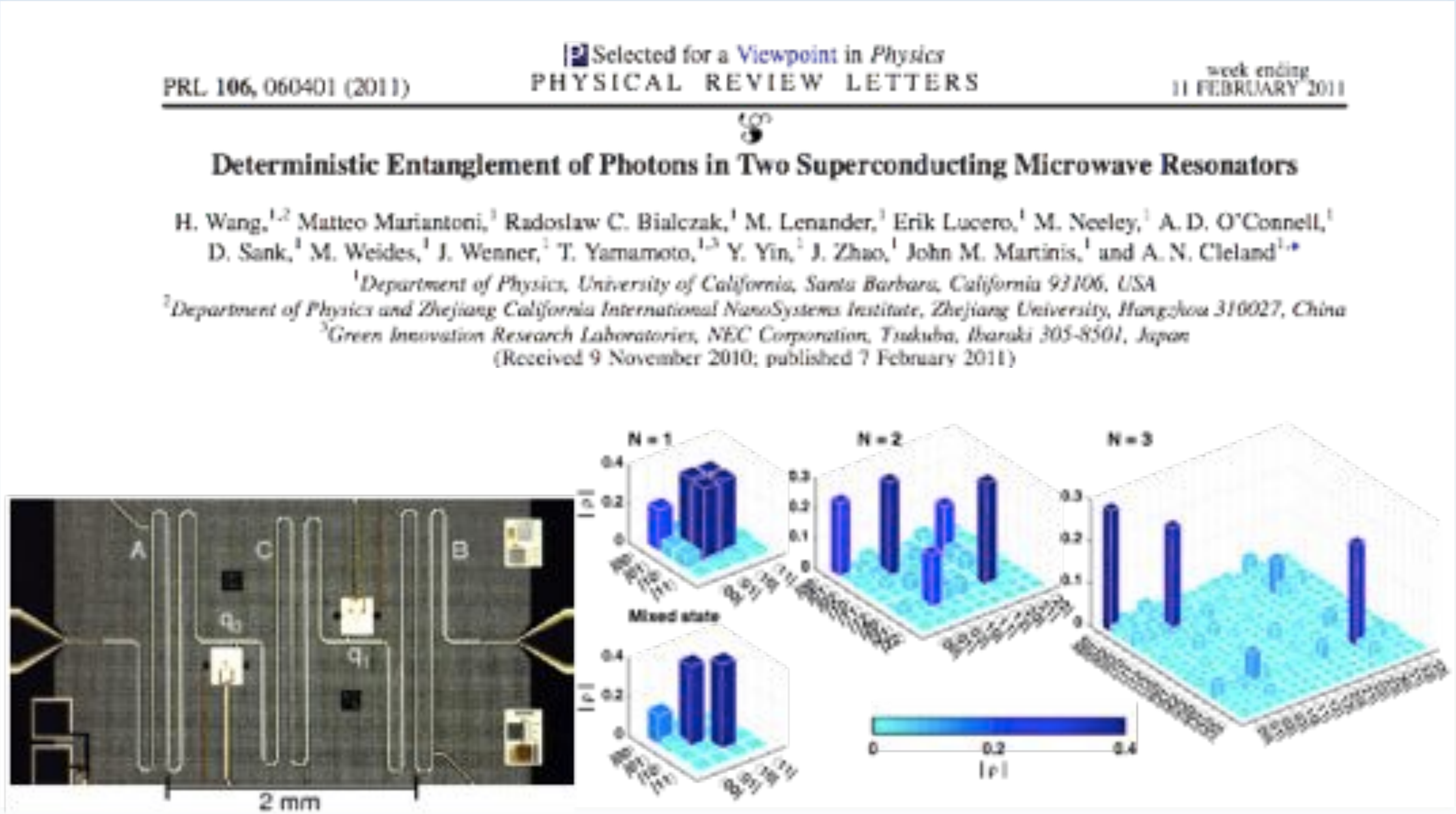
$$|3\rangle_A|0\rangle_B + |2\rangle_A|1\rangle_B + |1\rangle_A|2\rangle_B + |0\rangle_A|3\rangle_B$$



Work ***Backwards***
to
$|0\rangle_{qubit}|0\rangle_A|0\rangle_B$

Experimental Results

Martinis Group, UCSB



Selected for a Viewpoint in *Physics*

PRL **106,** 060401 (2011) PHYSICAL REVIEW LETTERS week ending 11 FEBRUARY 2011

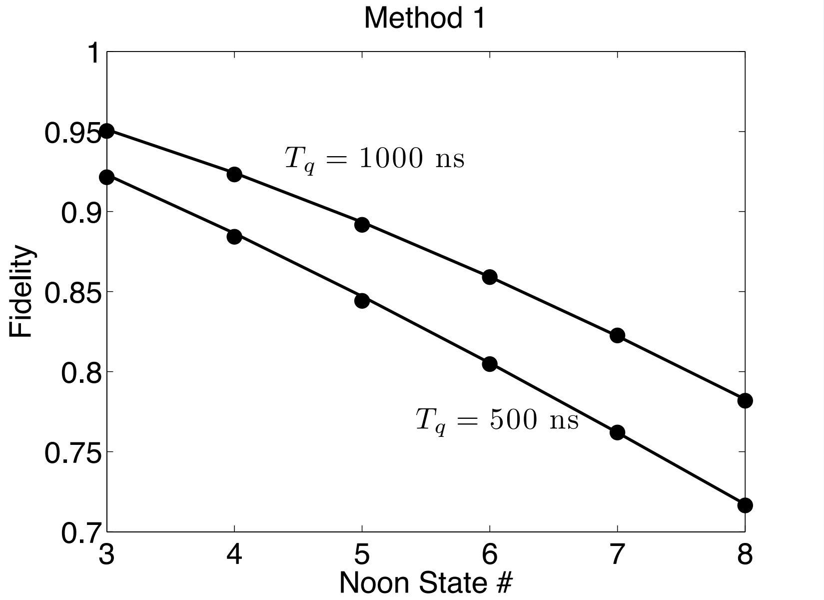
Deterministic Entanglement of Photons in Two Superconducting Microwave Resonators

H. Wang,[1,2] Matteo Mariantoni,[1] Radoslaw C. Bialczak,[1] M. Lenander,[1] Erik Lucero,[1] M. Neeley,[1] A. D. O'Connell,[1] D. Sank,[1] M. Weides,[1] J. Wenner,[1] T. Yamamoto,[1,3] Y. Yin,[1] J. Zhao,[1] John M. Martinis,[1] and A. N. Cleland[1,*]

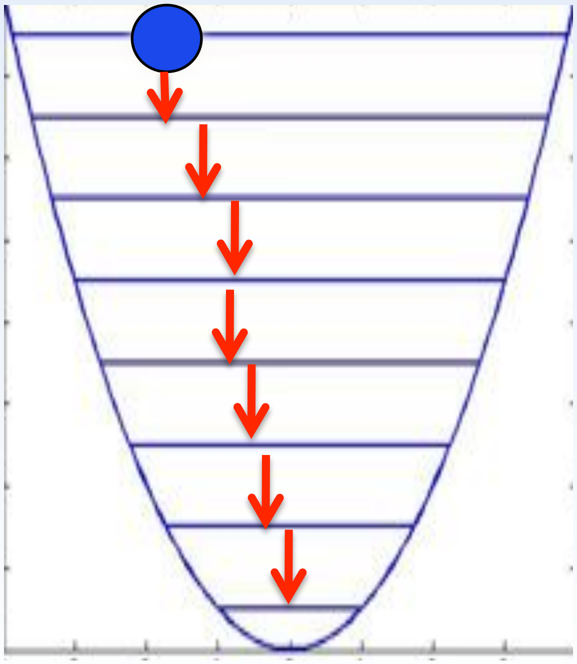
[1]*Department of Physics, University of California, Santa Barbara, California 93106, USA*
[2]*Department of Physics and Zhejiang California International NanoSystems Institute, Zhejiang University, Hangzhou 310027, China*
[3]*Green Innovation Research Laboratories, NEC Corporation, Tsukuba, Ibaraki 305-8501, Japan*
(Received 9 November 2010; published 7 February 2011)

Decoherence for NOON States

- Superconducting resonators are super, but not perfect---they lose *quanta* to the environment in time 100-10000 ns $F \sim e^{-t/T_q} e^{-Nt/T_r}$



Fidelity = Probability of Success



FWS, *Douglas Onyango* '11, K Jacobs, and RW Simmonds, In preparation

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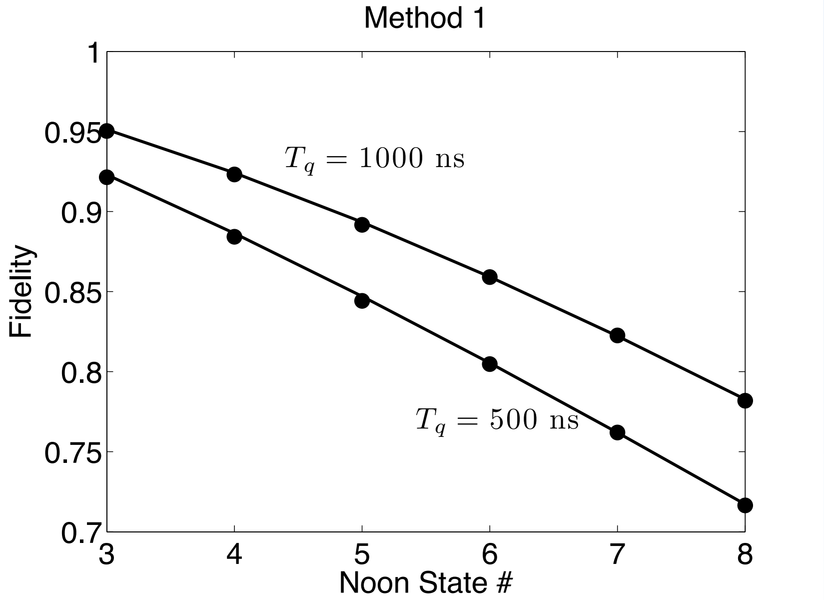


Fidelity = Probability of Success

FWS, *Douglas Onyango* '11, K Jacobs, and RW Simmonds, In preparation

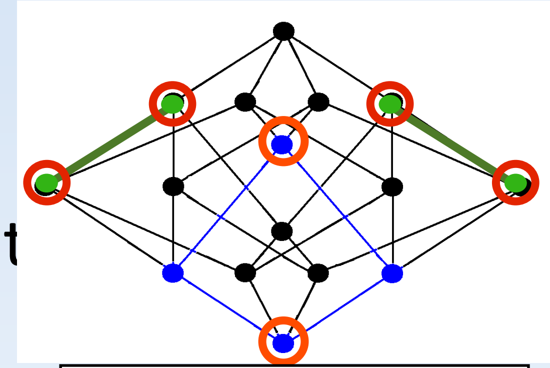
Schrödinger Cats are hard to build!

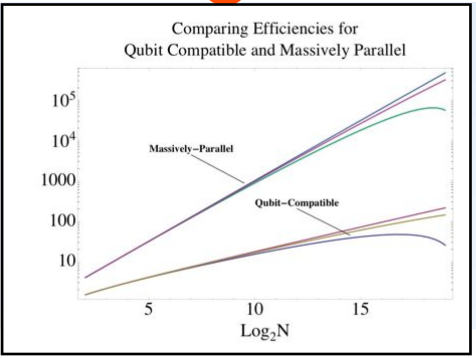
Outline

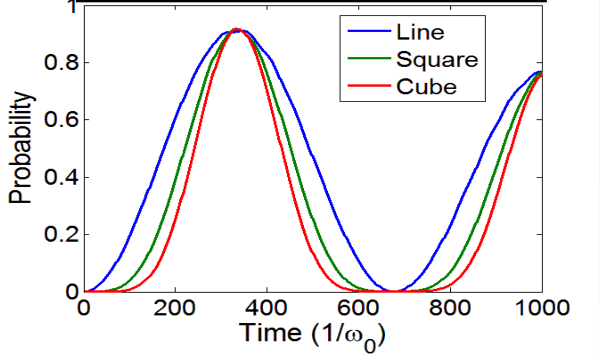
- Superconducting Quantum Circuits
  - LC oscillators + Qubits
- Entangled Qubits
  - through Capacitors, Oscillators, + Resonators
- Entangled Resonators
- **Resonator Networks,**
  Quantum Machines and Beyond

Quantum Routing on Resonator Networks

- **Programmable**: Any two nodes can communicate by programming the qubit frequencies in the network.
- **Parallel**: Multiple quantum states can be transferred at the same time.
- **Efficient**: Transfer time is independent of the distance between nodes!
- **High Fidelity:** F > 90% possible using existin technology, modest dimensions!





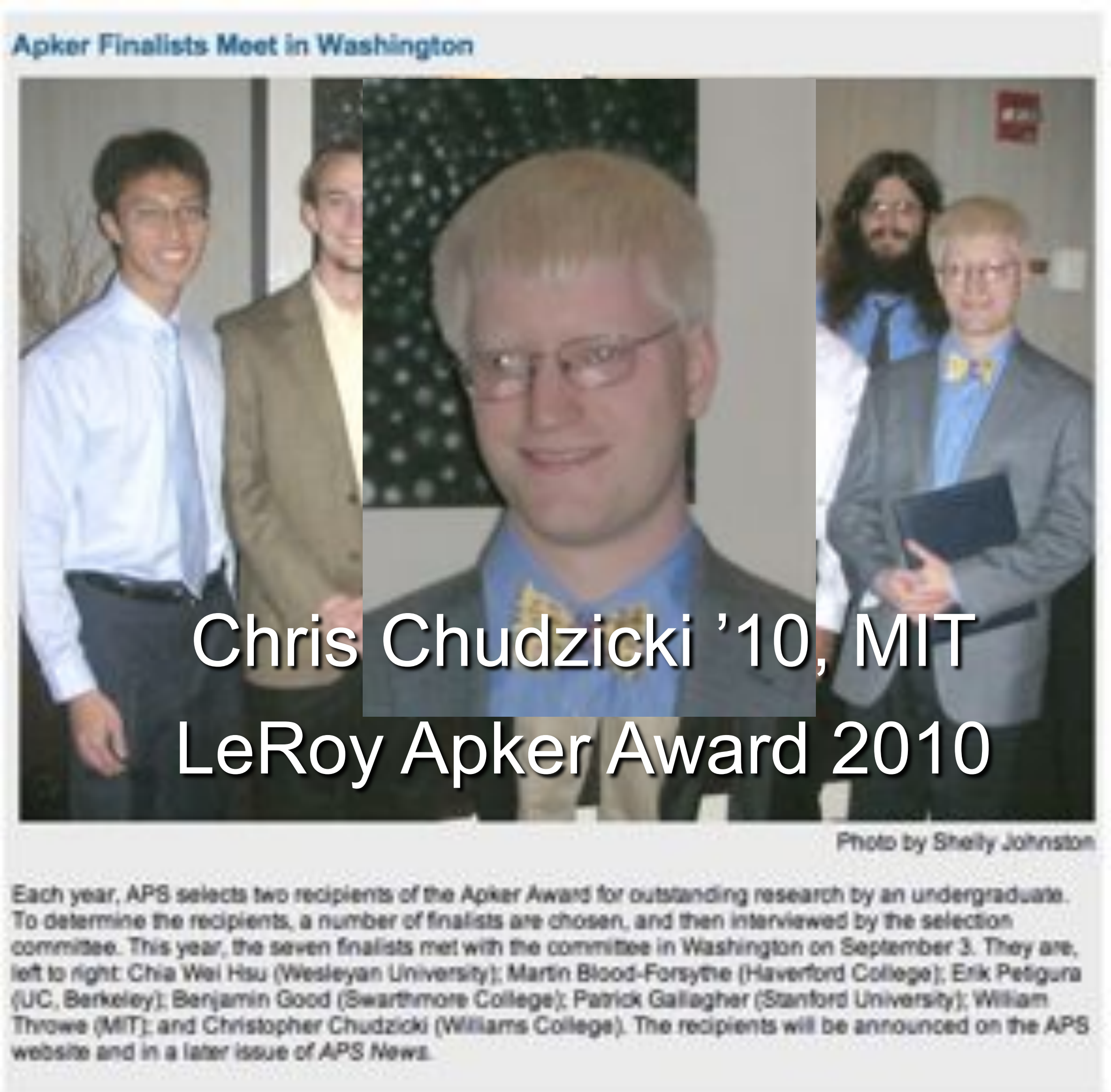


Requires Study of Disorder and Decoherence

FWS and C.J. Williams,
Phys. Rev. B **78**, 094516 (2008)

Chris Chudzicki and FWS,
Phys. Rev. Lett. **105**, 260501 (2010)

Quantum Routing on Resonator Networks


Apker Finalists Meet in Washington

Chris Chudzicki '10, MIT

LeRoy Apker Award 2010

Photo by Shelly Johnston

Each year, APS selects two recipients of the Apker Award for outstanding research by an undergraduate. To determine the recipients, a number of finalists are chosen, and then interviewed by the selection committee. This year, the seven finalists met with the committee in Washington on September 3. They are, left to right: Chia Wei Hsu (Wesleyan University); Martin Blood-Forsythe (Haverford College); Erik Petigura (UC, Berkeley); Benjamin Good (Swarthmore College); Patrick Gallagher (Stanford University); William Throwe (MIT); and Christopher Chudzicki (Williams College). The recipients will be announced on the APS website and in a later issue of *APS News*.

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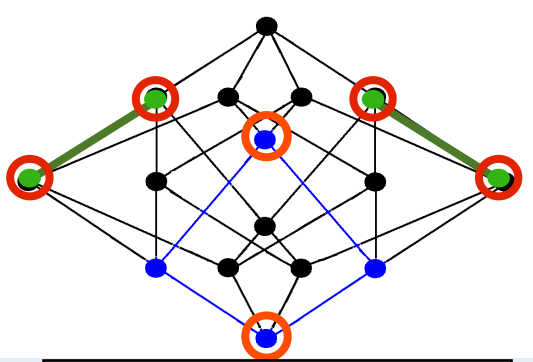

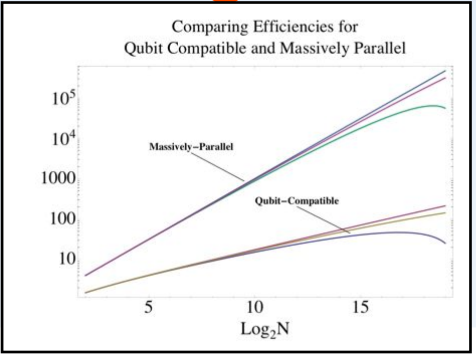

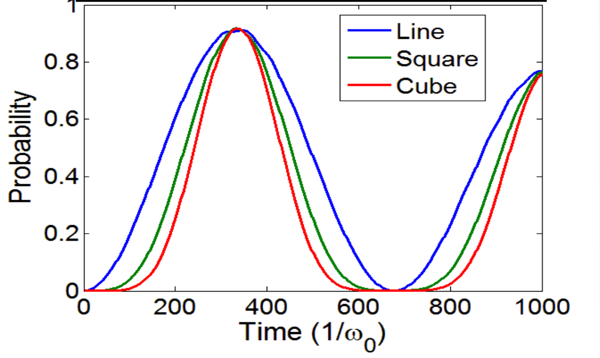

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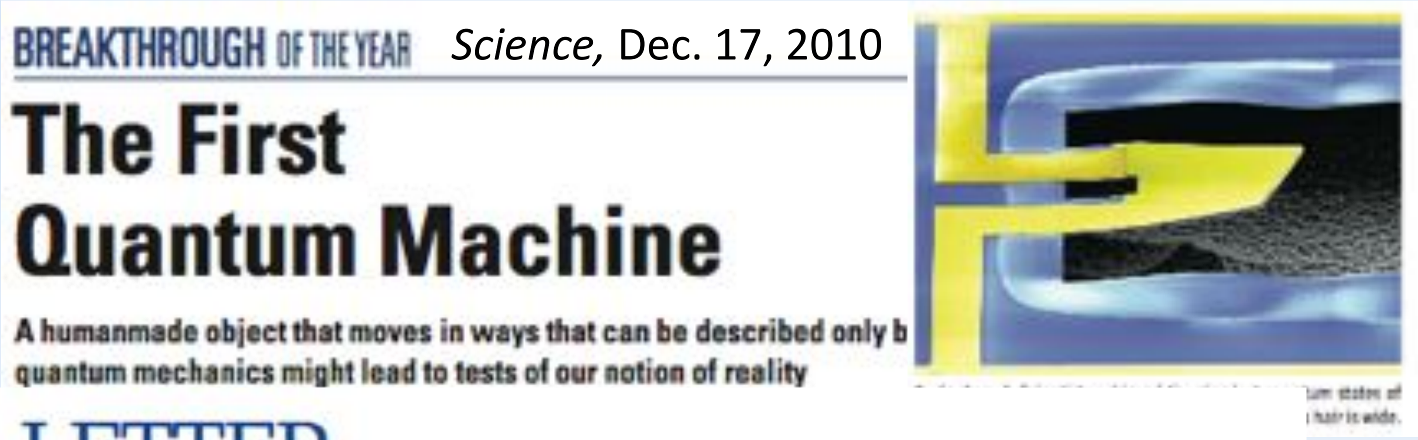
Phys. Rev. B **78**, 094516 (2008)

Chudzicki and FWS,

Phys. Rev. Lett. **105**, 260501 (2010)

Quantum Machines?



BREAKTHROUGH OF THE YEAR

Science, Dec. 17, 2010

The First Quantum Machine

A humanmade object that moves in ways that can be described only b quantum mechanics might lead to tests of our notion of reality

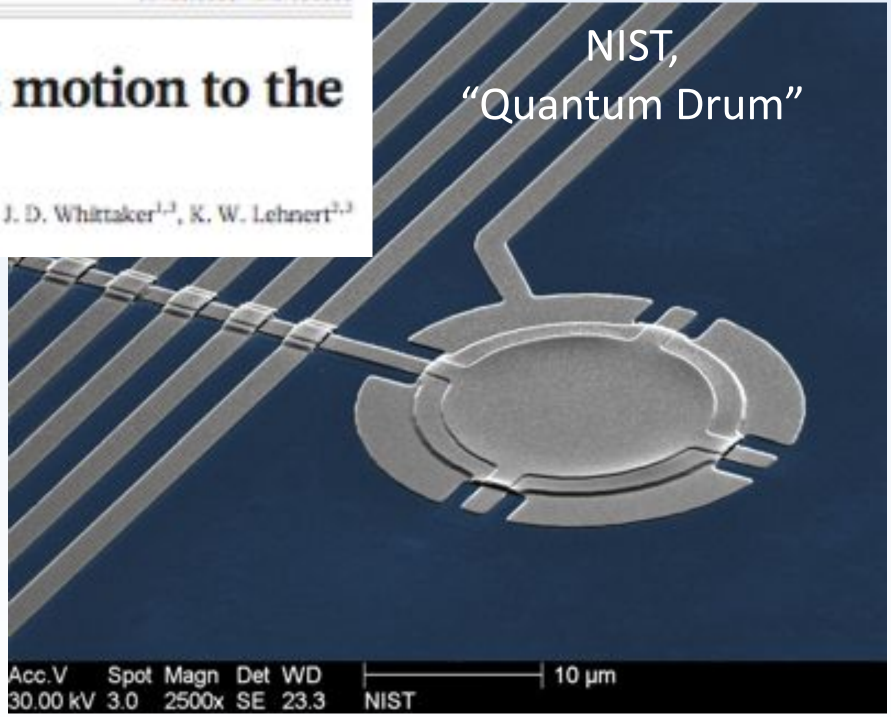
Martinis Group, UCSB

LETTER

doi:10.1038/nature10261

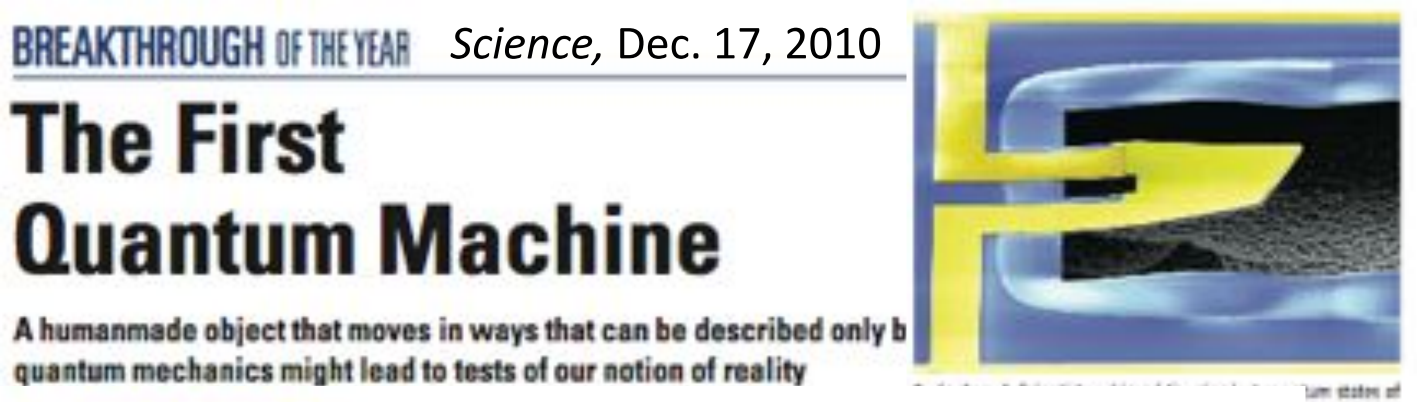
Sideband cooling of micromechanical motion to the quantum ground state

J. D. Teufel[1], T. Donner[2,3], Dale Li[1], J. W. Harlow[2,3], M. S. Allman[1,3], K. Cicak[1], A. J. Sirois[1,3], J. D. Whittaker[1,3], K. W. Lehnert[2,3] & R. W. Simmonds[1]



NIST,
“Quantum Drum”

Quantum Machines?



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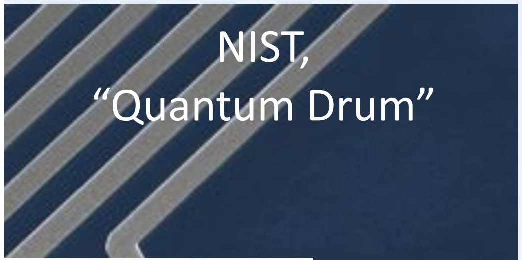

NIST,
“Quantum Drum”

30.00 kV 3.0 2500x SE 23.3 NIST

PRL **107,** 177204 (2011) PHYSICAL REVIEW LETTERS week ending 21 OCTOBER 2011

Ultraefficient Cooling of Resonators: Beating Sideband Cooling with Quantum Control

Xiaoting Wang,[1,2] Sai Vinjanampathy,[2] Frederick W. Strauch,[3] and Kurt Jacobs[2,4]

[1]*Department of Applied Mathematics & Theoretical Physics, University of Cambridge, Cambridge, CB3 0WA, United Kingdom*
[2]*Department of Physics, University of Massachusetts at Boston, Boston, Massachusetts 02125, USA*
[3]*Department of Physics, Williams College, Williamstown, Massachusetts 01267, USA*
[4]*Hearne Institute for Theoretical Physics, Louisiana State University, Baton Rouge, Louisiana 70803, USA*
(Received 28 March 2011; revised manuscript received 22 August 2011; published 19 October 2011)

Quantum Computing?

- Q: When will we have a quantum computer?

A: $|2020\rangle + |2030\rangle + |2040\rangle + |2050\rangle + \cdots ?$

Thank you very much!

- Williams College
- Physics Department
- Jay Pasachoff + Sigma Xi
- Students, especially:
  - *Teng Jian Khoo '09*
  - *Chris Chudzicki '10*
  - *Steve Jackson '10*
  - *Samyam Rajbhandari '11*
  - Douglas Onyango '11
  - Hai Zhou '11
  - Ben Athiwaratkun '12
  - Qiao Zhang '13

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Collaborators:

Kurt Jacobs
(UMass Boston)

Ray Simmonds
(NIST Boulder)