



Quantum Routing and Beyond with Superconducting Resonators

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Funding: NSF, Research Corporation

Collaborators



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'10, MIT



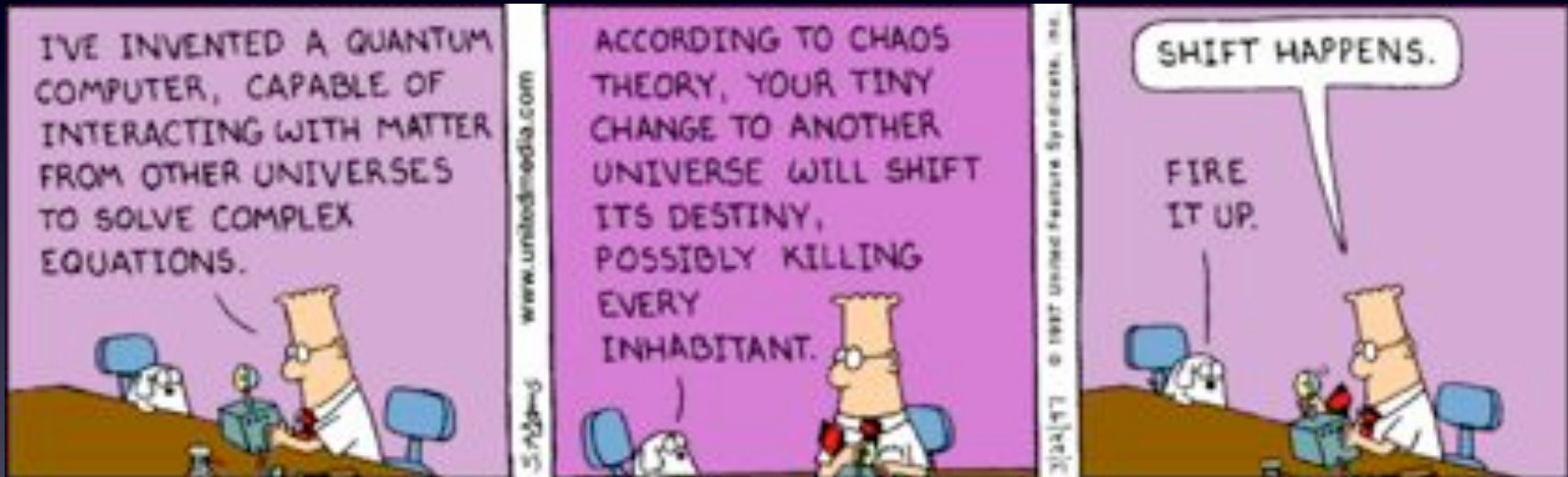
Ray Simmonds,
NIST Boulder

Outline

- **Superconducting Qubits and Resonators**
- **Quantum Routing on Networks**
 - Perfect State Transfer on Hypercube Networks
 - Parallel State Transfer and Efficient Quantum Routing
C. Chudzicki '10 + FWS, Phys. Rev. Lett. **105**, 260501 (2010)
- **Quantum Computing with Superconducting Resonators**
 - Entangled 'NOON' States FWS, K Jacobs, and RW Simmonds, Phys. Rev. Lett. **105**, 050501 (2010)
 - Multi-level quantum logic FWS + K Jacobs, in preparation

Quantum Computing

Executive Summary



Quantum Computing

Extended Abstract

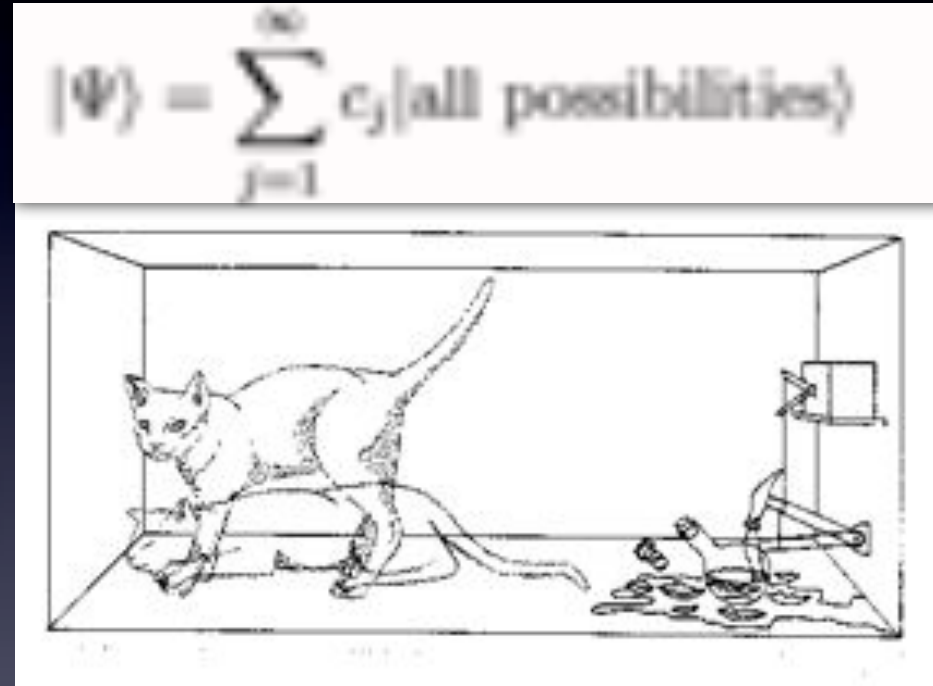
Quantum Computer:

A hypothetical device which uses the intrinsic weirdness of the universe (?multiverse?) to speed up computational tasks.

Quantum Algorithms:

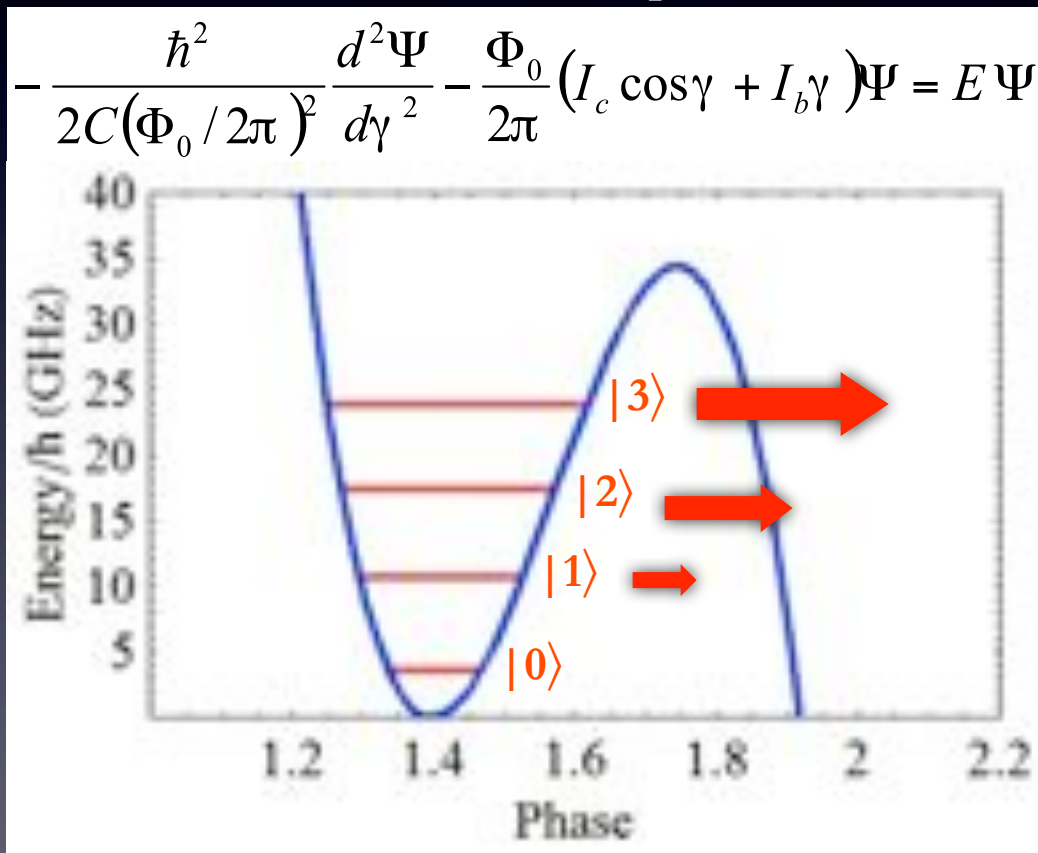
Factoring, Search, Simulation (+ a few more)

Quantum Bit: A bit (binary digit) that can be in a superposition of both 0 and 1



Phase Qubit

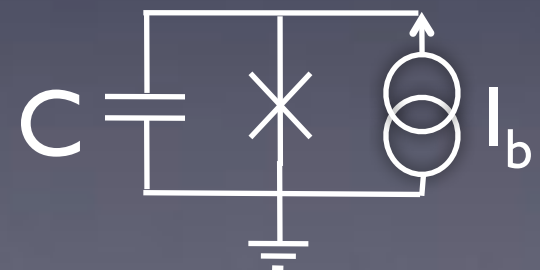
**Artificial Atom,
Controlled by Wires!**



Josephson Junction

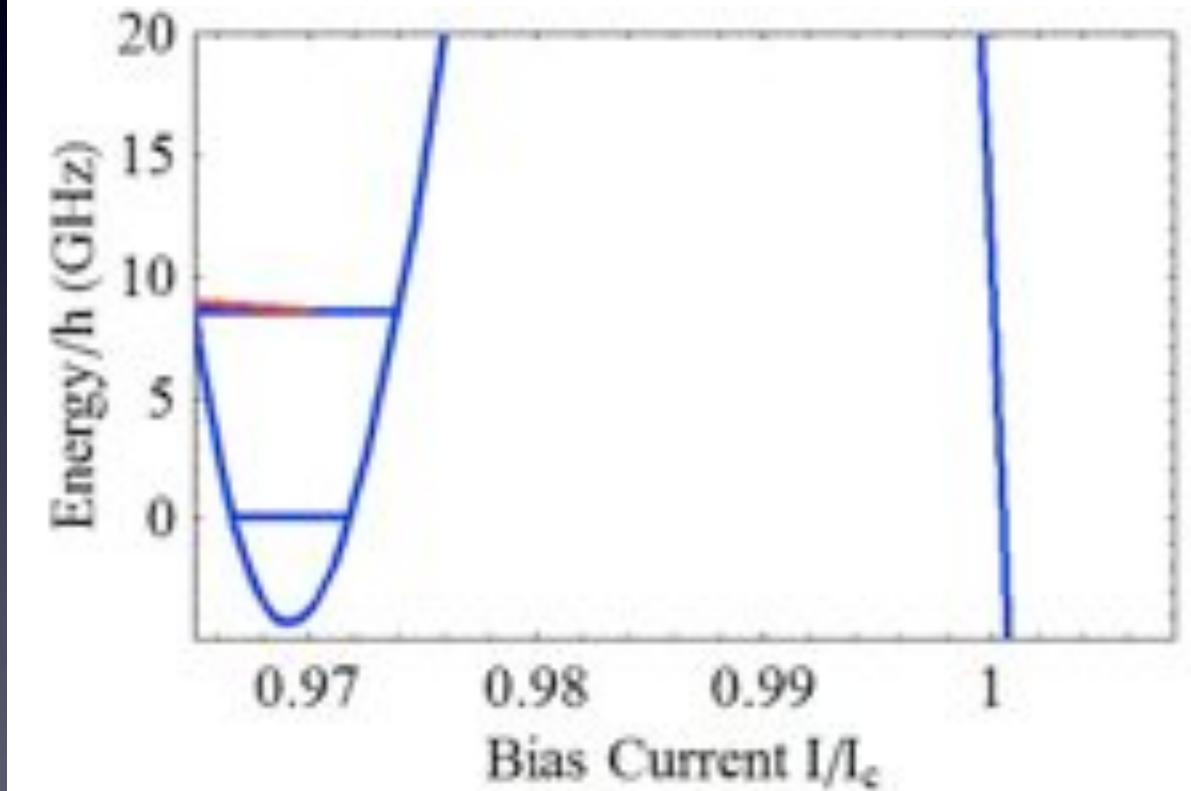


AC Circuits:
Capacitor, Inductor + JJunction



Tunable Oscillator

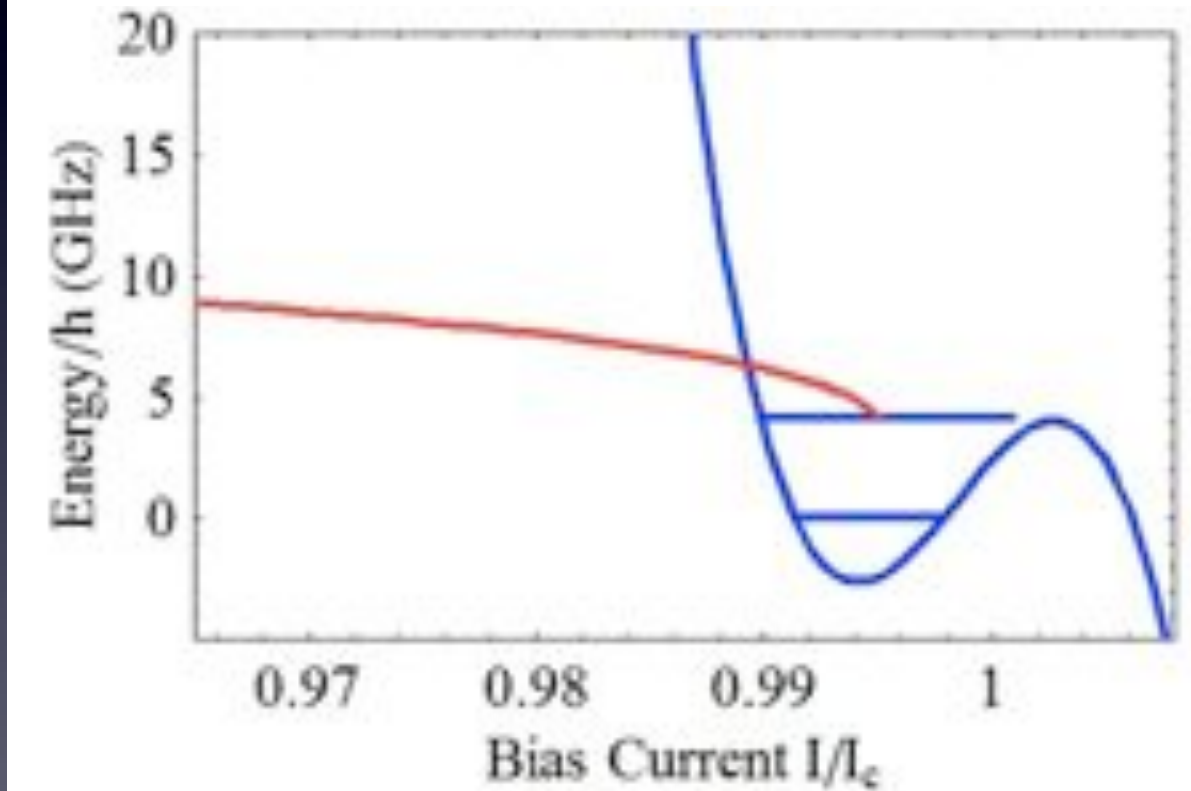
$$-\frac{\hbar^2}{2C(\Phi_0/2\pi)^2} \frac{d^2\Psi}{d\gamma^2} - \frac{\Phi_0}{2\pi} (I_c \cos\gamma + I_b\gamma) \Psi = E\Psi$$



Sweep of bias current allows experimental control of energy levels.

Tunable Oscillator

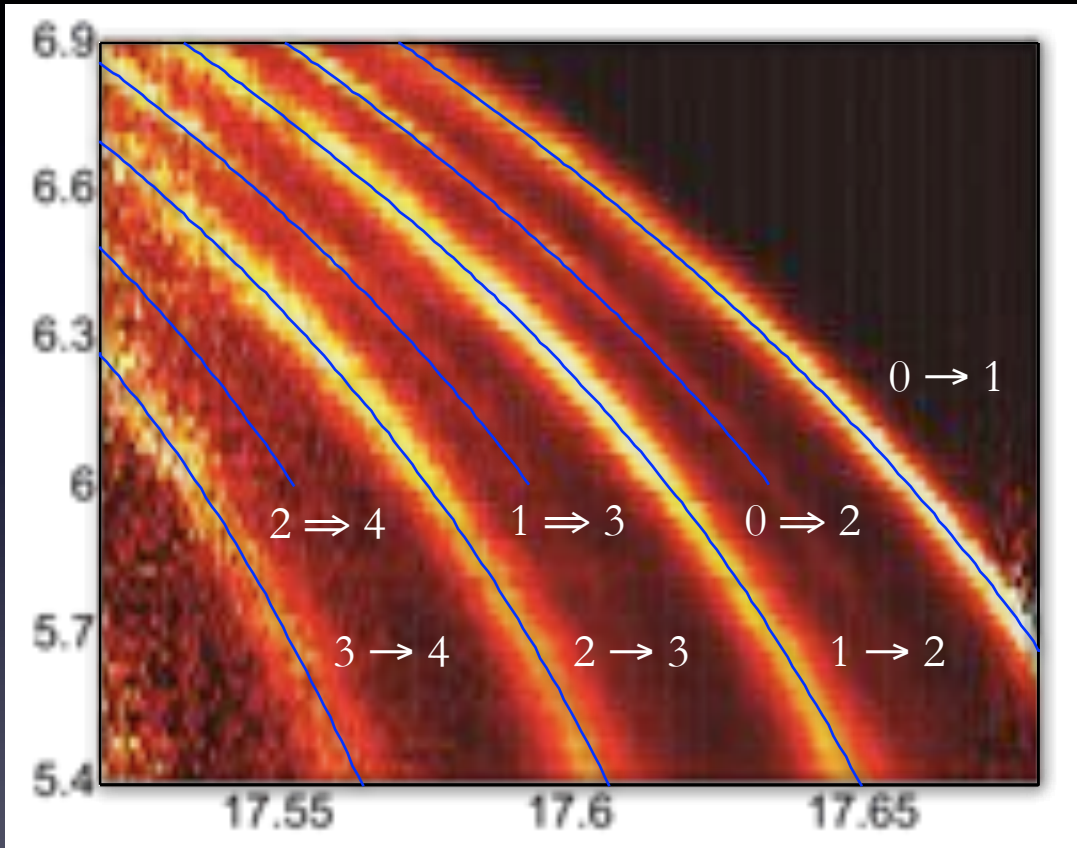
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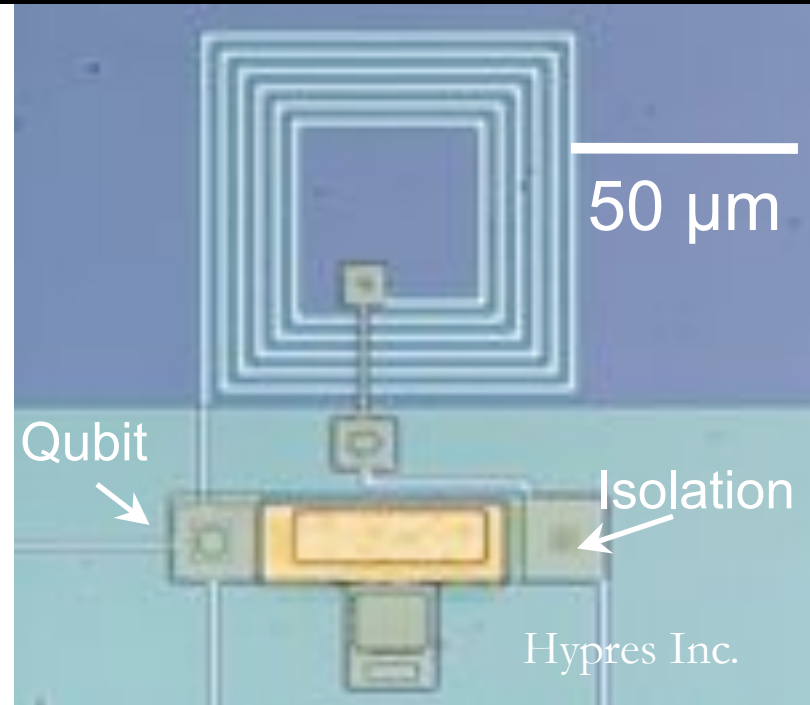
Phase Qubit Spectroscopy

f (GHz)



I (μA)

Each microwave transition is an excitation of the junction with an increased tunneling rate. Bright indicates a large number of tunneling events, dark a small number of events.

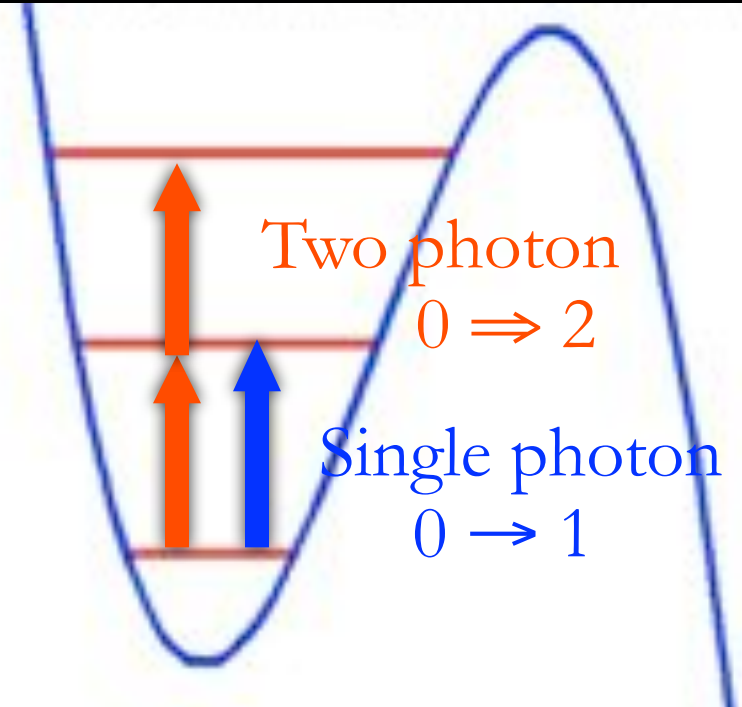
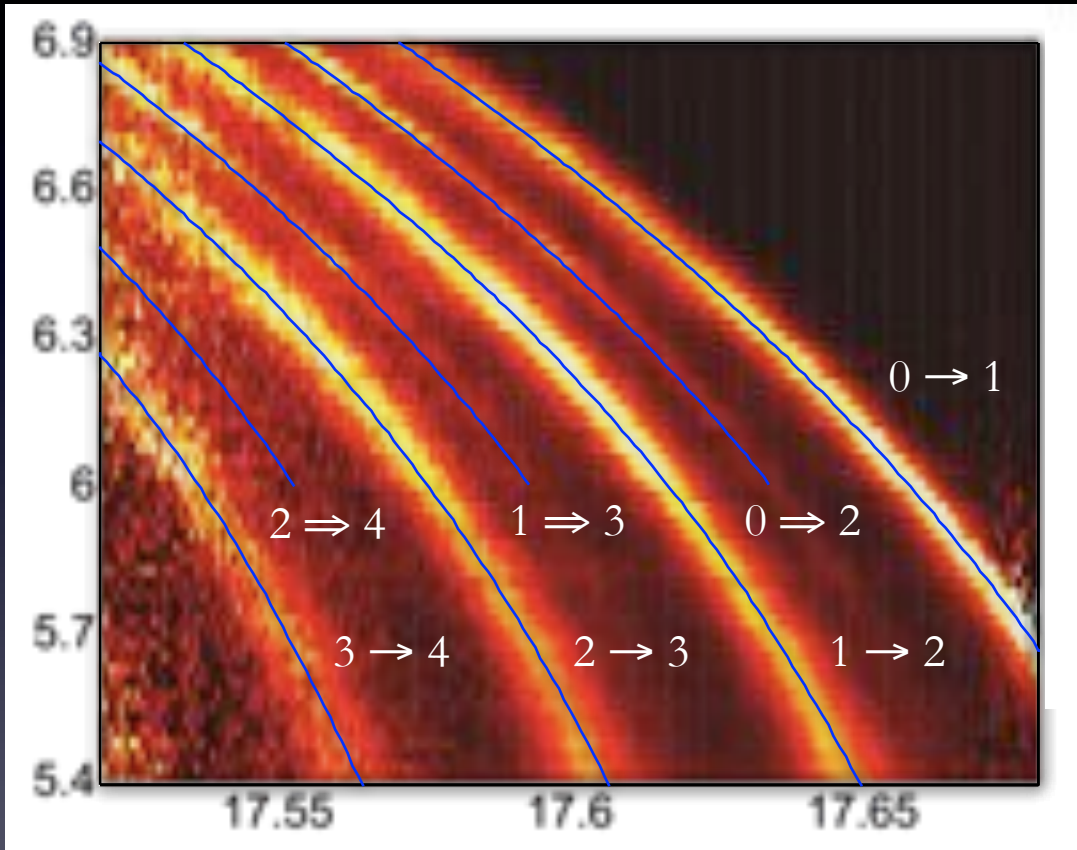


Sudeep Dutta et al. (Univ. Maryland)



Phase Qubit Spectroscopy

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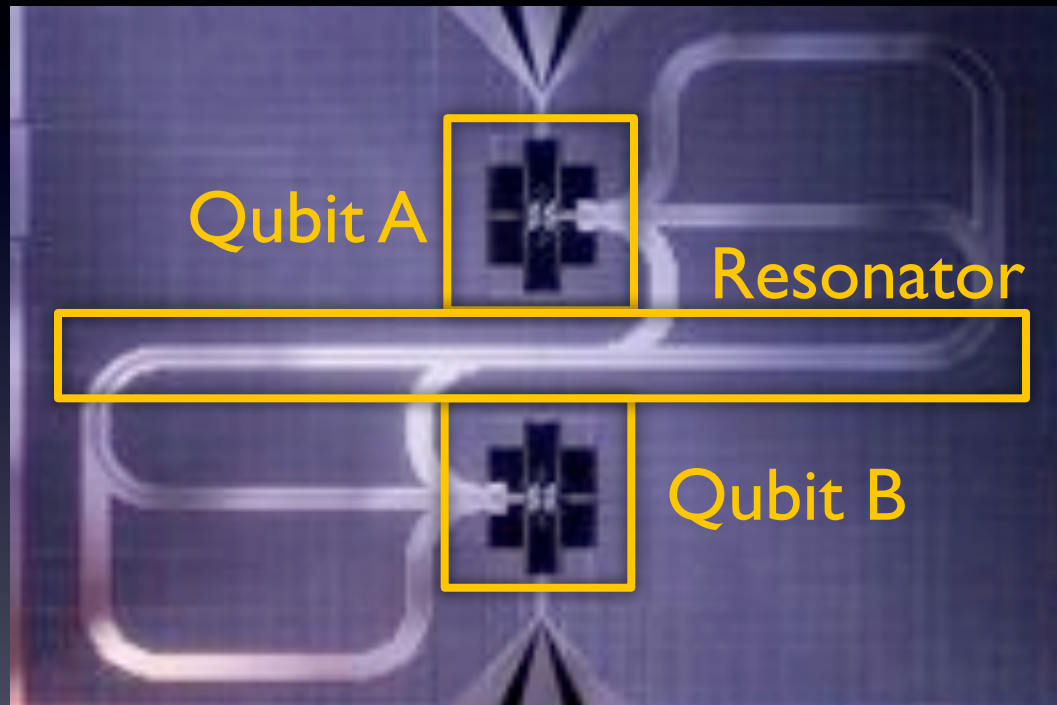
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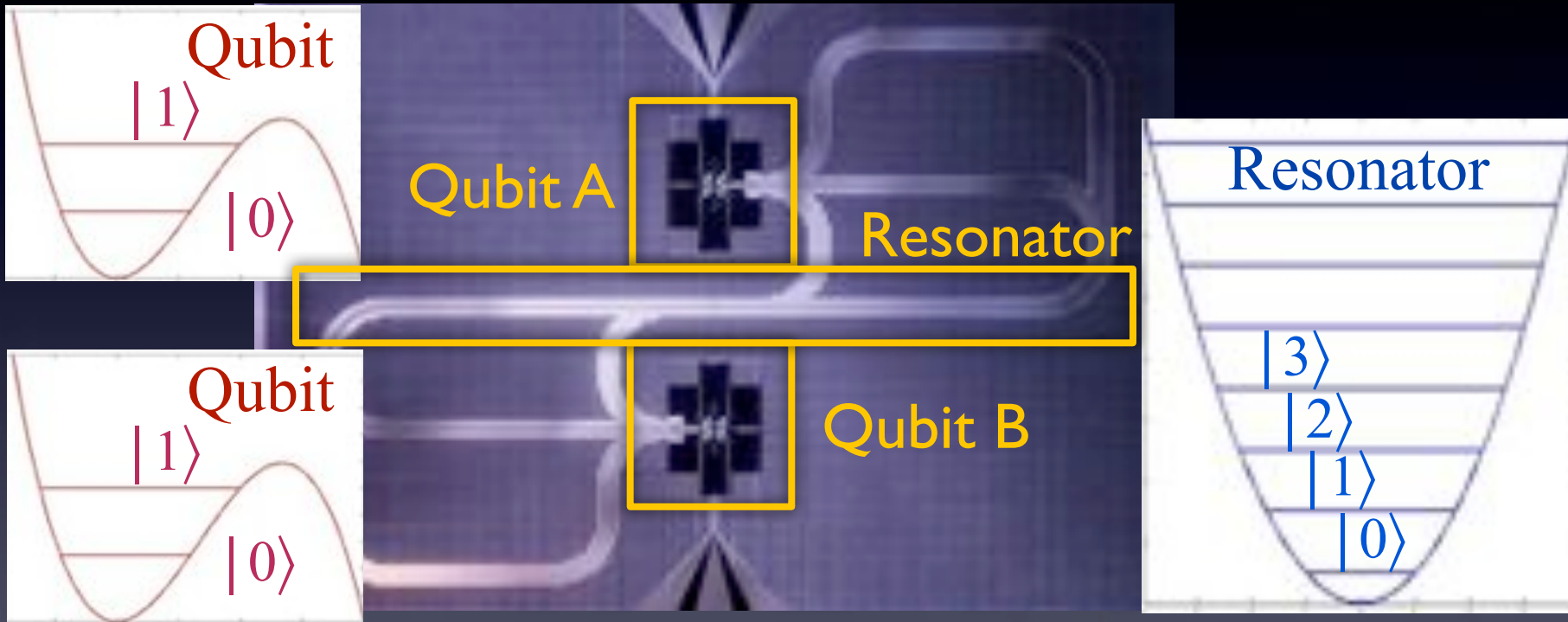


Coupling Qubits by Cavities



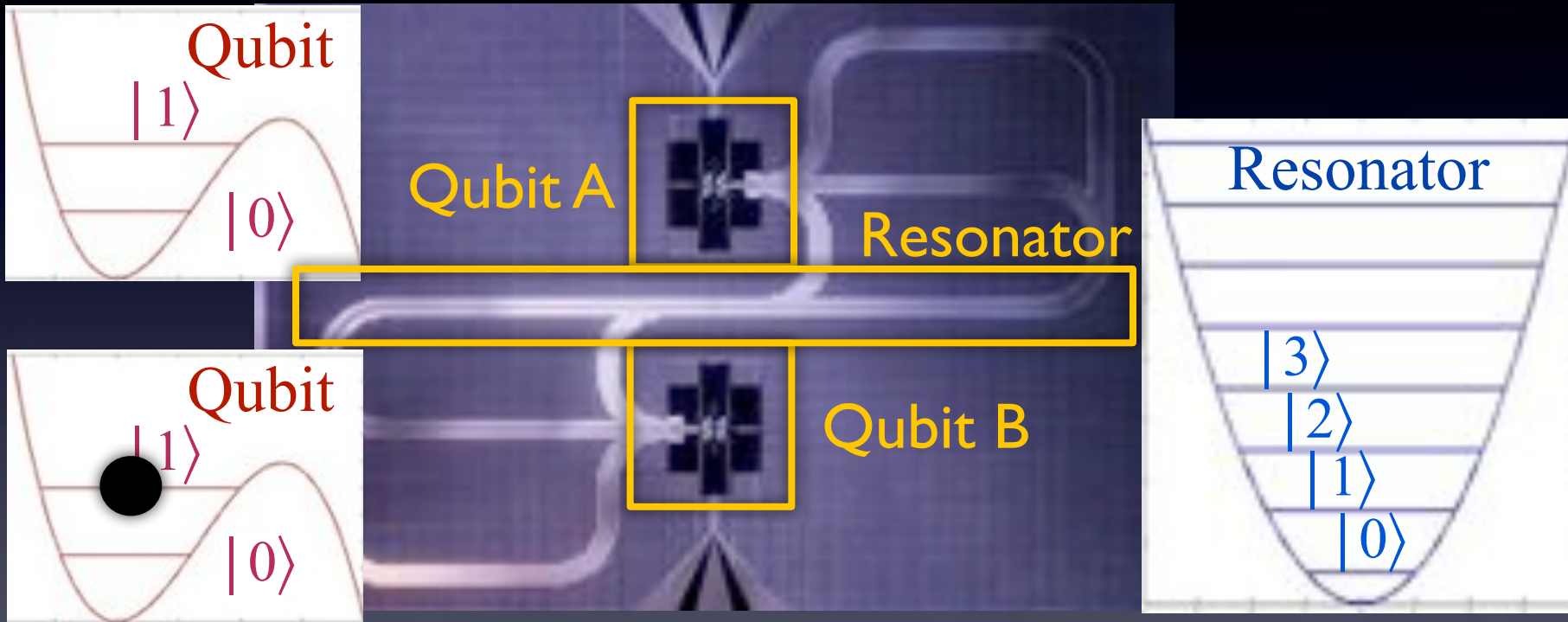
“Coherent quantum state storage and transfer between two phase qubits via a resonant cavity”, M. Sillanpaa, J. I. Park, and R.W. Simmonds, *Nature* 449, 438 (2007)

Coupling Qubits by Cavities



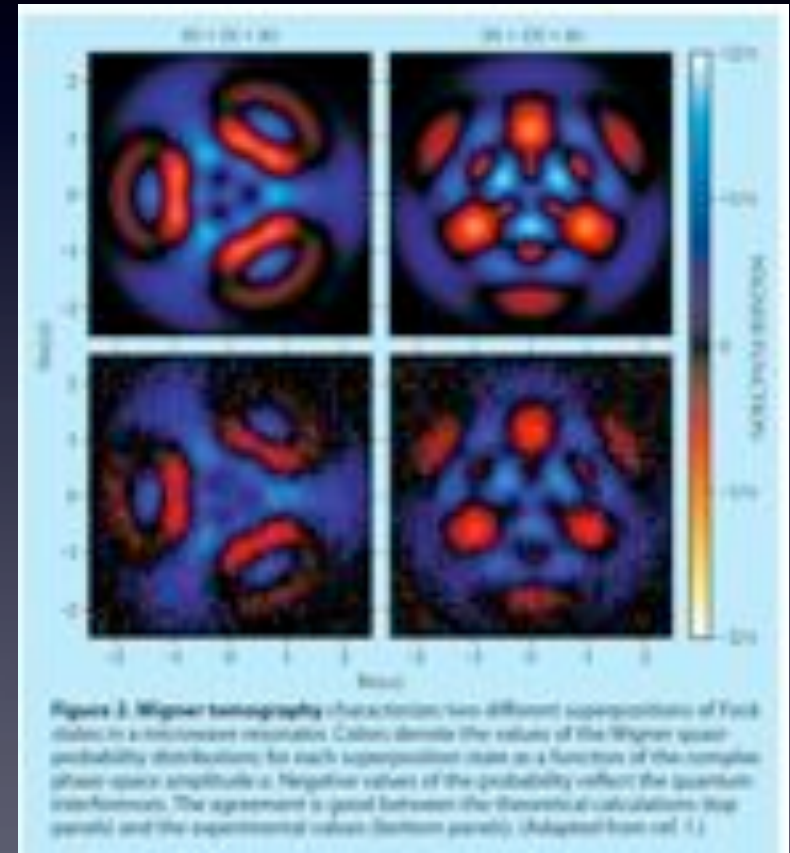
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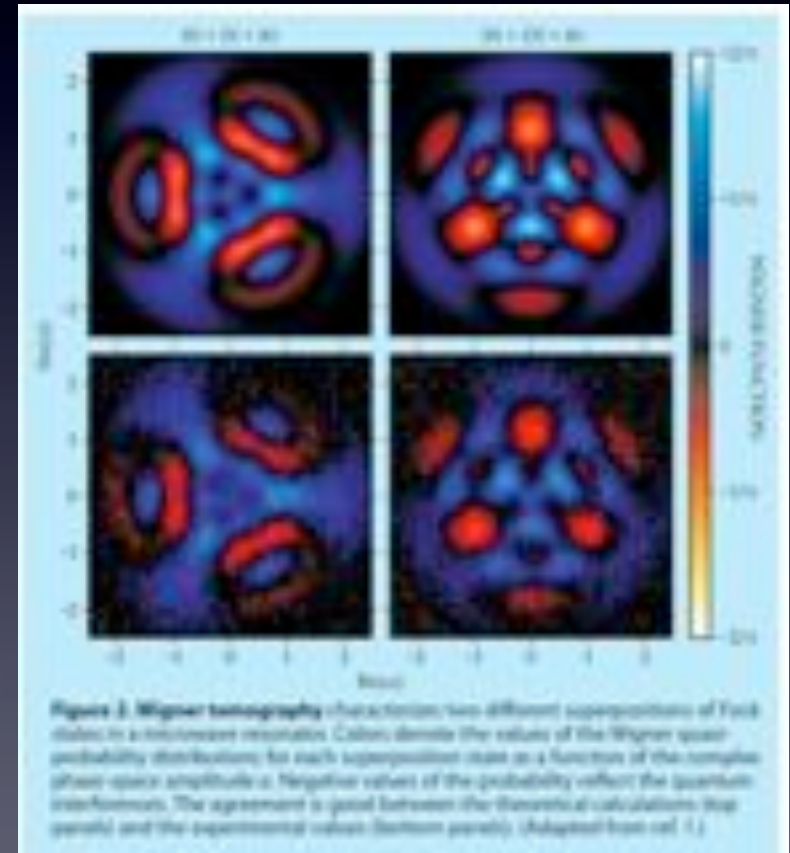
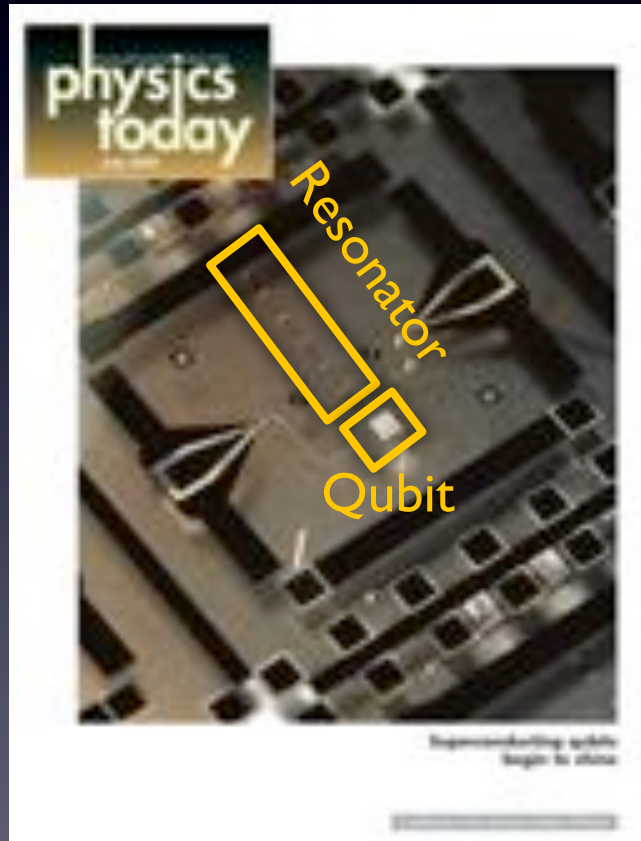
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Arbitrary Control of a Superconducting Resonator



- Martinis Group, UC Santa Barbara (2008)

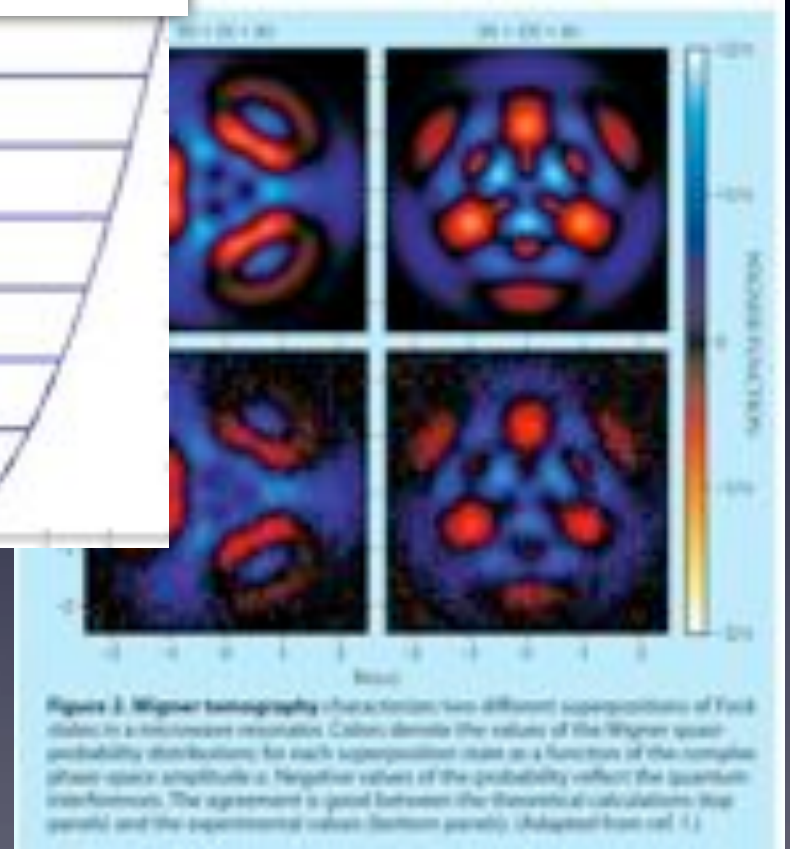
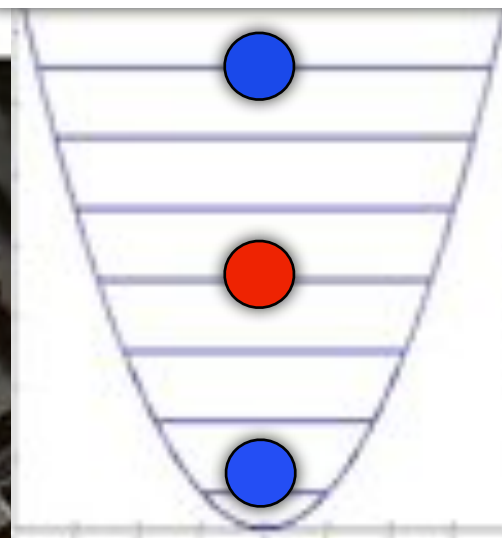
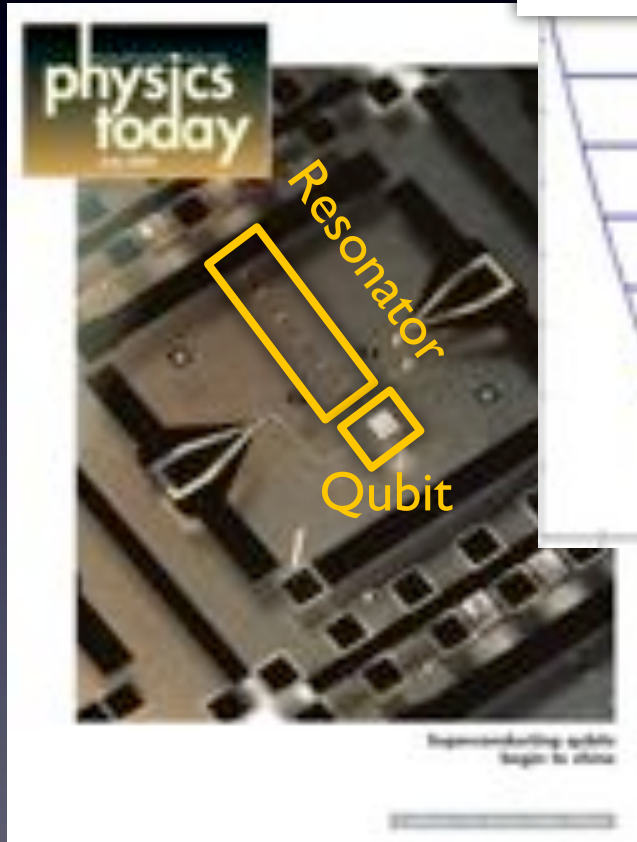
Arbitrary Control of a Superconducting Resonator



- Martinis Group, UC Santa Barbara (2008)

Arbitrary Control of a Superconducting Resonator

$$|\Psi\rangle = |0\rangle + |3\rangle + |6\rangle$$



- Martinis Group, UC Santa Barbara (2008)

Outline

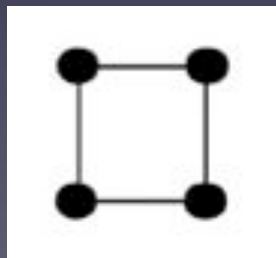
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- Quantum Routing on Networks
- Perfect State Transfer on Hypercube Networks
- Parallel State Transfer and Efficient Quantum Routing
- Quantum Computing with Superconducting Resonators

Quantum Networks

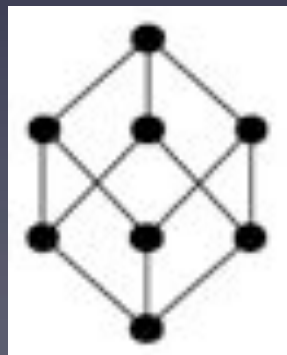
- Quantum computers require **many qubits** that can quickly communicate with each other.
- A possible solution is to couple qubits (or oscillators) as a **hypercube network**. (each node represents a qubit, coupled to some other qubits)
- These networks could be implemented using superconducting qubits!



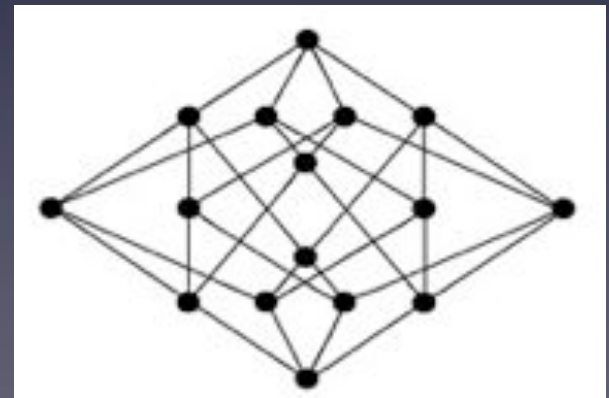
$d = 1$



$d = 2$



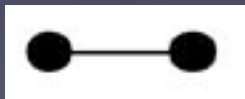
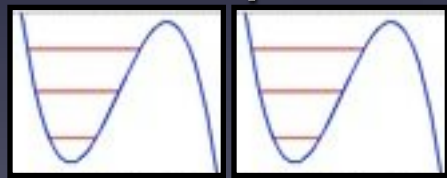
$d = 3$



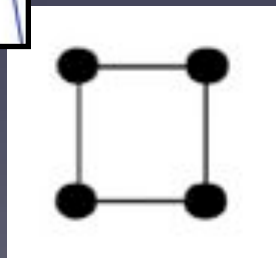
$d = 4$

Quantum Networks

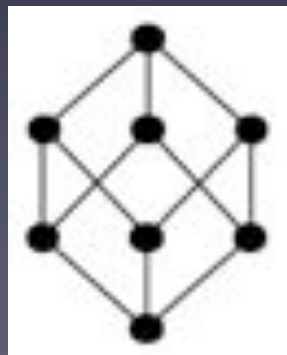
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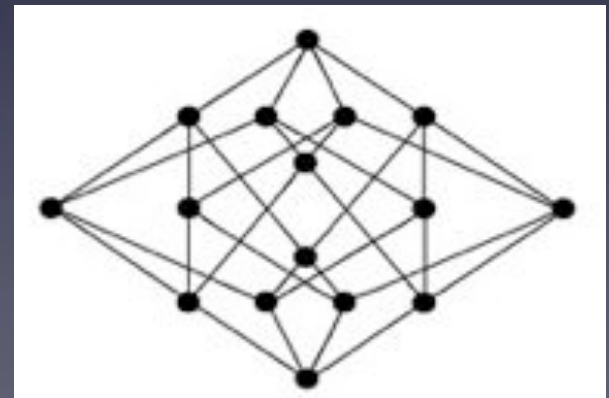
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$d = 2$



$d = 3$



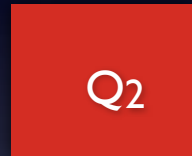
$d = 4$

Quantum Routing: Schematic

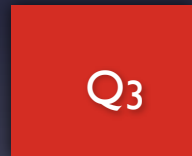
Qubits



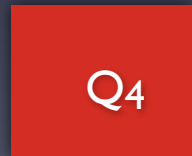
Q1



Q2



Q3



Q4

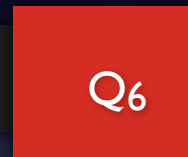


Interconnection
Network

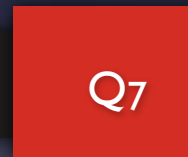
Qubits



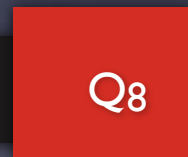
Q5



Q6



Q7



Q8

Quantum Routing: Schematic

Qubits

Q1

Q2

Q3

Q4

Interconnection
Network

Qubits

Q5

Q6

Q7

Q8

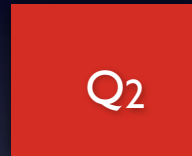
$|\Psi\rangle$

Quantum Routing: Schematic

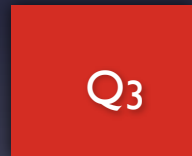
Qubits



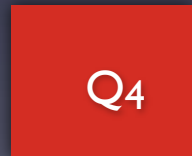
Q1



Q2



Q3

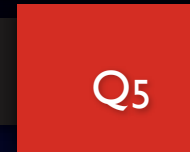


Q4

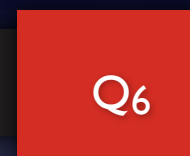


Interconnection
Network

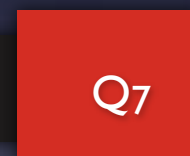
Qubits



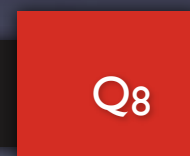
Q5



Q6



Q7



Q8

$|\Psi\rangle$

Quantum Routing: Schematic

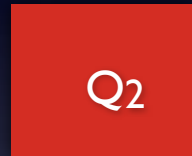


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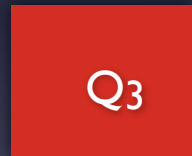
Qubits



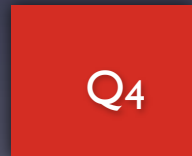
Q1



Q2



Q3

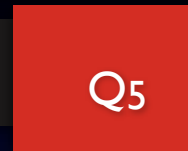


Q4



Interconnection
Network

Qubits



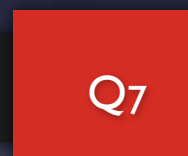
Q5

$|\Psi\rangle$

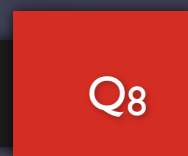


Q6

$|\Psi_2\rangle$



Q7



Q8

$|\Psi_1\rangle$

Quantum Routing

- **Goal:** Use a network of elements (qubits or resonators) to transfer quantum information.
- **Programmable**---send information between any two nodes.
- **Parallel**---information between different pairs of nodes can be sent at the same time.
- Ideally suited for **entanglement distribution** between distinct registers for **teleportation**, error detection, ancilla preparation, and other steps toward fault tolerance.

Entanglement \Rightarrow Teleportation

- Discovered by Bennett, Brassard, Crépeau, Jozsa, Peres, Wootters
- Very useful for quantum computers!



Very Unlikely

Entanglement \Rightarrow Teleportation

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Very Unlikely



but who knows?

Why Hypercubes?

- Transfer of a single quantum state is governed by a matrix differential equation:


$$i \frac{d\Psi}{dt} = \omega A \Psi$$

- ω is a frequency
- A is the adjacency matrix of the network (graph)

- $A_{jk} = 1$ if nodes j and k are connected

- $A_{jk} = 0$ if nodes j and k are not connected

- Two nodes:
(pair):


$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad A_{\text{pair}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Quantum State Transfer

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad \mathbf{1} \bullet \text{---} \bullet \mathbf{2} \quad A_{\text{pair}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\Psi(t) = \exp(-i\omega A_{\text{pair}}t) \Psi(0)$$

$$\exp(-i\omega A_{\text{pair}}t) = \begin{pmatrix} \cos(\omega t) & -i \sin(\omega t) \\ -i \sin(\omega t) & \cos(\omega t) \end{pmatrix}$$

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Quantum State Transfer

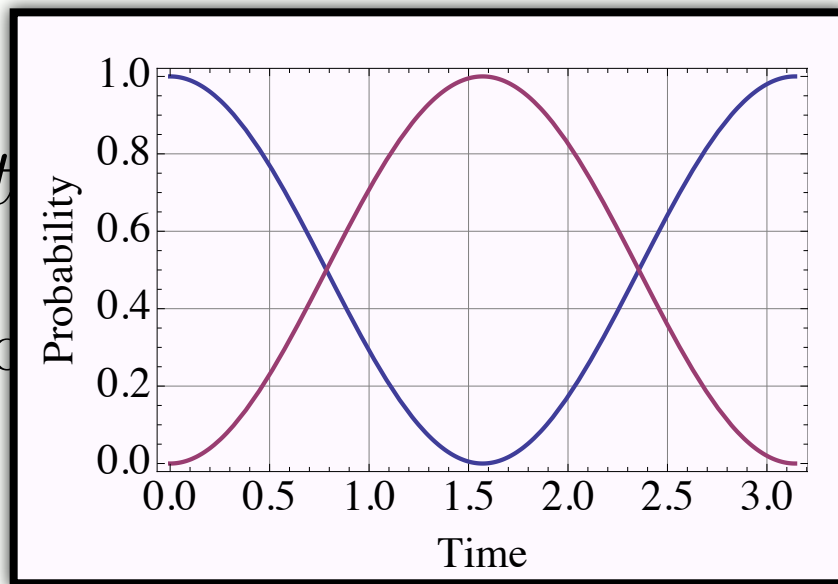
Quantum state oscillates (swaps) between 1 and 2

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad \mathbf{1} \quad \bullet \text{---} \bullet \quad \mathbf{2} \quad A_{\text{pair}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

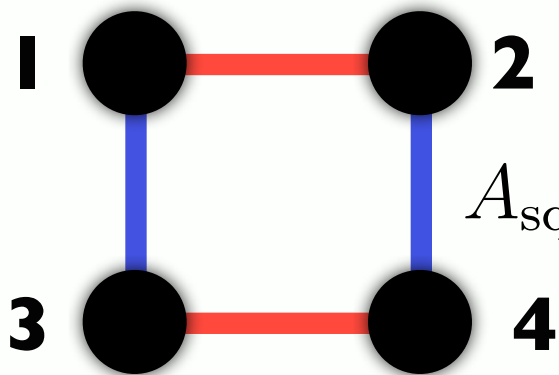
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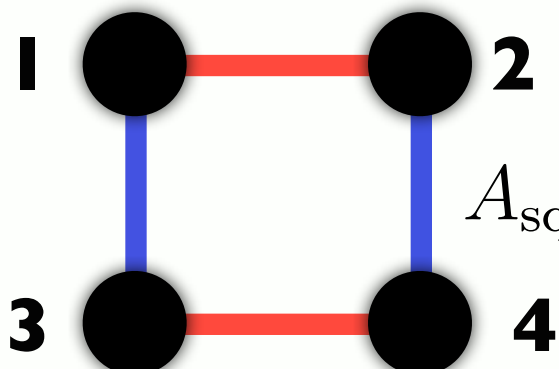
Why Hypercubes ??

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \quad \begin{array}{cc} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{array} \quad A_{\text{square}} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$


$$\exp(-i\omega A_{\text{square}} t) = \begin{pmatrix} c & -is & 0 & 0 \\ -is & c & 0 & 0 \\ 0 & 0 & c & -is \\ 0 & 0 & -is & c \end{pmatrix} \times \begin{pmatrix} c & 0 & -is & 0 \\ 0 & c & 0 & -is \\ -is & 0 & c & 0 \\ 0 & -is & 0 & c \end{pmatrix} \quad (7)$$

$$s = \sin(\omega t) \quad c = \cos(\omega t)$$

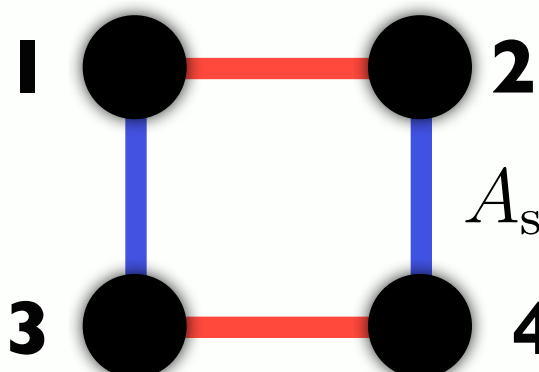
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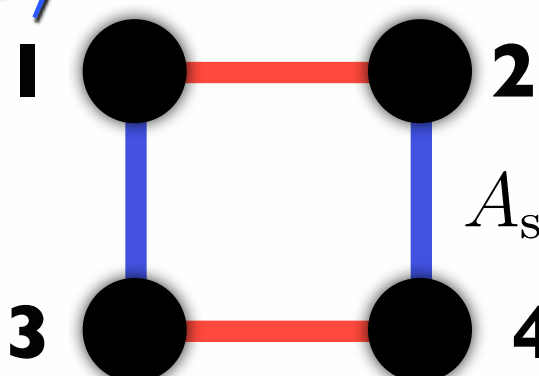
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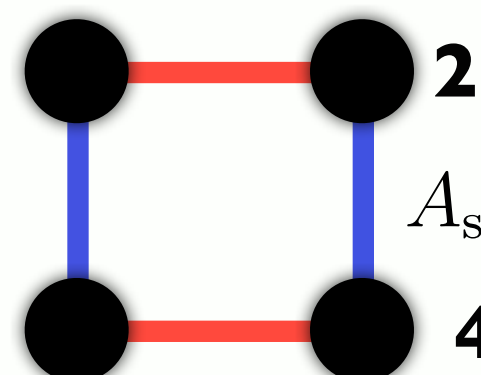
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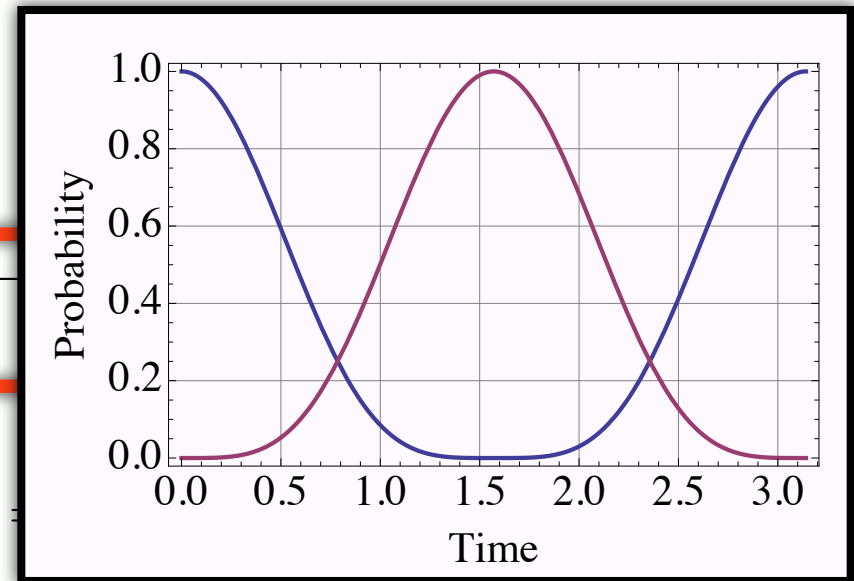
$|\Psi\rangle$
 $\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$



$A_{\text{square}} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$

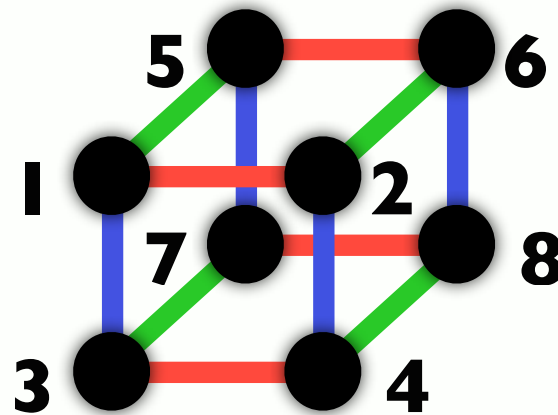
$\exp(-i\omega A_{\text{square}} t) = \begin{pmatrix} c & -is & 0 & 0 \\ -is & c & 0 & 0 \\ 0 & 0 & c & -is \\ 0 & 0 & -is & c \end{pmatrix}$

$s = \sin(\omega t)$ $c = \cos(\omega t)$



Why Hypercubes ???

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \\ \psi_6 \\ \psi_7 \\ \psi_8 \end{pmatrix}$$

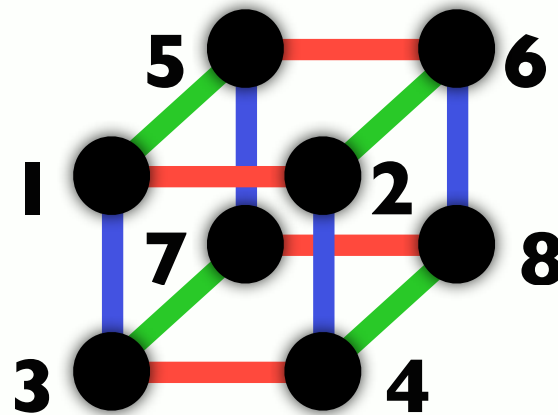


$$A_{\text{cube}} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\exp(-i\omega A_{\text{cube}} t) = \begin{pmatrix} c & -is & 0 & 0 & 0 & 0 & 0 & 0 \\ -is & c & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c & -is & 0 & 0 & 0 & 0 \\ 0 & 0 & -is & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & -is & 0 & 0 \\ 0 & 0 & 0 & 0 & -is & c & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c & -is \\ 0 & 0 & 0 & 0 & 0 & 0 & -is & c \end{pmatrix} \begin{pmatrix} c & 0 & -is & 0 & 0 & 0 & 0 & 0 \\ 0 & c & 0 & -is & 0 & 0 & 0 & 0 \\ -is & 0 & c & 0 & 0 & 0 & 0 & 0 \\ 0 & -is & 0 & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & 0 & -is & 0 \\ 0 & 0 & 0 & 0 & 0 & c & 0 & -is \\ 0 & 0 & 0 & 0 & -is & 0 & c & 0 \\ 0 & 0 & 0 & 0 & 0 & -is & 0 & c \end{pmatrix} \begin{pmatrix} c & 0 & 0 & 0 & -is & 0 & 0 & 0 \\ 0 & c & 0 & 0 & 0 & -is & 0 & 0 \\ 0 & 0 & c & 0 & 0 & 0 & -is & 0 \\ 0 & 0 & 0 & c & 0 & 0 & 0 & -is \\ -is & 0 & 0 & 0 & c & 0 & 0 & 0 \\ 0 & -is & 0 & 0 & 0 & c & 0 & 0 \\ 0 & 0 & -is & 0 & 0 & 0 & c & 0 \\ 0 & 0 & 0 & -is & 0 & 0 & 0 & c \end{pmatrix}$$

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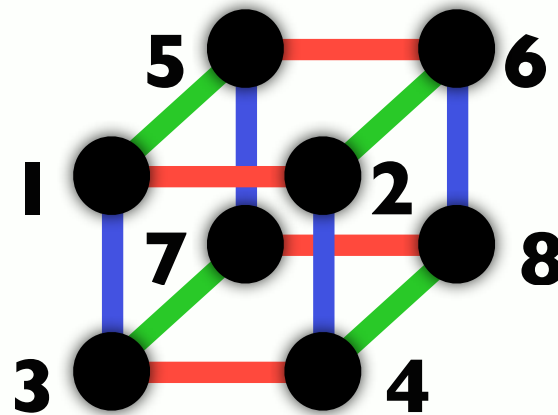
$A_{\text{cube}} =$

$$= \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\exp(-i\omega A_{\text{cube}}t) = \begin{pmatrix} c & -is & 0 & 0 & 0 & 0 & 0 & 0 \\ -is & c & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c & -is & 0 & 0 & 0 & 0 \\ 0 & 0 & -is & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & -is & 0 & 0 \\ 0 & 0 & 0 & 0 & -is & c & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c & -is \\ 0 & 0 & 0 & 0 & 0 & 0 & -is & c \end{pmatrix} \begin{pmatrix} c & 0 & -is & 0 & 0 & 0 & 0 & 0 \\ 0 & c & 0 & -is & 0 & 0 & 0 & 0 \\ -is & 0 & c & 0 & 0 & 0 & 0 & 0 \\ 0 & -is & 0 & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & 0 & -is & 0 \\ 0 & 0 & 0 & 0 & 0 & c & 0 & -is \\ 0 & 0 & 0 & 0 & 0 & -is & c & 0 \\ 0 & 0 & 0 & 0 & 0 & -is & 0 & c \end{pmatrix} \begin{pmatrix} c & 0 & 0 & 0 & -is & 0 & 0 & 0 \\ 0 & c & 0 & 0 & 0 & -is & 0 & 0 \\ 0 & 0 & c & 0 & 0 & 0 & -is & 0 \\ 0 & 0 & 0 & c & 0 & 0 & 0 & -is \\ -is & 0 & 0 & 0 & c & 0 & 0 & 0 \\ 0 & -is & 0 & 0 & 0 & c & 0 & 0 \\ 0 & 0 & -is & 0 & 0 & 0 & c & 0 \\ 0 & 0 & 0 & -is & 0 & 0 & 0 & c \end{pmatrix}$$

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$A_{\text{cube}} =$

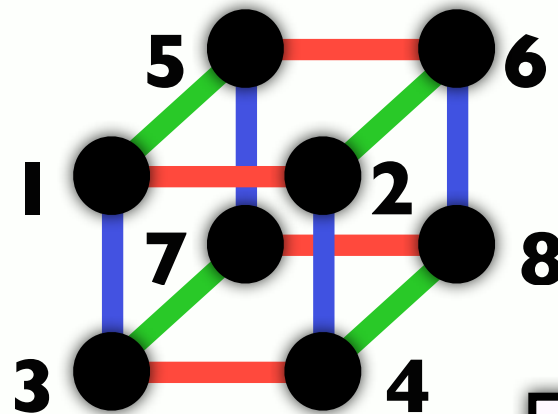
$$A_{\text{cube}} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

“Simple” matrices for each direction!

$$\exp(-i\omega A_{\text{cube}} t) = \begin{pmatrix} c & -is & 0 & 0 & 0 & 0 & 0 & 0 \\ -is & c & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c & -is & 0 & 0 & 0 & 0 \\ 0 & 0 & -is & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & -is & 0 & 0 \\ 0 & 0 & 0 & 0 & -is & c & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c & -is \\ 0 & 0 & 0 & 0 & 0 & 0 & -is & c \end{pmatrix} \begin{pmatrix} c & 0 & -is & 0 & 0 & 0 & 0 & 0 \\ 0 & c & 0 & -is & 0 & 0 & 0 & 0 \\ -is & 0 & c & 0 & 0 & 0 & 0 & 0 \\ 0 & -is & 0 & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & 0 & -is & 0 \\ 0 & 0 & 0 & 0 & 0 & c & 0 & -is \\ 0 & 0 & 0 & 0 & -is & 0 & c & 0 \\ 0 & 0 & 0 & 0 & 0 & -is & 0 & c \end{pmatrix} \begin{pmatrix} c & 0 & 0 & 0 & -is & 0 & 0 & 0 \\ 0 & c & 0 & 0 & 0 & -is & 0 & 0 \\ 0 & 0 & c & 0 & 0 & 0 & -is & 0 \\ 0 & 0 & 0 & c & 0 & 0 & 0 & -is \\ -is & 0 & 0 & 0 & c & 0 & 0 & 0 \\ 0 & -is & 0 & 0 & 0 & c & 0 & 0 \\ 0 & 0 & -is & 0 & 0 & 0 & c & 0 \\ 0 & 0 & 0 & -is & 0 & 0 & 0 & c \end{pmatrix}$$

Why Hypercubes ???

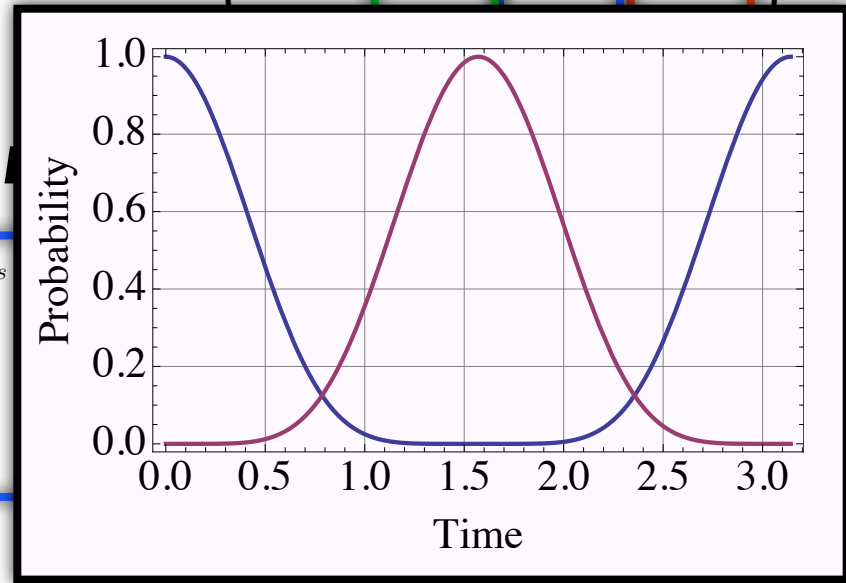
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$$A_{\text{cube}} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

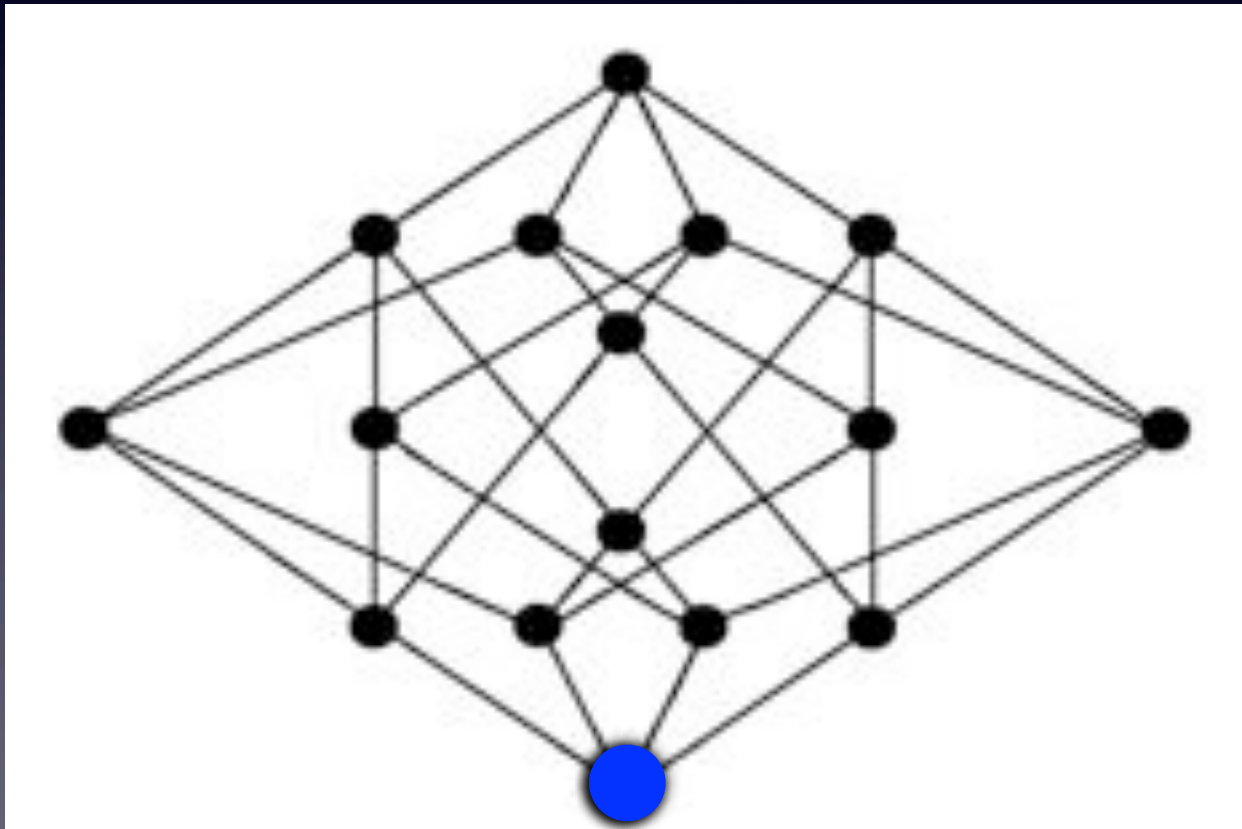
“Simple” matrices for

$$\exp(-i\omega A_{\text{cube}} t) = \begin{pmatrix} c & -is & 0 & 0 & 0 & 0 & 0 & 0 \\ -is & c & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c & -is & 0 & 0 & 0 & 0 \\ 0 & 0 & -is & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & -is & 0 & 0 \\ 0 & 0 & 0 & 0 & -is & c & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c & -is \\ 0 & 0 & 0 & 0 & 0 & 0 & -is & c \end{pmatrix} \begin{pmatrix} c & 0 & -is \\ 0 & c & 0 \\ -is & 0 & c \\ 0 & -is & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



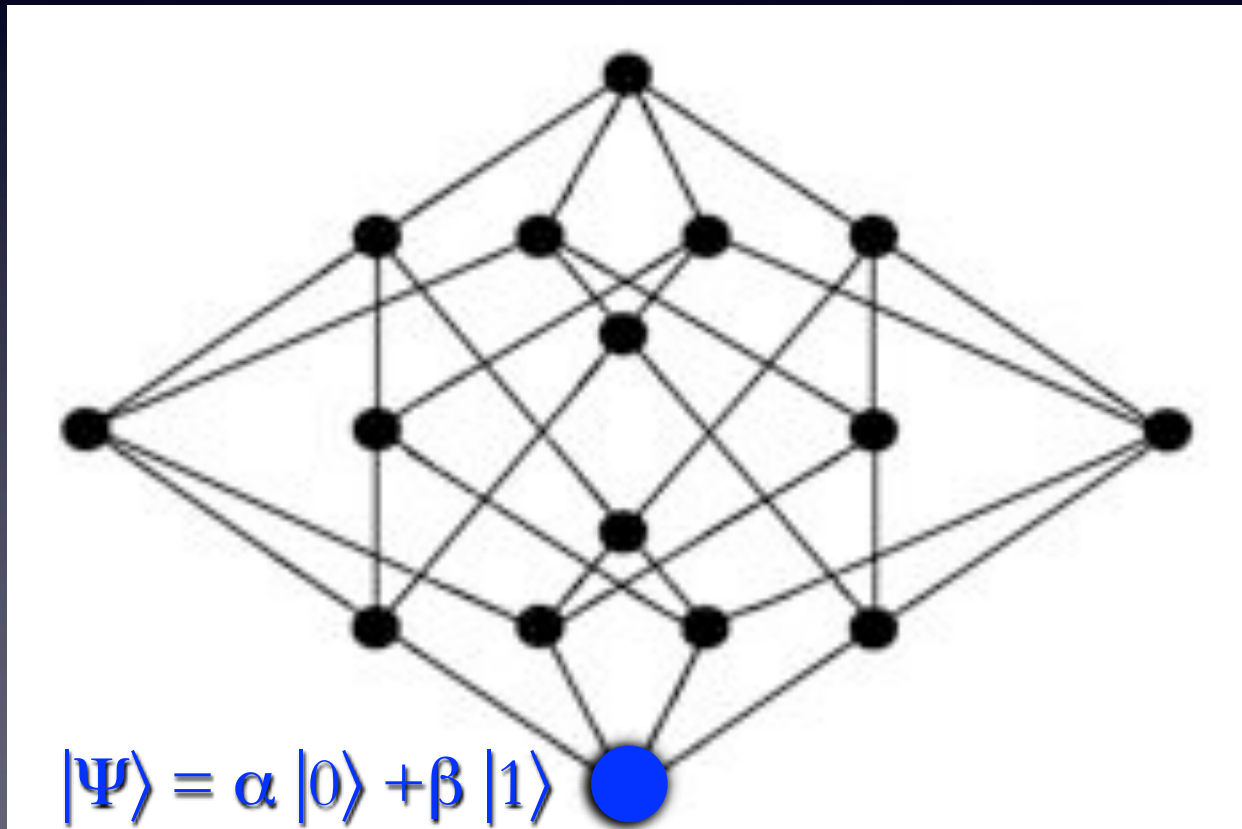
Hypercube State Transfer

- Each node represents a qubit. Quantum states travel along **all paths simultaneously** in superposition with **full constructive interference**, yielding **perfect state transfer**.



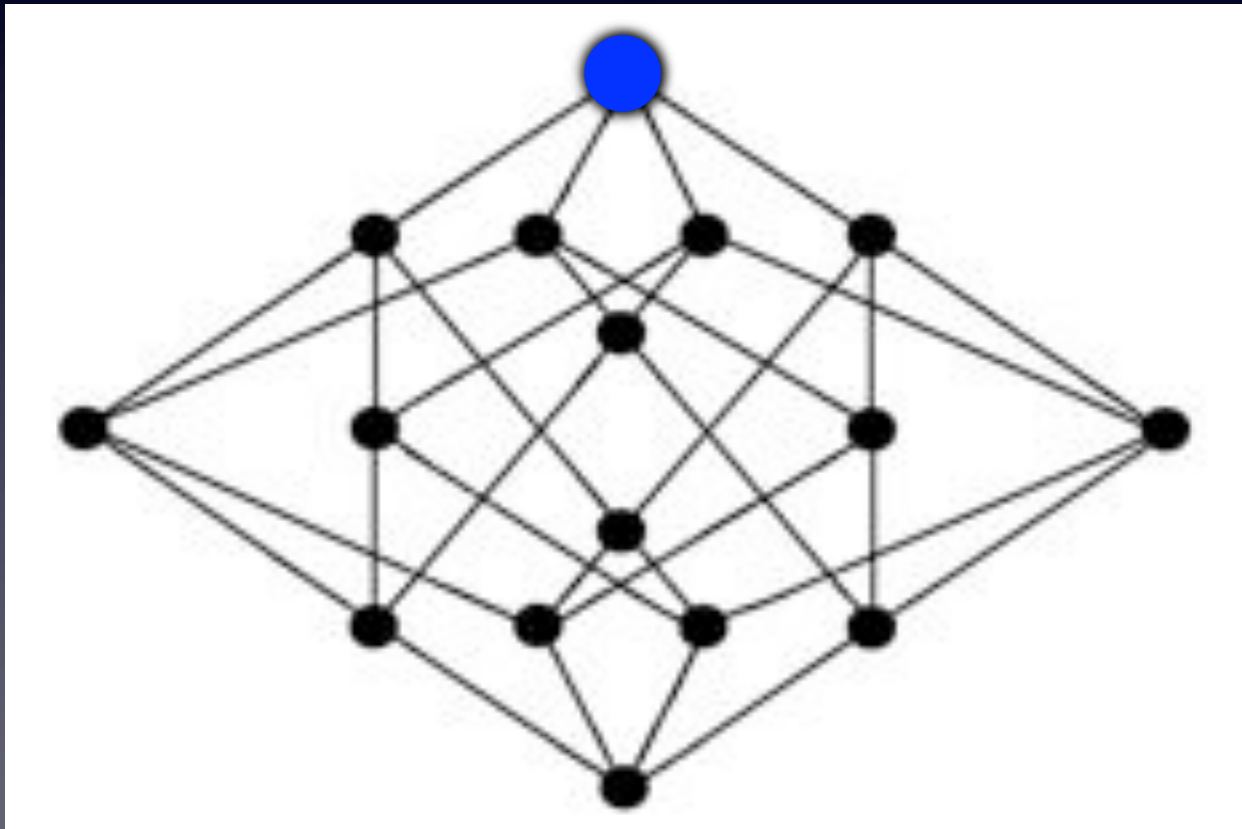
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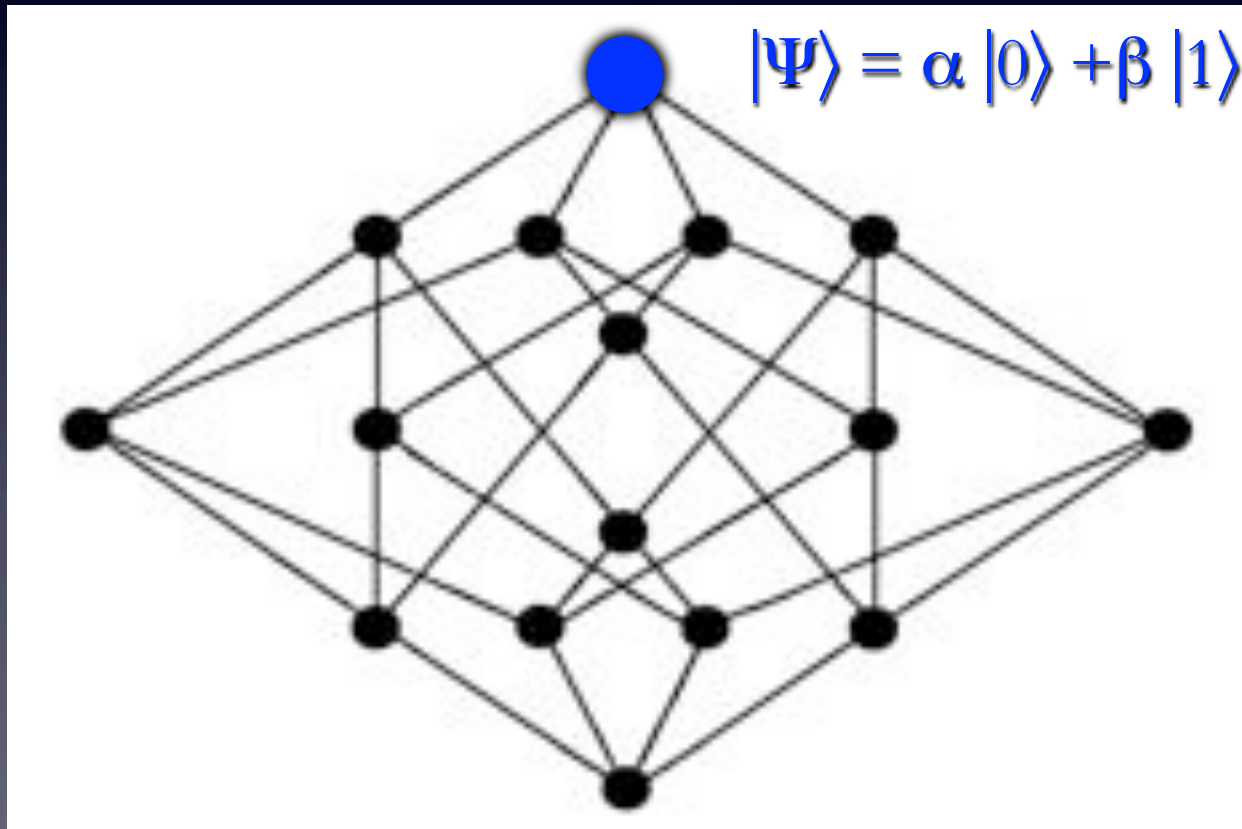
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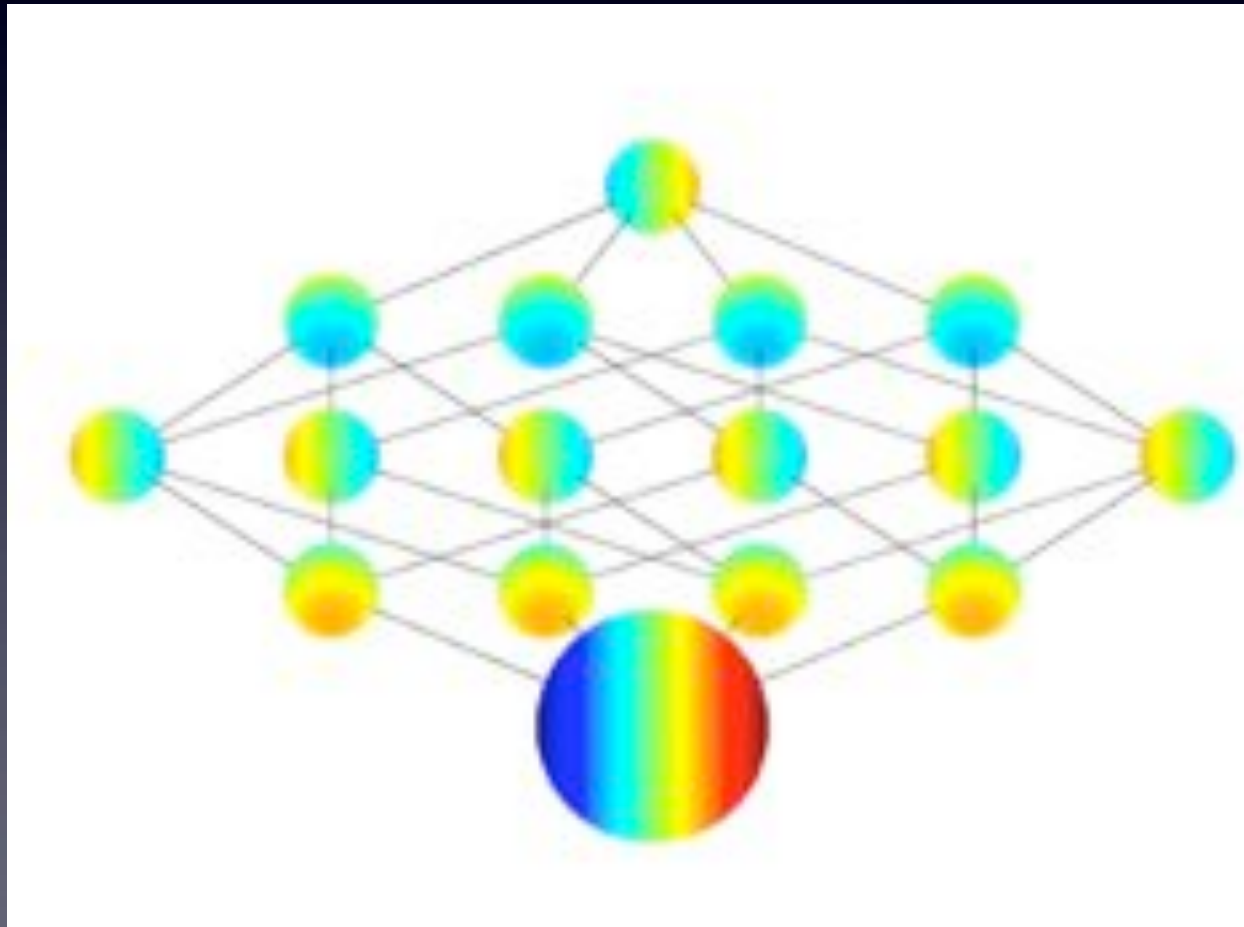
Exact Calculation

with Qiao Zhang '13

$$\Psi = |\Psi| e^{i\theta}$$

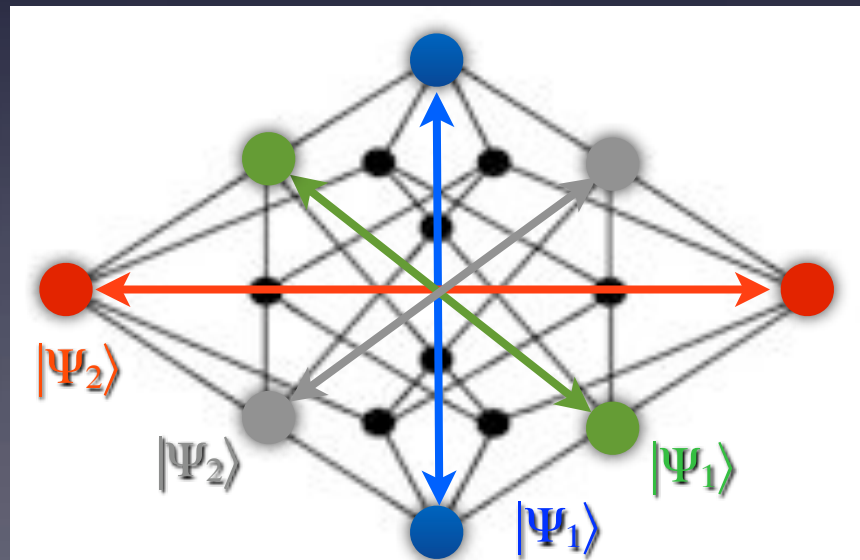
Size of Sphere $\sim |\Psi|$

Orientation of Sphere $\sim \theta$



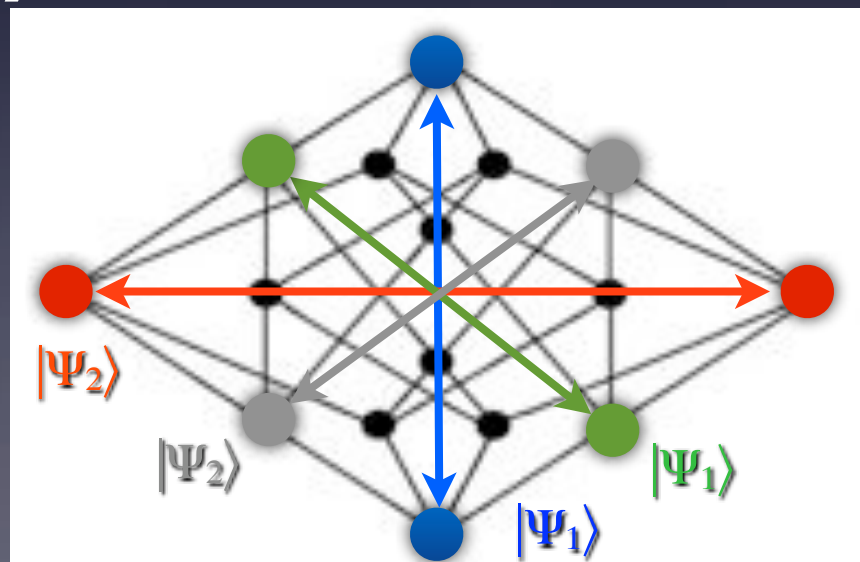
Parallel State Transfer

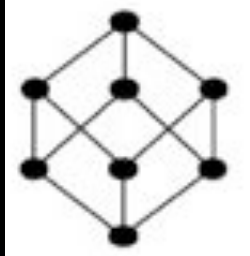
- Transmit multiple quantum states at the same time!
- Use Oscillator Networks:
Each node has an infinite number of states!
- **Calculation** not as simple, but still **exactly solvable**.



Parallel State Transfer

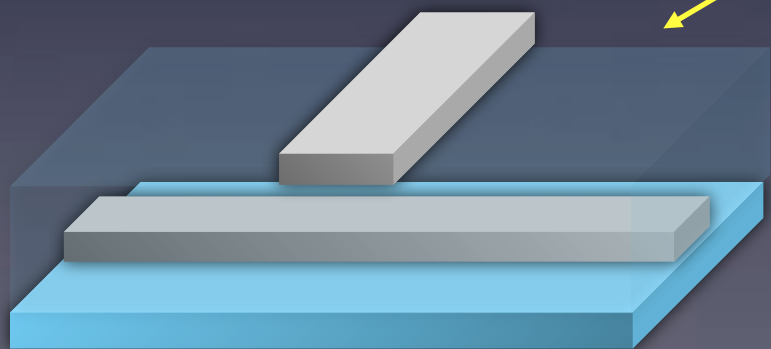
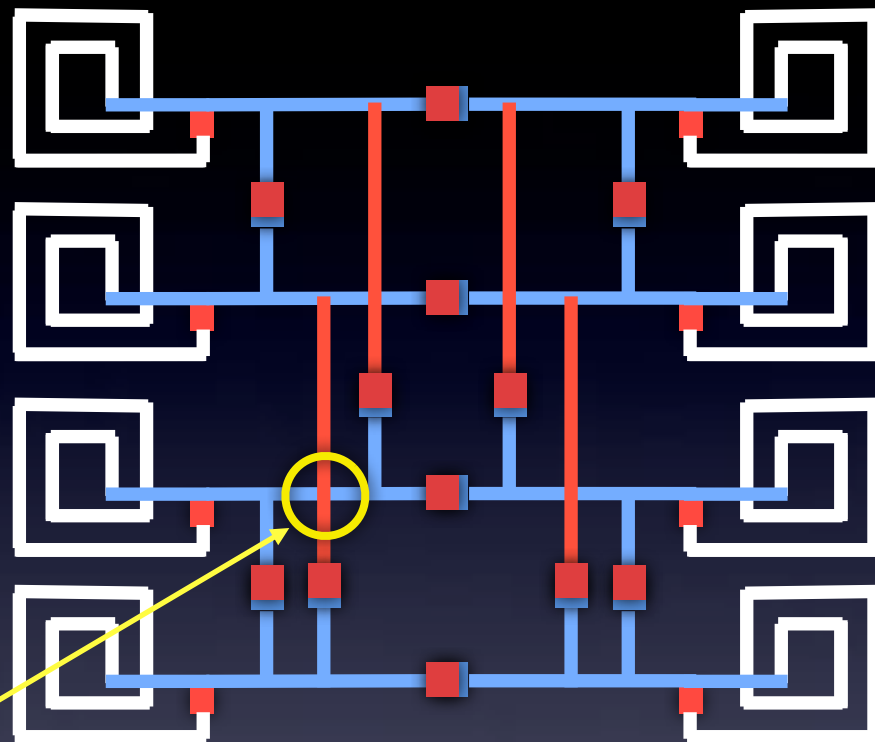
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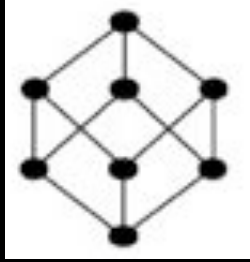




Phase Qubit Cube 1 2 3

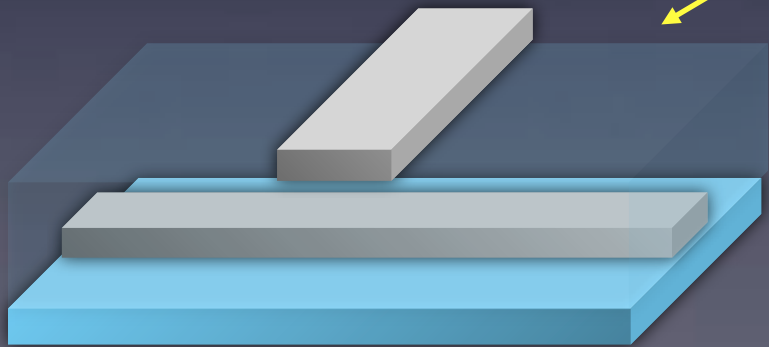
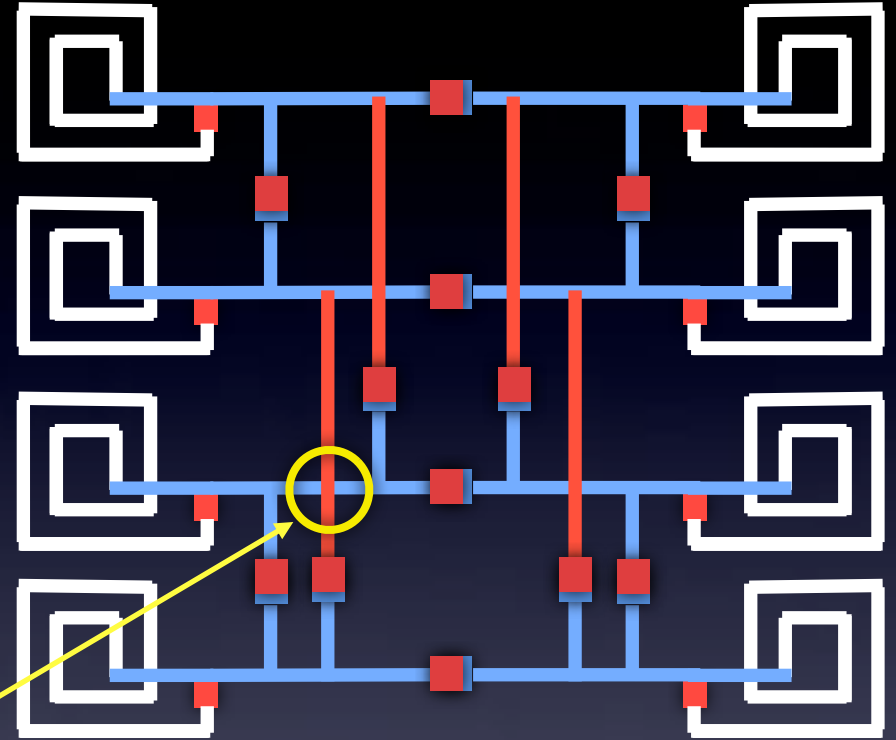
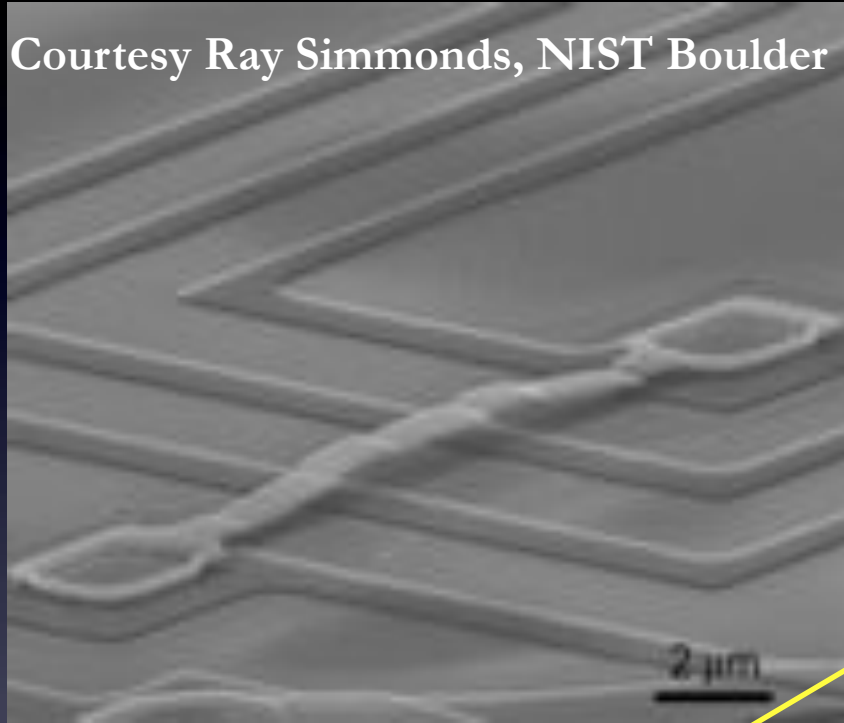
- *Circuits do not need to be simple two-dimensional layouts.*
- *Multi-layer interconnects allow many **crossovers** and complex couplings.*





Phase Qubit Cube 1 2 3

Courtesy Ray Simmonds, NIST Boulder



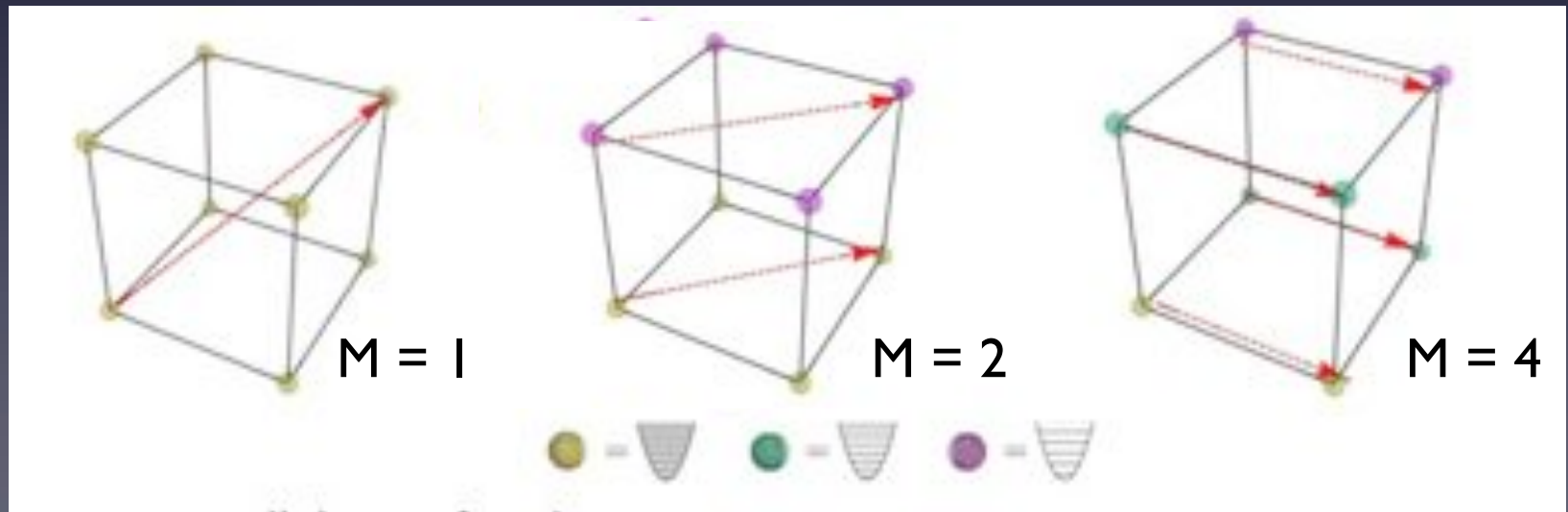
Outline

- ~~Superconducting Qubits and Resonators~~
- ~~Quantum Routing on Networks~~
- ~~Perfect State Transfer on Hypercube Networks~~
- **Parallel State Transfer and Efficient Quantum Routing**
- **Quantum Computing with Superconducting Resonators**

Chris Chudzicki's Senior Thesis

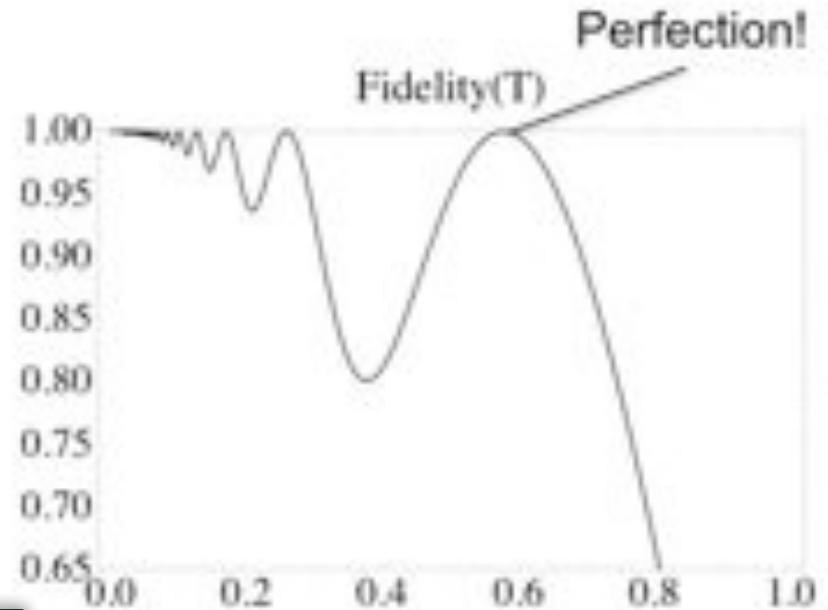
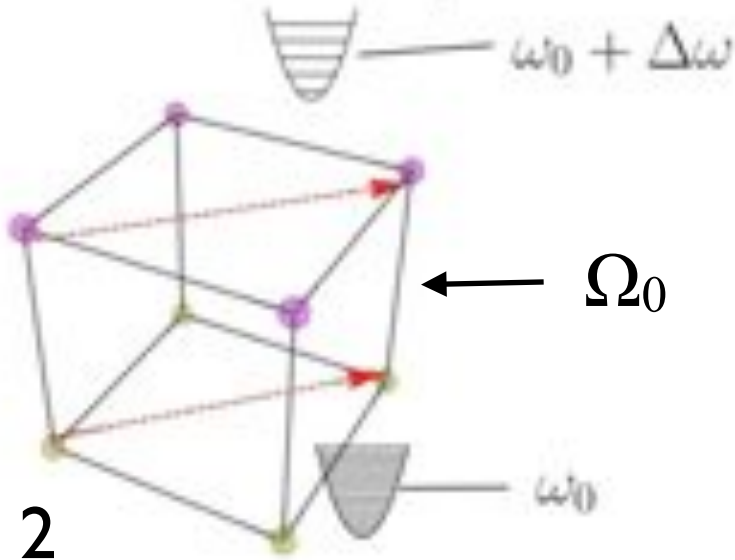


- Study tunable resonators: exactly solvable!
- Split cube into M subcubes by resonator frequencies.
(Matrices are still simple!)
- Send quantum states in parallel.



Parallel Transfer Fidelity

Fidelity = Probability of successfully transmitting a quantum state



$$F(T) \gtrsim 1 - \frac{3}{2} \log_2(M) \eta^2 \sin^2 \xi_T$$

$$\xi_T = \frac{\pi}{2} \sqrt{1 + \eta^{-2}}$$

$$\eta = 2\Omega_0 / \Delta\omega$$

Detuning parameter

$M = \#$ of parallel “messages”

Efficient Quantum Routing

Method to Characterize the Efficiency of Quantum Routing by
Parallel State Transfer

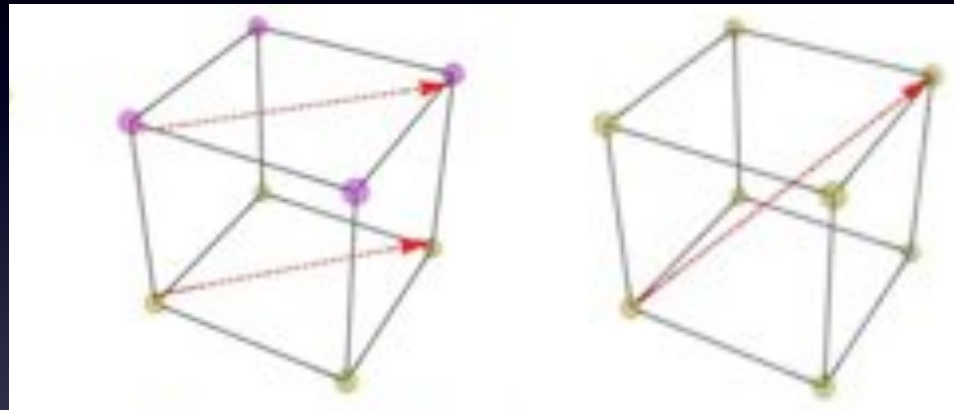
- **Goal** = Distribute entanglement between every pair of nodes ($N = 2^d$ total nodes for a hypercube network)
- Use **parallel state transfer**, sending states on each subcube
- Tune oscillators in a fixed frequency range (**finite bandwidth**)
- **Qubit-Compatible Scheme** = one state on each subcube
- **Massively Parallel Scheme** = multiple states on each subcube
- **Efficiency = Entanglement Distribution Rate**

Qubit-Compatible Scheme

One state per
subcube

$N = 2^d$ nodes total

Rate scales as:



$$\mathcal{R}^{(\text{QC})} \approx \frac{1}{T} N^{0.415} \left(1 - \frac{3}{4} \frac{\Omega_0^2}{(\omega_{\max} - \omega_{\min})^2} d^2 (d + 3) \right),$$

Rate calculation includes full set of “messages,”
weighted by their fidelities.

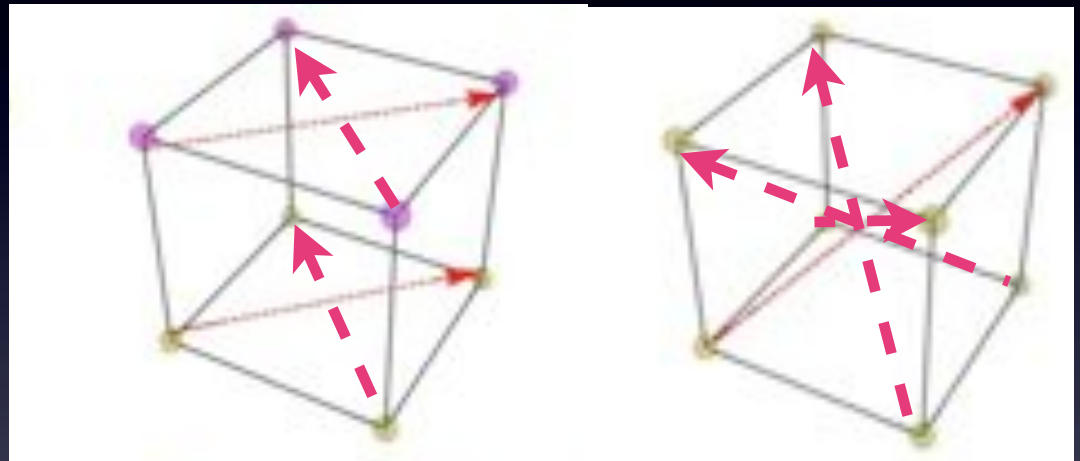
Massively Parallel Scheme

Multiple states per
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$N = 2^d$ nodes total

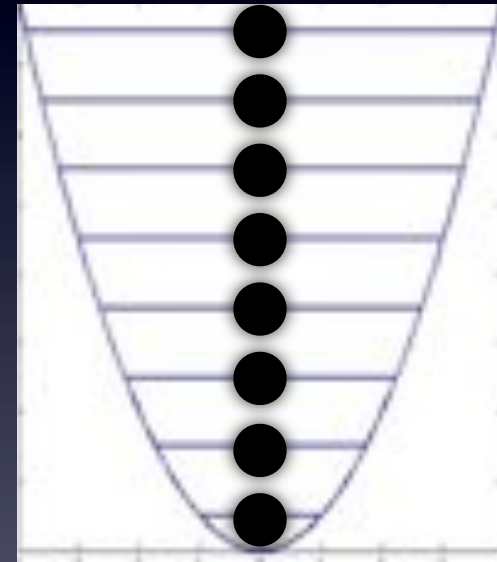
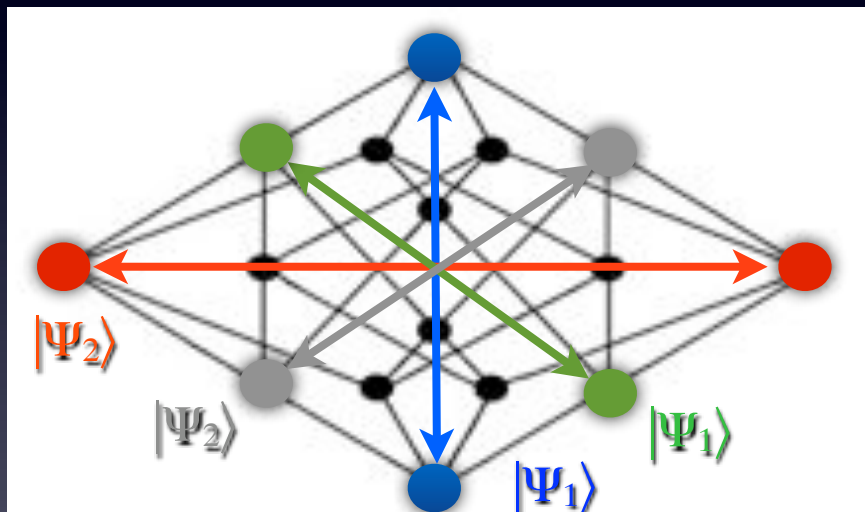
Rate scales as:

$$\mathcal{R}^{(\text{MP})} \approx \frac{1}{T} N \left(1 - \frac{3}{4} \frac{\Omega_0^2}{(\omega_{\max} - \omega_{\min})^2} d^2 (d + 3) \right).$$



Massively Parallel Distribution

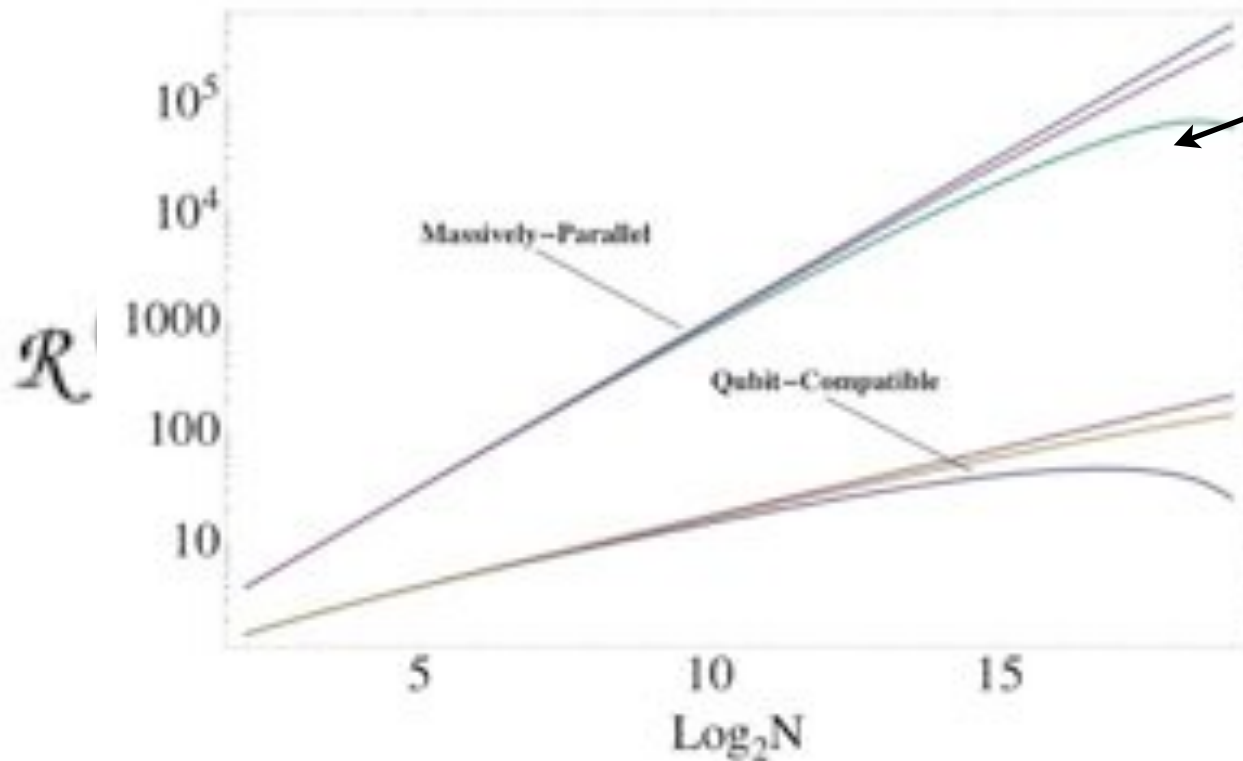
- Send oscillators' states between all corners simultaneously!



- Genuine Quantum Property!
- ***Excitations are noninteracting bosons: multiple photons just pass through each other.***

Massively Parallel Rate

Comparing Efficiencies for
Qubit Compatible and Massively Parallel



Various Bandwidths

$$\mathcal{R}^{(QC)} \propto N^{0.415}$$

$$\mathcal{R}^{(MP)} \propto N$$

(N registers)

Entanglement Distribution using Oscillator Networks is both optimally efficient and robust!

Apker Award

Apker Finalists Meet in Washington



Photo by Shelly Johnson

Each year, APS selects two recipients of the Apker Award for outstanding research by an undergraduate. To determine the recipients, a number of finalists are chosen, and then interviewed by the selection committee. This year, the seven finalists met with the committee in Washington on September 3. They are, left to right: Chia Wei Hsu (Wesleyan University); Martin Blood-Forsythe (Harvard College); Erik Pedgura (UC, Berkeley); Benjamin Good (Baltimore College); Patrick Gallagher (Stanford University); William Thrope (MIT); and Christopher Chulnick (Williams College). The recipients will be announced on the APS website and in a later issue of APS News.

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Beyond State Transfer

- **Superconducting resonators** could be used for quantum logic.
- Instead of qubits, these would be qudits (digits with arbitrary base)
- Resonators are significantly easier to fabricate and of higher quality than qubits!
- Results so far:
 - **Entangling Two Superconducting Resonators**
 - **Using Resonator as Multi-qubit Memory**
 - **Quantum Logic Using Resonator States**

Entangling Oscillators

FWS, K Jacobs, and RW Simmonds,
Phys. Rev. Lett. **105**, 050501 (2010)

Quantum Algorithm to generate highly non-classical “NOON” states:

$$|\Psi_{\text{NOON}}\rangle = |N,0\rangle + |0,N\rangle$$

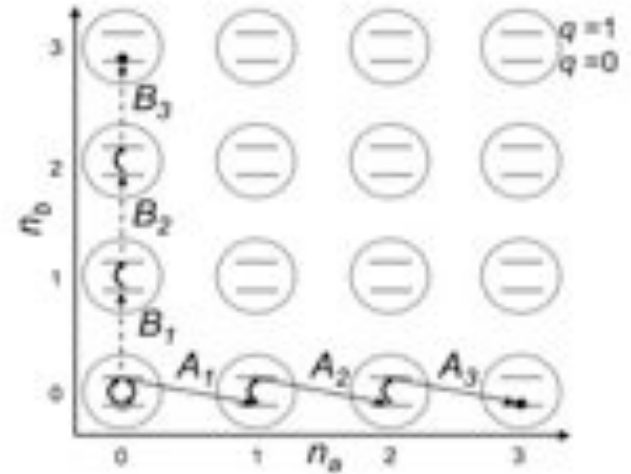
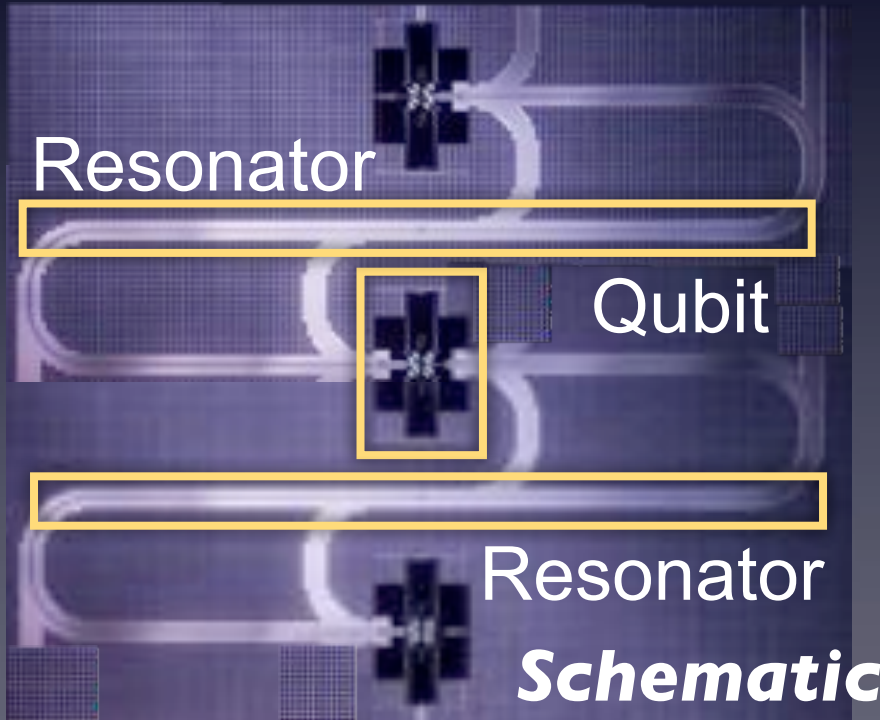


TABLE I: NOON State Synthesis Procedure

Step	Parameters	State
$R_{0,1}$	$\Omega_{q0,1} = \pi/2, \omega_1 = \omega_0$	$ 0,0,0\rangle - i 1,0,0\rangle$
A_1	$\varphi_1 \omega_1 = \pi$	$ 0,0,0\rangle - 0,1,0\rangle$
$R_{1,2}$	$\Omega_{q0,2} = \pi, \omega_2 = \omega_1$	$ 0,0,0\rangle + i 1,1,0\rangle$
A_2	$\varphi_2 \omega_2 = \pi/\sqrt{2}$	$ 0,0,0\rangle + 0,2,0\rangle$
$R_{2,3}$	$\Omega_{q0,3} = \pi, \omega_3 = \omega_2$	$ 0,0,0\rangle - i 1,2,0\rangle$
A_3	$\varphi_3 \omega_3 = \pi/\sqrt{3}$	$ 0,0,0\rangle - 0,3,0\rangle$
B_1	$\Omega_{q1,1} = \pi, \omega_1 = \omega_0$	$-i 1,0,0\rangle - 0,3,0\rangle$
B_2	$\varphi_2 \omega_2 = \pi$	$- 0,0,1\rangle - 0,3,0\rangle$
$R_{0,2}$	$\Omega_{q0,2} = \pi, \omega_2 = \omega_{-1}$	$i 1,0,1\rangle - 0,3,0\rangle$
B_2	$\varphi_2 \omega_2 = \pi/\sqrt{2}$	$ 0,0,2\rangle - 0,3,0\rangle$
$R_{0,3}$	$\Omega_{q0,3} = \pi, \omega_3 = \omega_{-2}$	$-i 1,0,2\rangle - 0,3,0\rangle$
B_3	$\varphi_3 \omega_3 = \pi/\sqrt{3}$	$- 0,0,3\rangle - 0,3,0\rangle$



Experimental Results

PRL **106**, 060401 (2011)

Selected for a **Viewpoint** in *Physics*
PHYSICAL REVIEW LETTERS

week ending
11 FEBRUARY 2011



Deterministic Entanglement of Photons in Two Superconducting Microwave Resonators

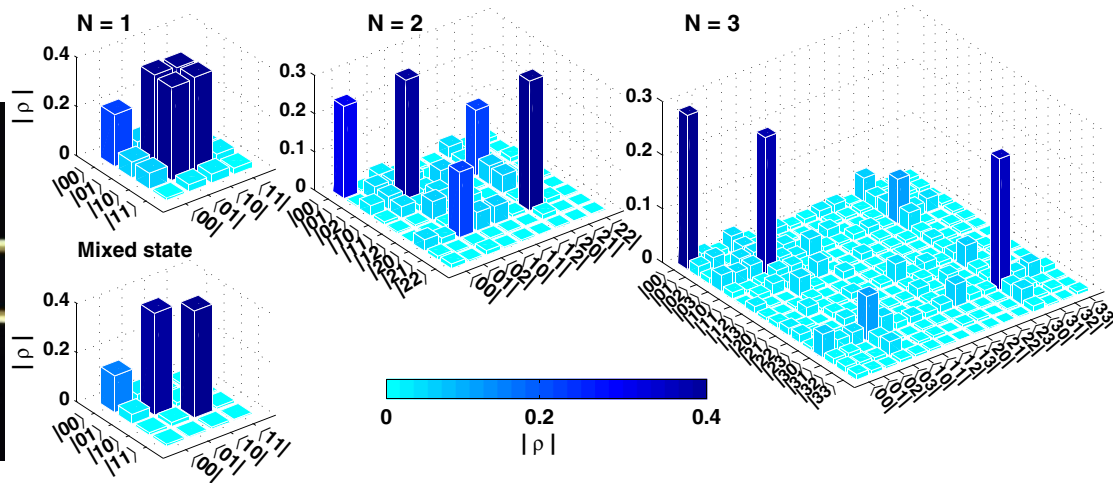
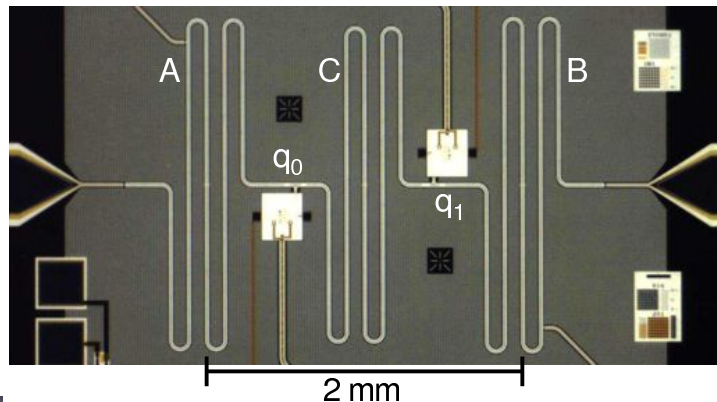
H. Wang,^{1,2} Matteo Mariani, ¹ Radoslaw C. Bialczak,¹ M. Lenander,¹ Erik Lucero,¹ M. Neeley,¹ A. D. O'Connell,¹ D. Sank,¹ M. Weides,¹ J. Wenner,¹ T. Yamamoto,^{1,3} Y. Yin,¹ J. Zhao,¹ John M. Martinis,¹ and A. N. Cleland^{1,*}

¹Department of Physics, University of California, Santa Barbara, California 93106, USA

²Department of Physics and Zhejiang California International NanoSystems Institute, Zhejiang University, Hangzhou 310027, China

³Green Innovation Research Laboratories, NEC Corporation, Tsukuba, Ibaraki 305-8501, Japan

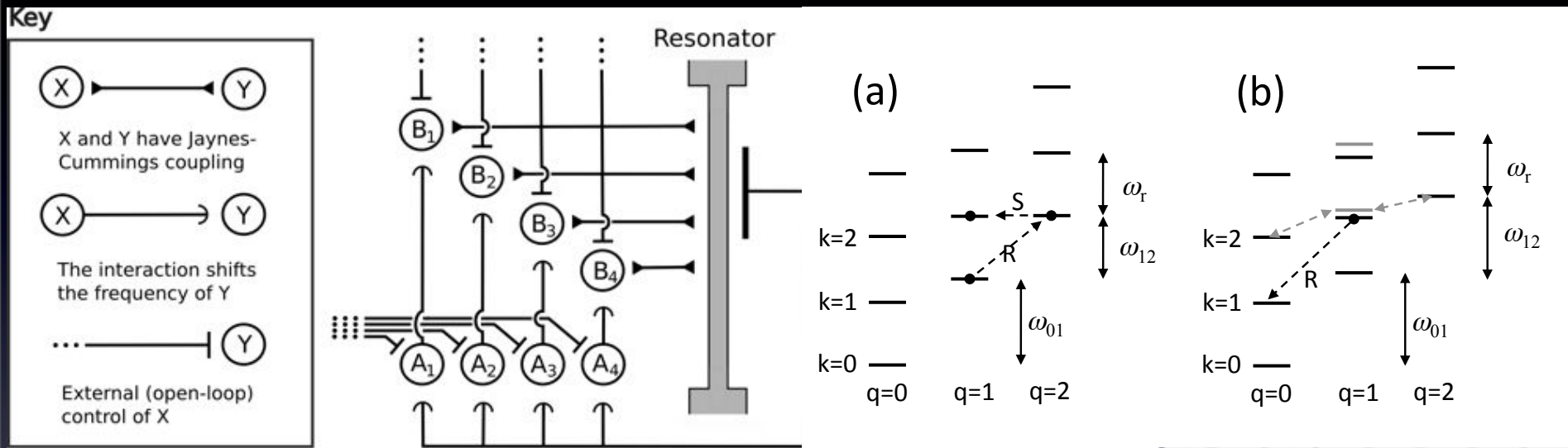
(Received 9 November 2010; published 7 February 2011)



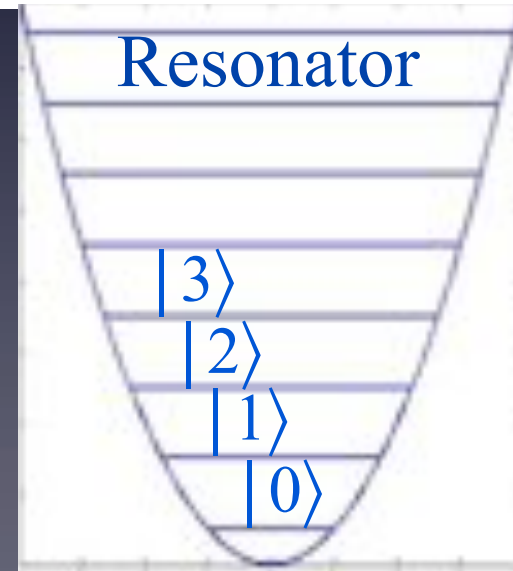
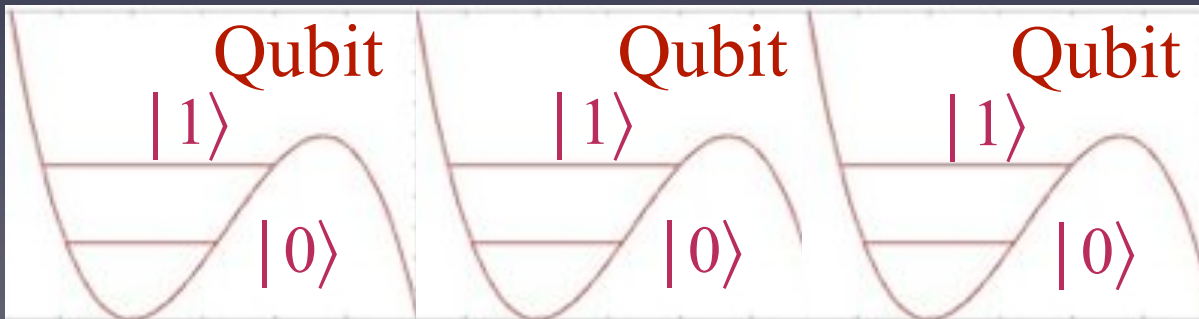
• Martinis Group, UC Santa Barbara

Multi-Qubit Memory

FWS and K Jacobs, in preparation

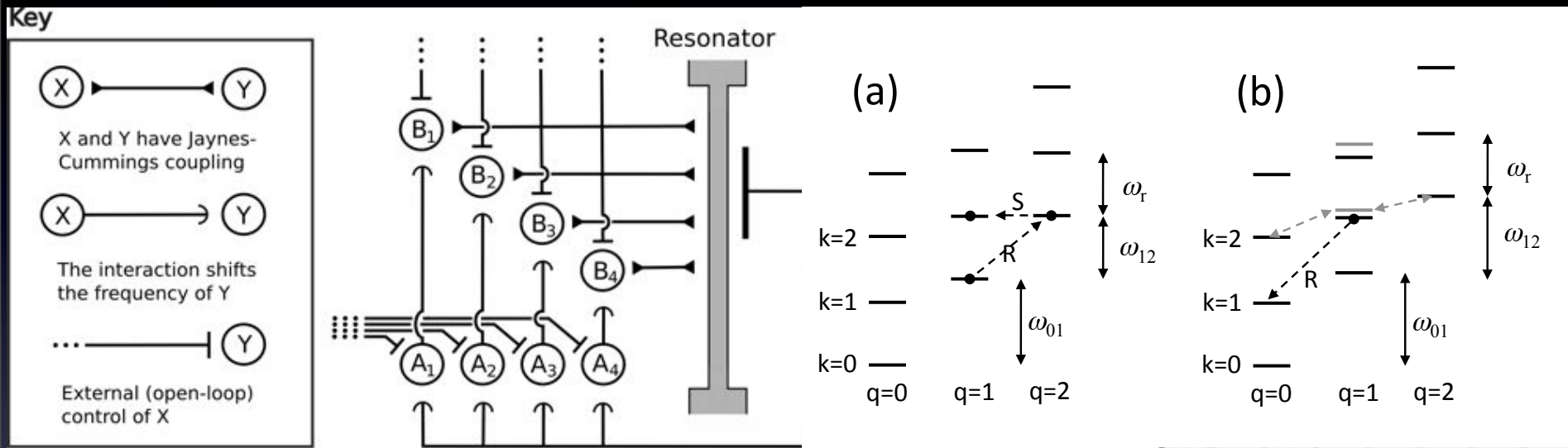


Swap Multiple Qubits into a Resonator

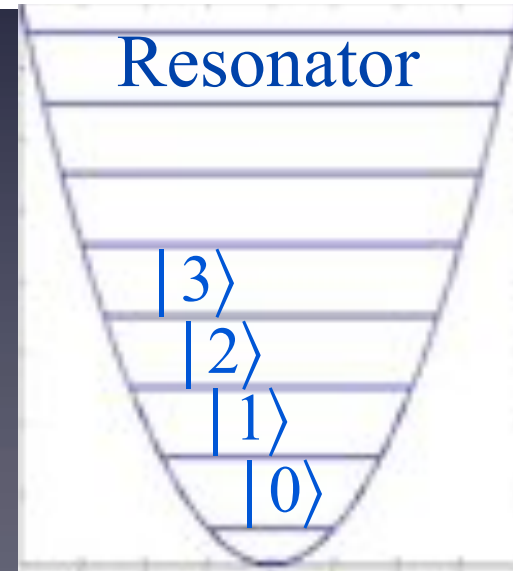
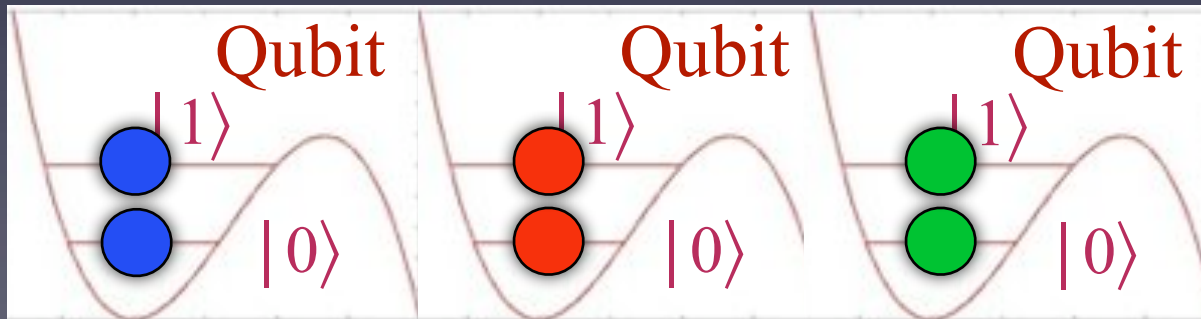


Multi-Qubit Memory

FWS and K Jacobs, in preparation

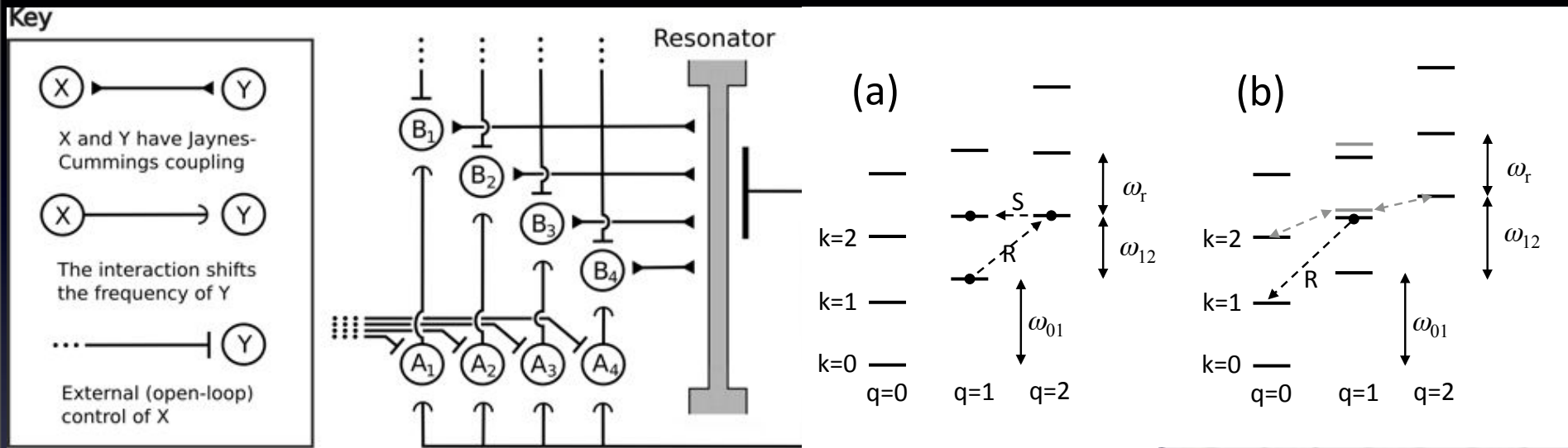


Swap Multiple Qubits into a Resonator

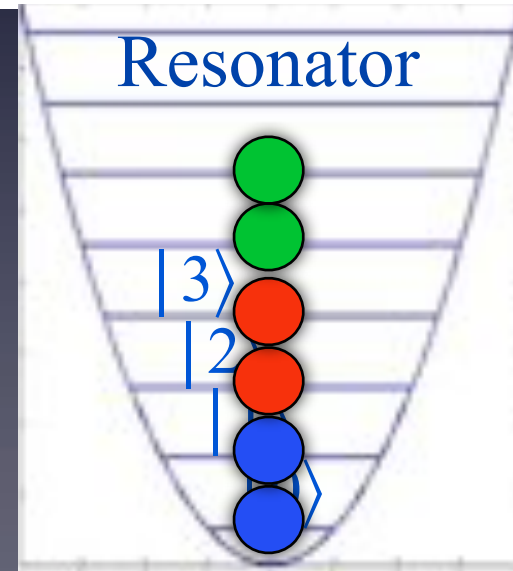
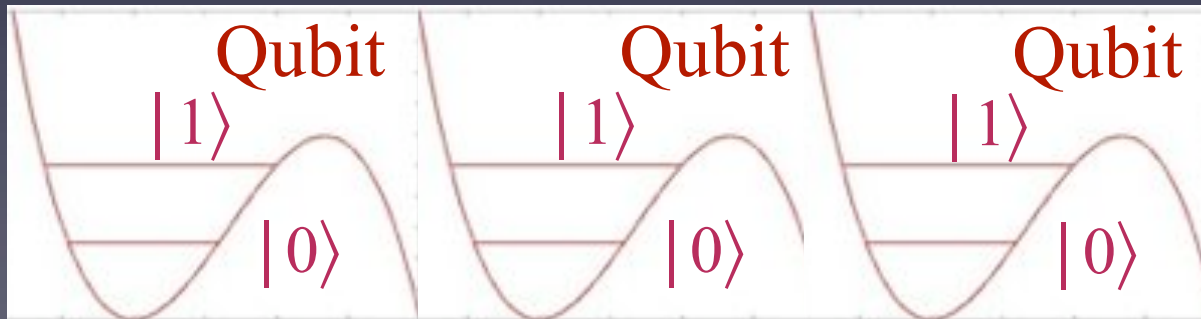


Multi-Qubit Memory

FWS and K Jacobs, in preparation

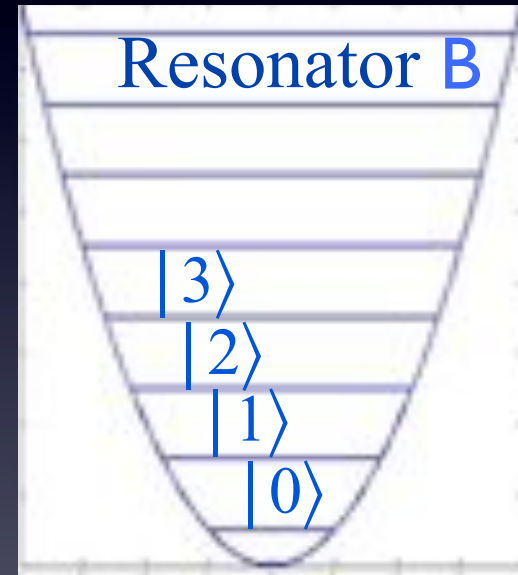
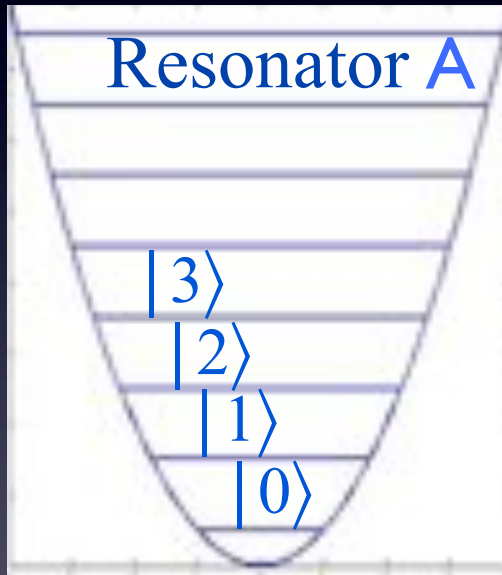


Swap Multiple Qubits into a Resonator



Quantum Logic

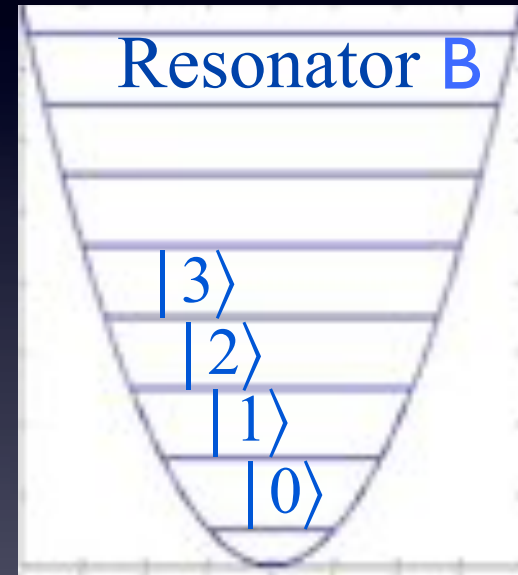
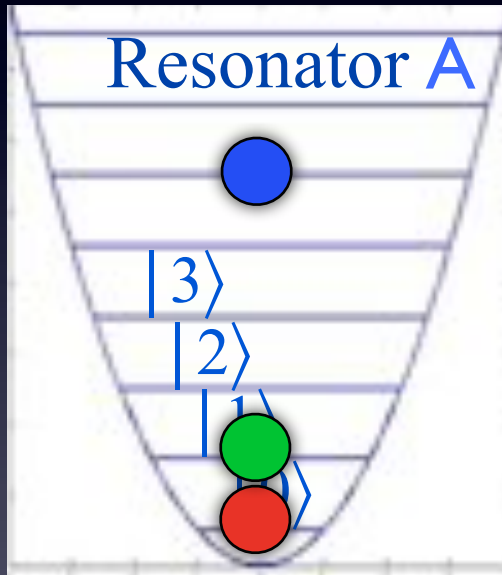
FWS and K Jacobs, in preparation



Same methods can be used to
Generate Arbitrary Transformations between Resonators

Quantum Logic

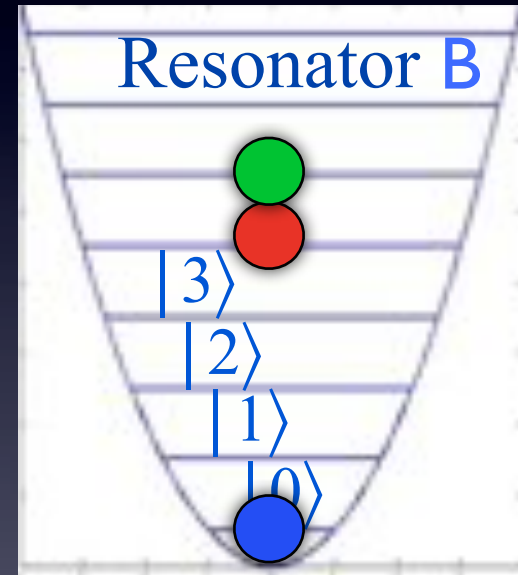
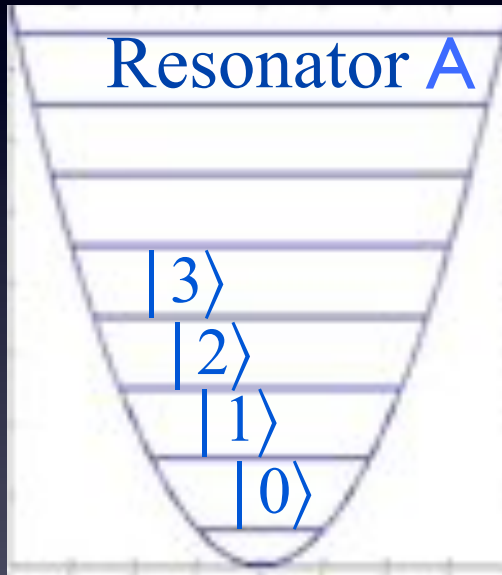
FWS and K Jacobs, in preparation



Same methods can be used to
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Quantum Logic

FWS and K Jacobs, in preparation



Same methods can be used to
Generate Arbitrary Transformations between Resonators

Conclusions

- Superconducting Circuits are an exciting testbed for quantum processes (state transfer) and other algorithms.
- Superconducting resonators are really interesting---let's use them!
- Quantum Routing is both optimal and robust for certain networks of oscillators
- Entangled Resonator States Demonstrated!
- Quantum State Synthesis and Logic Gates on the way---stay tuned!