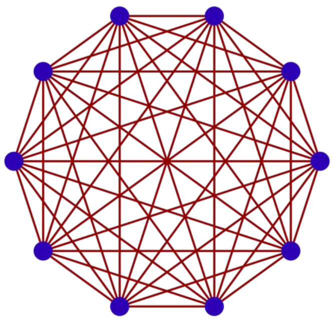
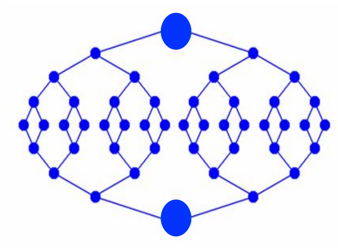


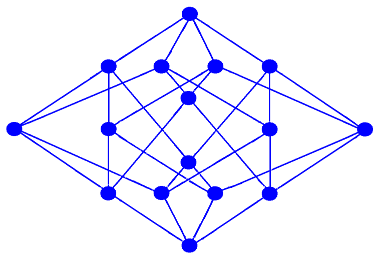
Quantum Walks in Discrete and Continuous Time

Frederick W. Strauch

Department of Physics

Williams College





Valencia 2011

NIST Physics Laboratory

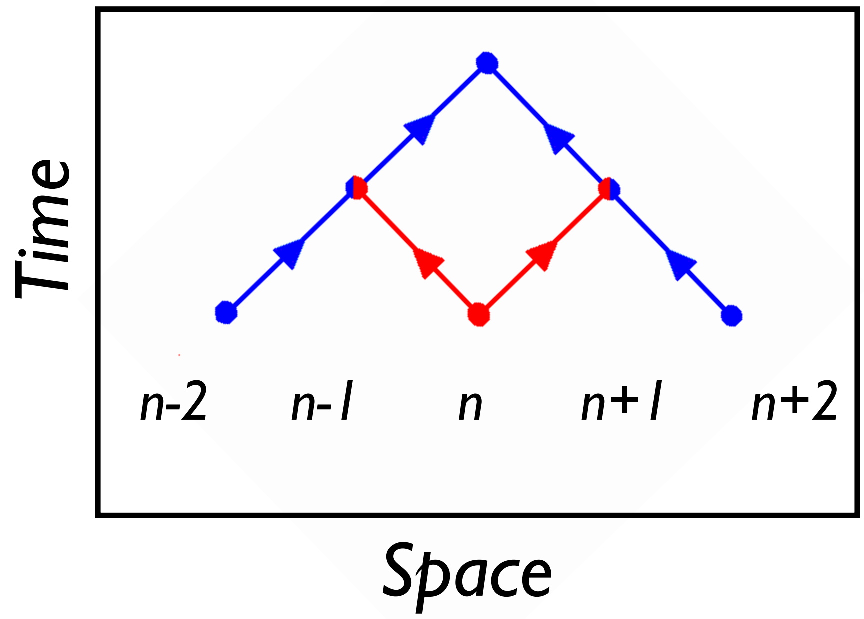
NIST
National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce

Outline

- Quantum Walks
  - DTQW & CTQW
- Quantum Algorithms
  - Searching, Mixing, Hitting, & Graph Traversal
- Connecting the quantum walks
  - Relativistic Limits & Other Approaches
- Applications Present and Future
  - Algorithms, Implementations, Quantum State Transfer

Classical Random Walk

- Consider a process in which a particle can move either left or right, in one dimension, based on an internal state (coin).
- Before each step, the coin is flipped.
- This generates a stochastic process, governed by the following equations for the probability:

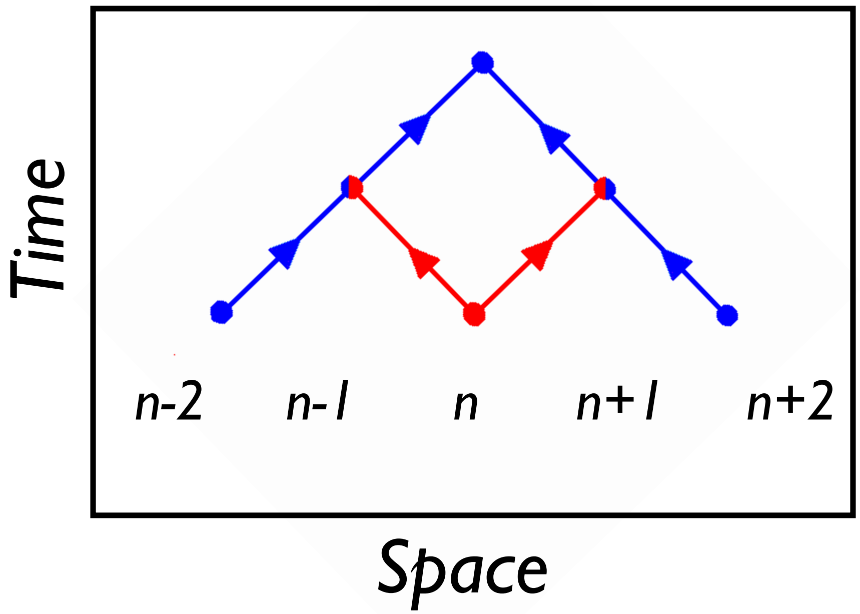


$$p_R(n,t+1) = \frac{1}{2} p_R(n-1,t) + \frac{1}{2} p_L(n-1,t)$$

$$p_L(n,t+1) = \frac{1}{2} p_L(n+1,t) + \frac{1}{2} p_R(n+1,t)$$

Quantum Random Walk

- *Change probabilities to probability amplitudes.*
- *Change coin flip to unitary transformation.*
- E.g. Hadamard Walk:



$$\psi_R(n,t+1) = \frac{1}{\sqrt{2}}\psi_R(n-1,t) + \frac{1}{\sqrt{2}}\psi_L(n-1,t)$$

$$\psi_L(n,t+1) = \frac{1}{\sqrt{2}}\psi_R(n+1,t) \enclose{circle}{-} \frac{1}{\sqrt{2}}\psi_L(n+1,t)$$

Interference!

Aharonov, Davidovich, and Zagury (1993)

Quantum Walk vs. Classical Walk

QW Probabilities

CW Probabilities

Time

1	0	11	0	4	0	4	0	11	0	1	/32	1	0	5	0	10	0	10	0	5	0	1
0	1	0	6	0	2	0	6	0	1	0	/16	0	1	0	4	0	6	0	4	0	1	0
0	0	1	0	3	0	3	0	1	0	0	/8	0	0	1	0	3	0	3	0	1	0	0
0	0	0	1	0	2	0	1	0	0	0	/4	0	0	0	1	0	2	0	1	0	0	0
0	0	0	0	1	0	1	0	0	0	0	/2	0	0	0	0	1	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	/1	0	0	0	0	0	1	0	0	0	0	0

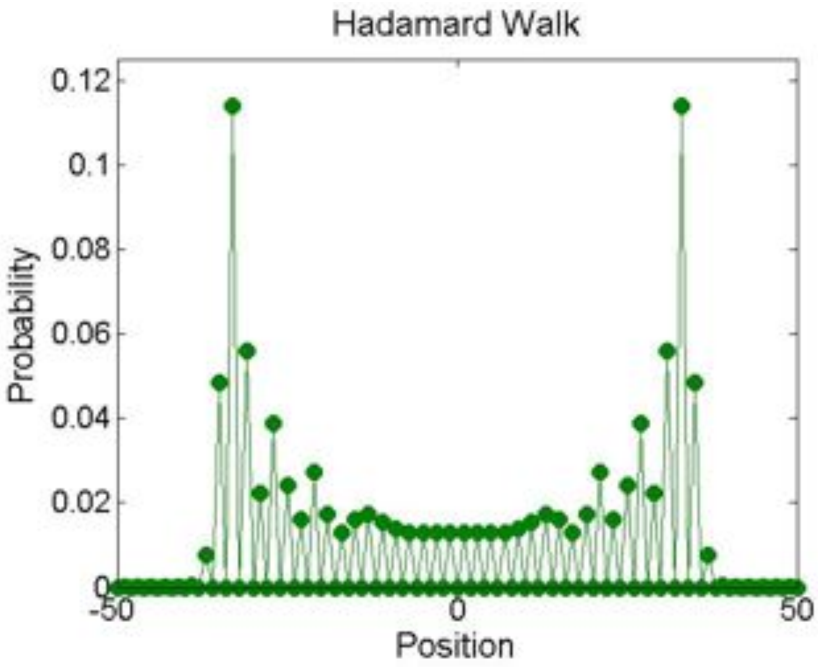
Space

$$\psi_R(n,t=0) = \tfrac{1}{\sqrt{2}}\delta_{n,0} \qquad p_R(n,t=0) = \tfrac{1}{2}\delta_{n,0}$$

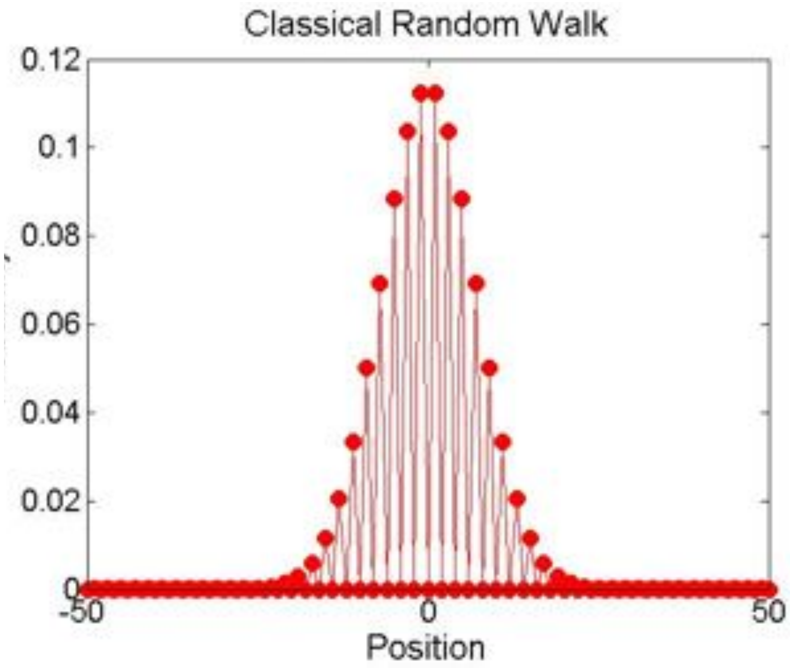
$$\psi_L(n,t=0) = \tfrac{1}{\sqrt{2}}i\,\delta_{n,0} \qquad p_L(n,t=0) = \tfrac{1}{2}\delta_{n,0}$$

Ensures symmetric dynamics

Quantum Interference = Quadratic Speed-up



$$\Delta x^2 \sim \Delta t^2$$



$$\Delta x^2 \sim \Delta t$$

Interesting Probability Distribution

constant for $-T/\sqrt{2} < x < T/\sqrt{2}$

Wave equation w/ Dispersion (Knight, Roldan, & Sipe 2003)

DTQWs on General Graphs

Discrete-Time Quantum Walk

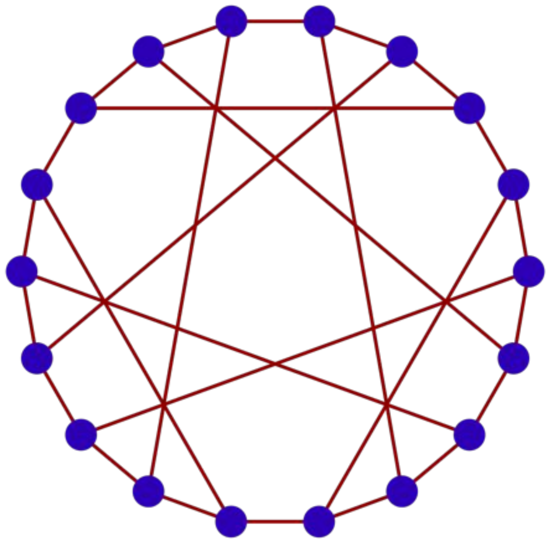
Use *d*-dimensional coin operator **C** for vertices of degree *d*

Conditional shift operator ***S***, depends on each vertex

$$S|c,v\rangle = |c,w(c,v)\rangle$$

w(c,v) = vertex connected to *v* along edge *c*

Other definitions possible



Unitary Mapping:

$$U = S(C \otimes I)$$

$$|\Psi(T)\rangle = U^T|\Psi(0)\rangle$$

DTQWs on General Graphs

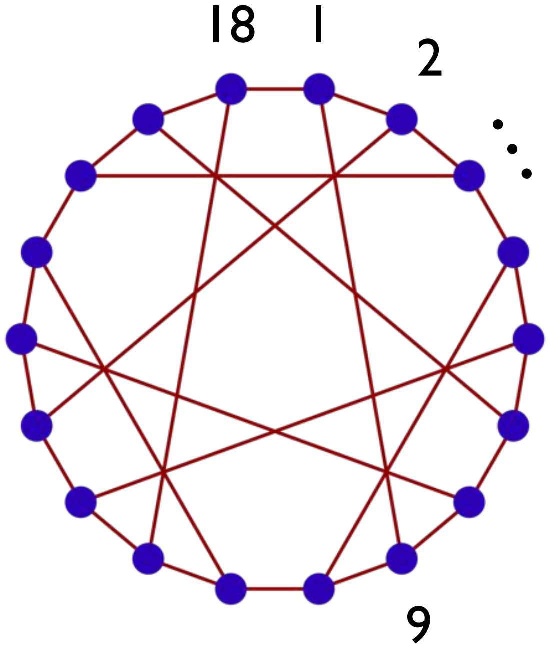
Discrete-Time Quantum Walk

Use *d*-dimensional coin operator **C** for vertices of degree *d*

Conditional shift operator ***S***, depends on each vertex

$$S|c,v\rangle = |c,w(c,v)\rangle$$

w(c,v) = vertex connected to *v* along edge *c*



E.g. *d=3*

$$S|c=1,v=1\rangle = |c=1,v=2\rangle$$
$$S|c=2,v=1\rangle = |c=2,v=9\rangle$$
$$S|c=3,v=1\rangle = |c=3,v=18\rangle$$

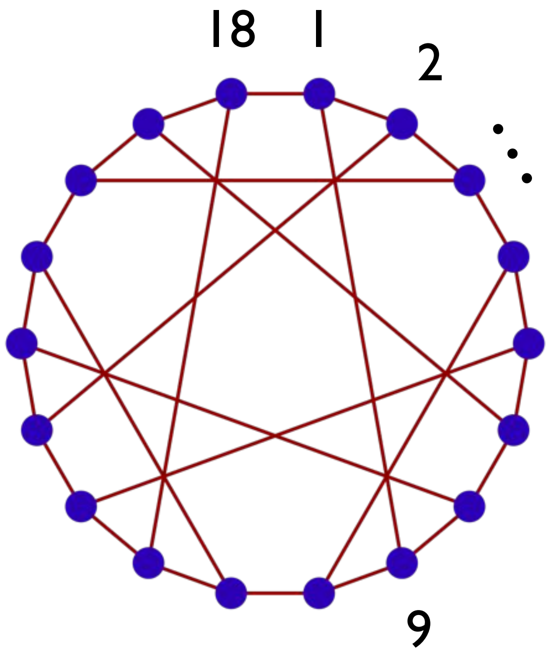
Diffusion on General Graphs

Use *Laplacian matrix* for graph

L = A – D
A = adjacency matrix
D = degree matrix

$$D_{i,i} = d_i,\ d_i = \text{degree of vertex } i$$

$$A_{i,j} = \begin{cases} 1 & \text{if } (i,j) \text{ are connected} \\ 0 & \text{otherwise} \end{cases}$$

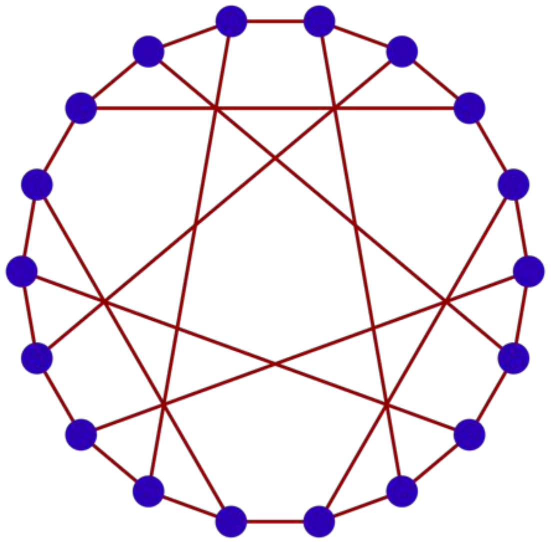


$$D_{i,i} = 3 \ \forall i$$

$$A_{1,2} = A_{1,9} = A_{1,18} = 1$$

Diffusion on General Graphs

Use *Laplacian matrix* for graph



L = A – D

A = adjacency matrix

D = degree matrix

p = probability vector

$$\sum_{j=1}^{|G|} p_j = 1 \qquad \frac{dp}{dt} = Lp \qquad p(t) = e^{Lt}p(0)$$

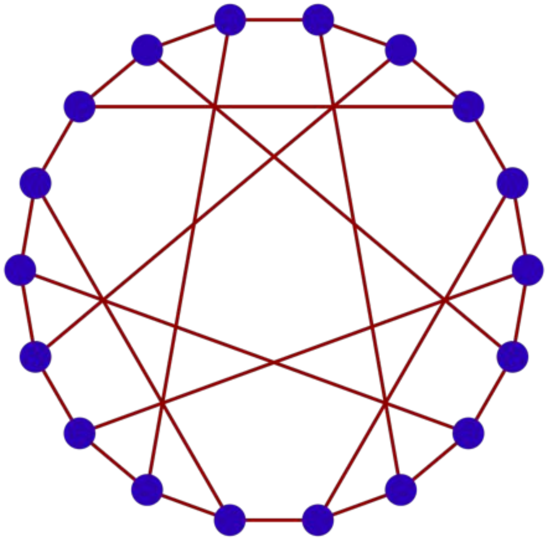
CTQW on General Graphs

Continuous-Time Quantum Walk

Use *Laplacian matrix* as Hamiltonian:

H = A – D

Sometimes $H=A$



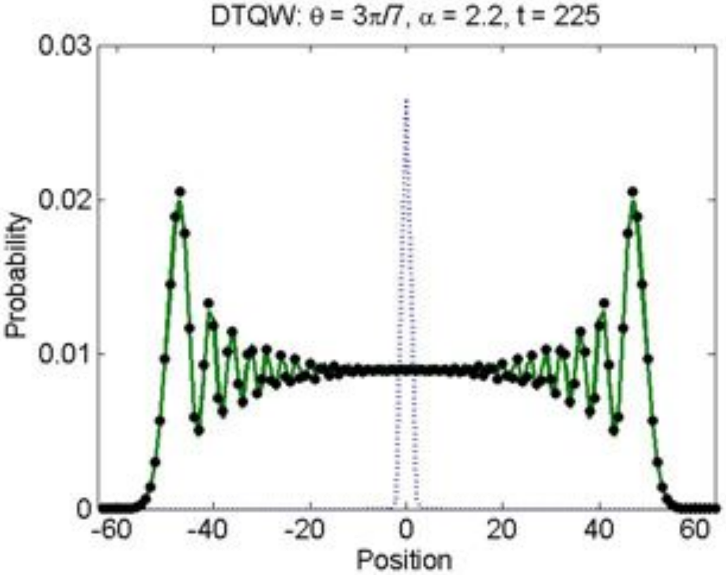
$$\Psi = \left\{\psi_1, \psi_2, \cdots, \psi_{|G|}\right\}$$

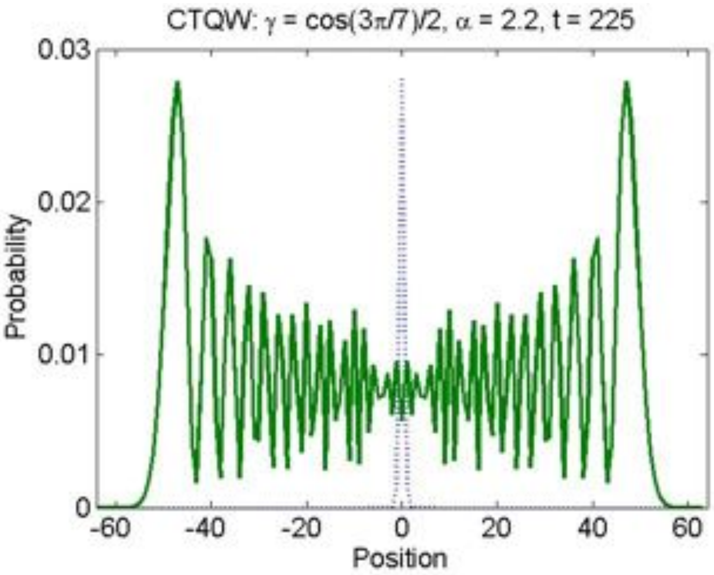
= state vector (probability *amplitudes*)

$$\sum_{j=1}^{|G|} \left|\psi_j\right|^2 = 1 \qquad i\frac{d\Psi}{dt} = H\Psi \qquad \Psi(t) = e^{-iHt}\Psi(0)$$

DTQW vs CTQW

One-dimensional Line





Jacobi polynomial
Airy, Bessel fxn
approximations

Bessel function

Dynamics can be very similar!

Outline

- Quantum Walks
  - DTQW & CTQW
- Quantum Algorithms
  - Search, Mixing, Hitting, & Graph Traversal
- Connecting the quantum walks
  - Relativistic Limits & Other Approaches
- Applications Present and Future
  - Algorithms, Implementations, Quantum State Transfer

Quantum Walk Search

Grover Search

Let *w* be the marked vertex

= item of interest in database

$$|\psi\rangle = \frac{1}{\sqrt{N}}\sum_{v=1}^{N}|v\rangle$$

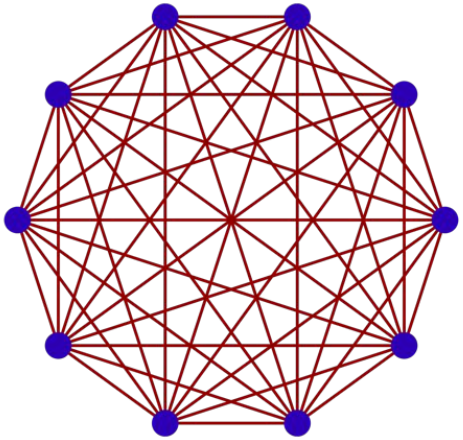
= uniform superposition

$$U = \underbrace{e^{i\pi|\psi\rangle\langle\psi|}}_{\text{Inversion about average}}\underbrace{e^{i\pi|w\rangle\langle w|}}_{\text{Oracle}}$$

$$U^T|\psi\rangle \rightarrow |w\rangle$$

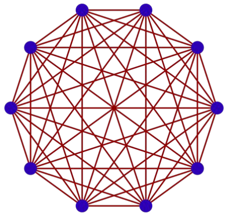
$$\boxed{|\psi\rangle\langle\psi| = \frac{1}{N}(A-I)}$$

Similar results by **CTQW** on complete graph

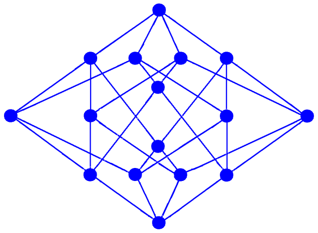


$$H = |\psi\rangle\langle\psi| + |w\rangle\langle w|$$

$$e^{-iHT}|\psi\rangle \rightarrow |w\rangle$$



QW Local Search



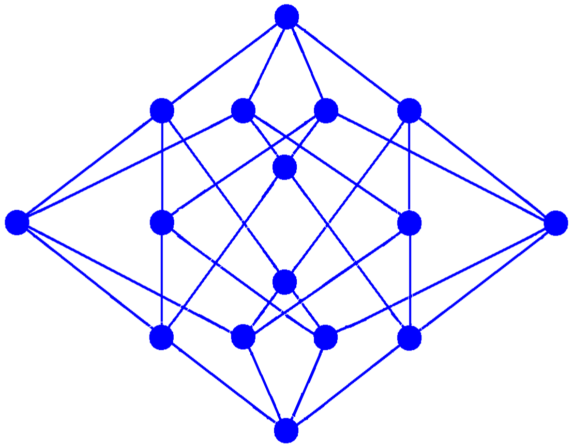
Authors	**Graph**	**Walk**	**Time**
Farhi & Gutmann (1998)	Complete	CTQW	$N^{1/2}$
Shenvi, Kempe, & Whaley (2003)	Hypercube	DTQW by coin	$N^{1/2}$
Aaronson & Ambainis (2003)	D-dim Lattice	DTQW by coin	$N^{1/2}$ $(\log N)^{3/2}$ D=2 $N^{1/2}$ D≥3
Childs & Goldstone (2003)	D-dim Lattice	CTQW	N D=2 $N^{5/6}$ D=3 $N^{1/2}$ log N D=4 $N^{1/2}$ D>4
Ambainis, Kempe, & Rivosh (2004)	D-dim Lattice	DTQW by coin	$N^{1/2}$ log N D=2 $N^{1/2}$ D≥3
Childs & Goldstone (2004)	D-dim Lattice	CTQW + Spin	$N^{1/2}$ log N D=2 $N^{1/2}$ D≥3
Magniez, Nayak, Roland, & Santha (2007)	D-dim Lattice	DTQW by reflection	$N^{1/2}$ $(\log N)^{1/2}$ D=2 $N^{1/2}$ D ≥3

Mixing on Quantum Walks

Mixing time = time when probability is essentially uniform over graph

Quantum walks mix faster (often quadratically) than classical walks---and decoherence helps!

Kendon & Tregenna (2003)



Hypercube

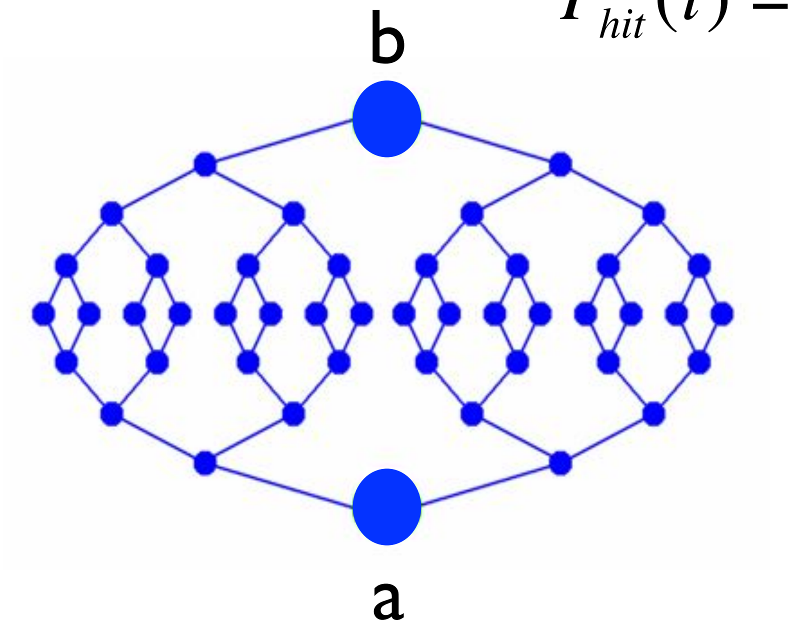
Moore & Russell (2001)

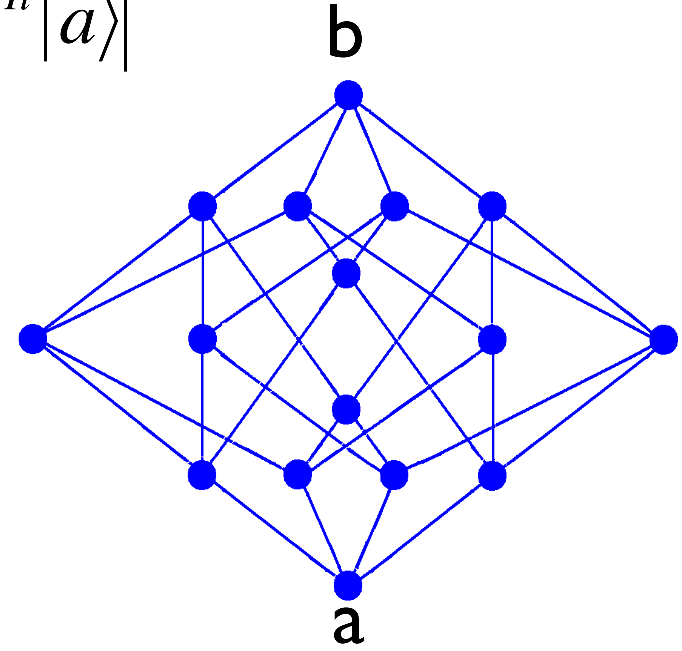
$$P_v(t) = \left|\langle v|e^{-iHt}|a\rangle\right|^2 \rightarrow \frac{1}{2^d} \text{at time } \omega t = \pi/4$$

Hitting on Quantum Walks

Hitting time = time when probability is large (>1/log|G|)

$$P_{hit}(t) = \left|\langle b|e^{-iHt}|a\rangle\right|^2$$





Quantum walks hit ***exponentially faster*** than random walks on glued trees and hypercube graphs

(One-shot hitting time; others have been studied by Kempe, Kendon, and Brun)

Hypercube Quantum Walk

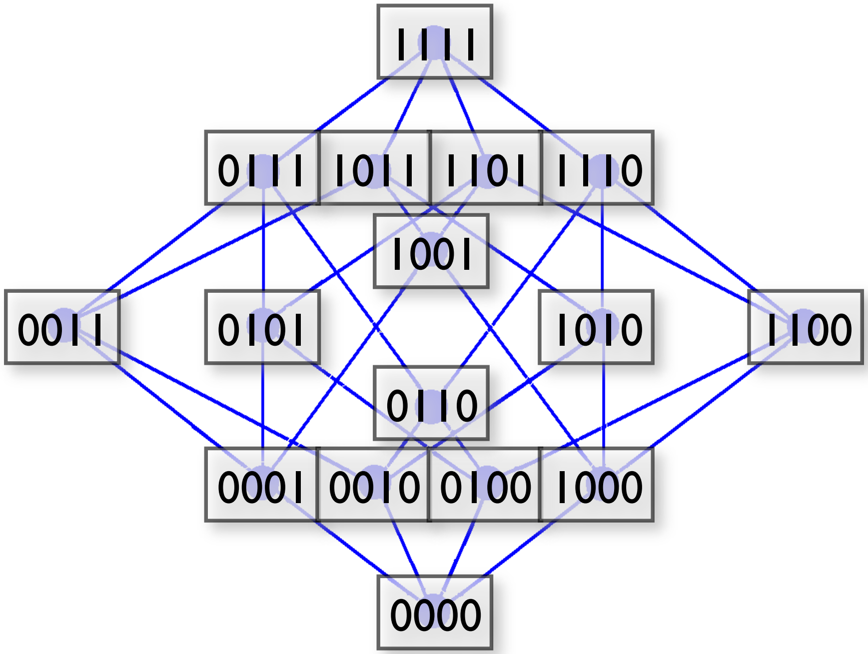
- Each vertex of the d-dimensional hypercube can be encoded by a set of ***d*** bits (E.g. $x = x_1x_2x_3x_4$).
- Hamiltonian (~Adjacency Matrix) simply flips each bit!

$$H = \omega \sum_{j=1}^{d} \sigma_x^{(j)}$$

Wavefunction evolves simply:

$$|\Psi(t)\rangle = \left(e^{-i\omega t \sigma_x}\right)^{\otimes d} |\Psi(0)\rangle$$

At time $T = \dfrac{\pi}{2\omega}$ all bits flip!

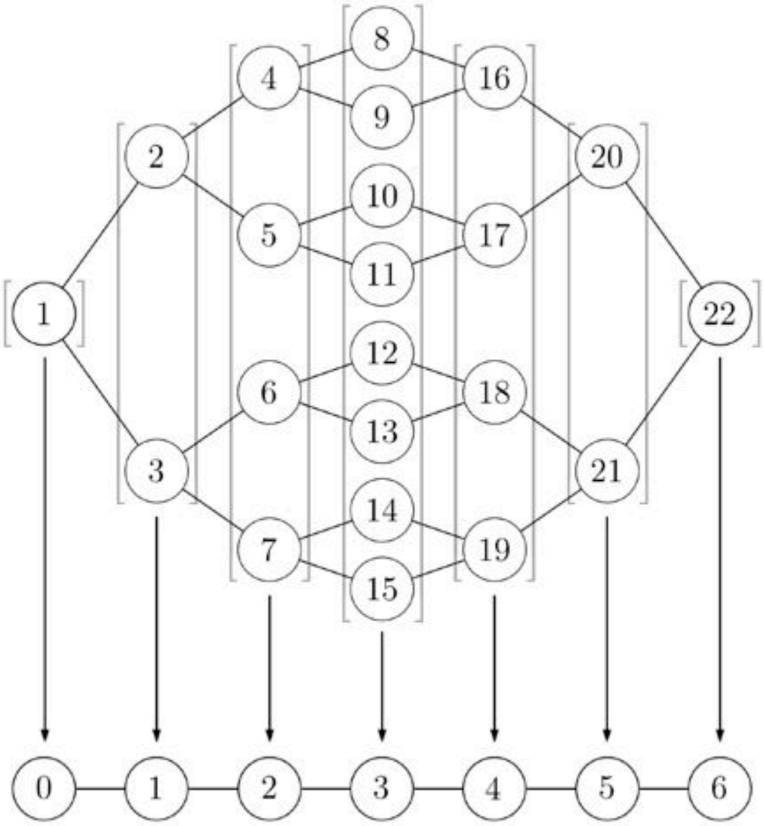


Exponential Speedup:

Quantum states propagate from corner to corner in time independent of the hypercube dimension ***d***!

Not a computational speedup

Glued Trees Quantum Walk

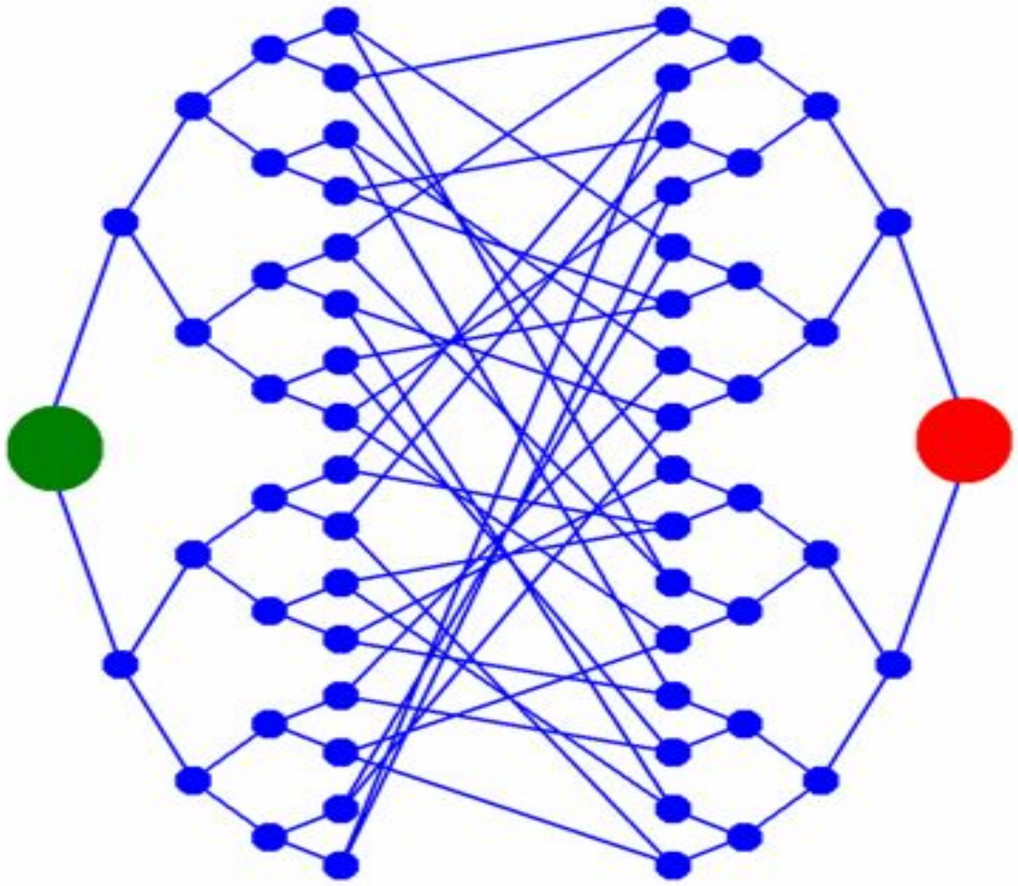


Childs et al. (2001)

QW on graph of size ~ 2^d can be represented by walk on line with 2d+1 sites

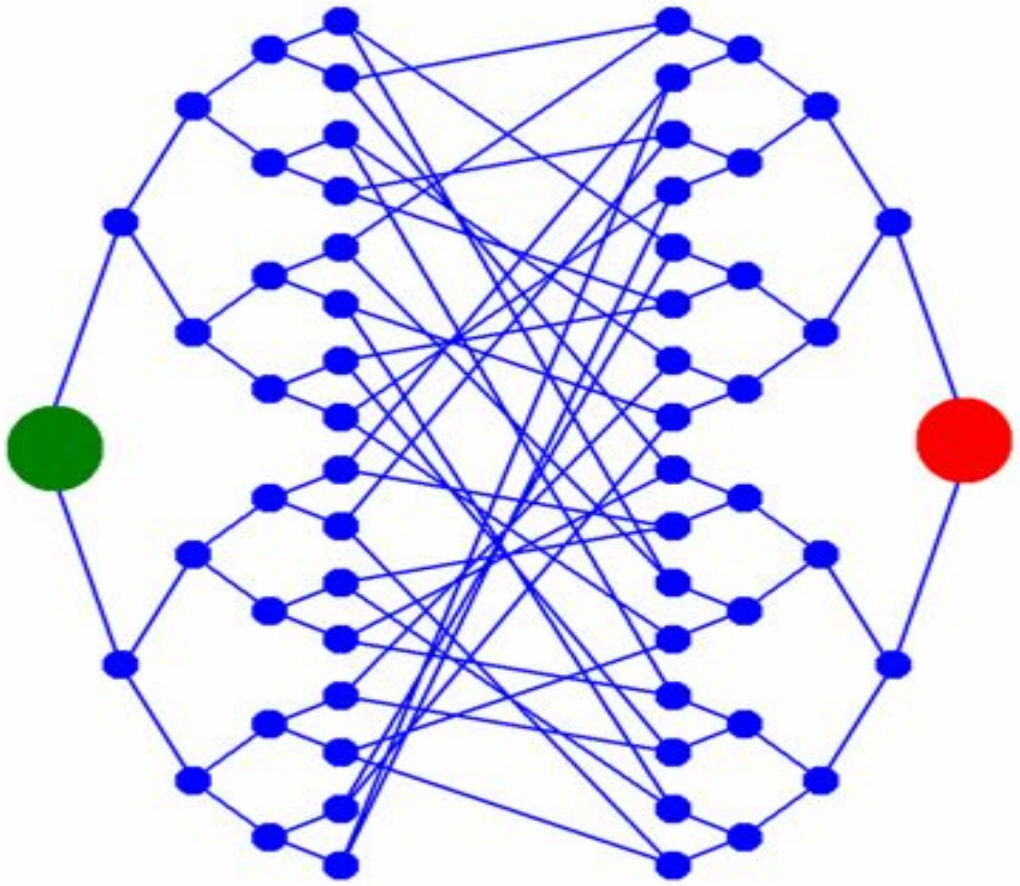
QW hits in time ~ 2d+1, while RW takes time ~ 2^d

Graph Traversal Problem



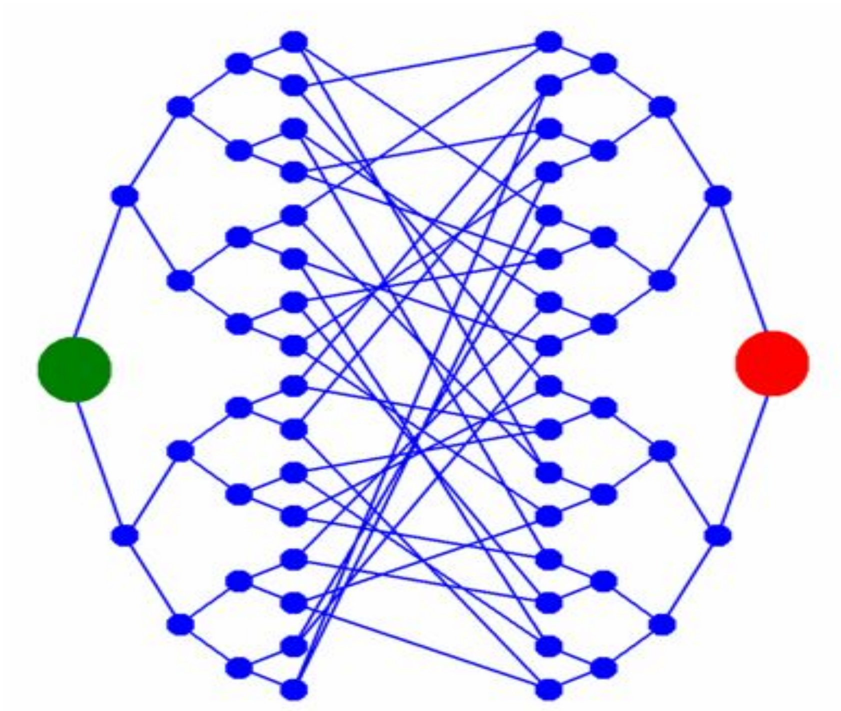
- Problem: Starting from the entrance (green), find the exit (red).
- Resources: Local queries of an oracle.
- Classical Running Time: polynomial in *N* (number of nodes)

Graph Traversal Problem



- Problem: Starting from the entrance (green), find the exit (red).
- Resources: Local queries of an oracle (unitary operator).
- Quantum Running Time: polynomial in *log N*.

Graph Traversal Problem



Exponential Algorithmic Speedup by Quantum Walk!
Childs (2003)

Reduction leads to QW on line with a defect

Significant amplitude is transmitted through!

Outline

- Quantum Walks
  - DTQW & CTQW
- Quantum Algorithms
  - Search, Mixing, Hitting, & Graph Traversal
- Connecting the quantum walks
  - Relativistic Limits & Other Approaches
- Applications Present and Future
  - Algorithms, Implementations, Quantum State Transfer

Connecting QWs

DTQW (Discrete-Time QW)

$$\psi_R(n,t+1) = \cos\theta\psi_R(n-1,t) - i\sin\theta\psi_L(n-1,t)$$

$$\psi_L(n,t+1) = \cos\theta\psi_L(n+1,t) - i\sin\theta\psi_R(n+1,t)$$

θ =coin rotation angle

CTQW (Continuous-Time QW)

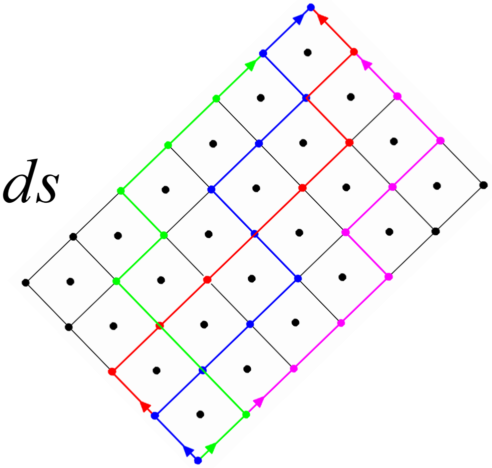
$$i\partial_t\psi(n,t) = -\gamma\left[\psi(n-1,t) - 2\psi(n,t) + \psi(n+1,t)\right]$$

- *Can one get one from the other?*
- *What physics drives these walks?*

Feynman Path Integral

- DTQW = Feynman's Checkerboard

$$\sum_{paths} (i\varepsilon m)^{\# bends}$$



- Feynman's Checkboard
  = Dirac Equation

$$i\hbar\partial_t \Psi = \left(-i\hbar c\vec{\alpha}\cdot\nabla + \beta\, mc^2\right)\Psi$$

D. Meyer (1996)

Feynman’s Proof

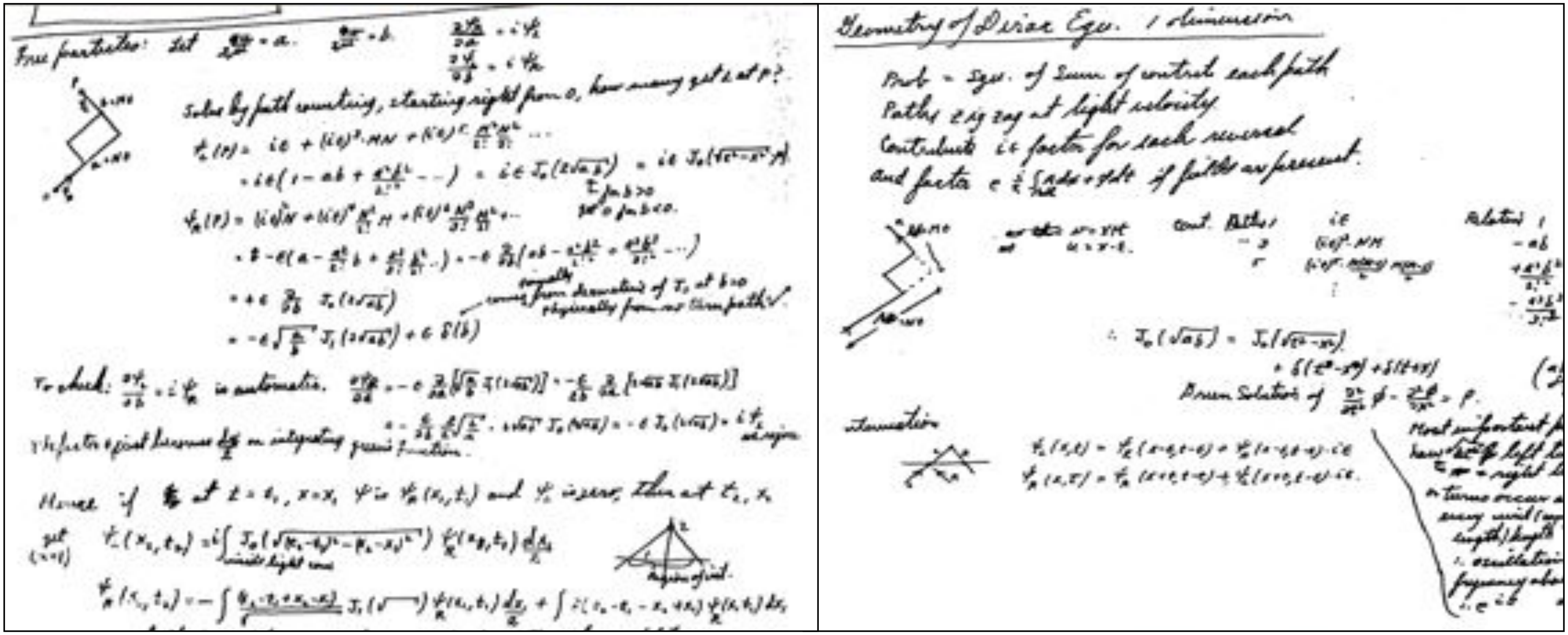
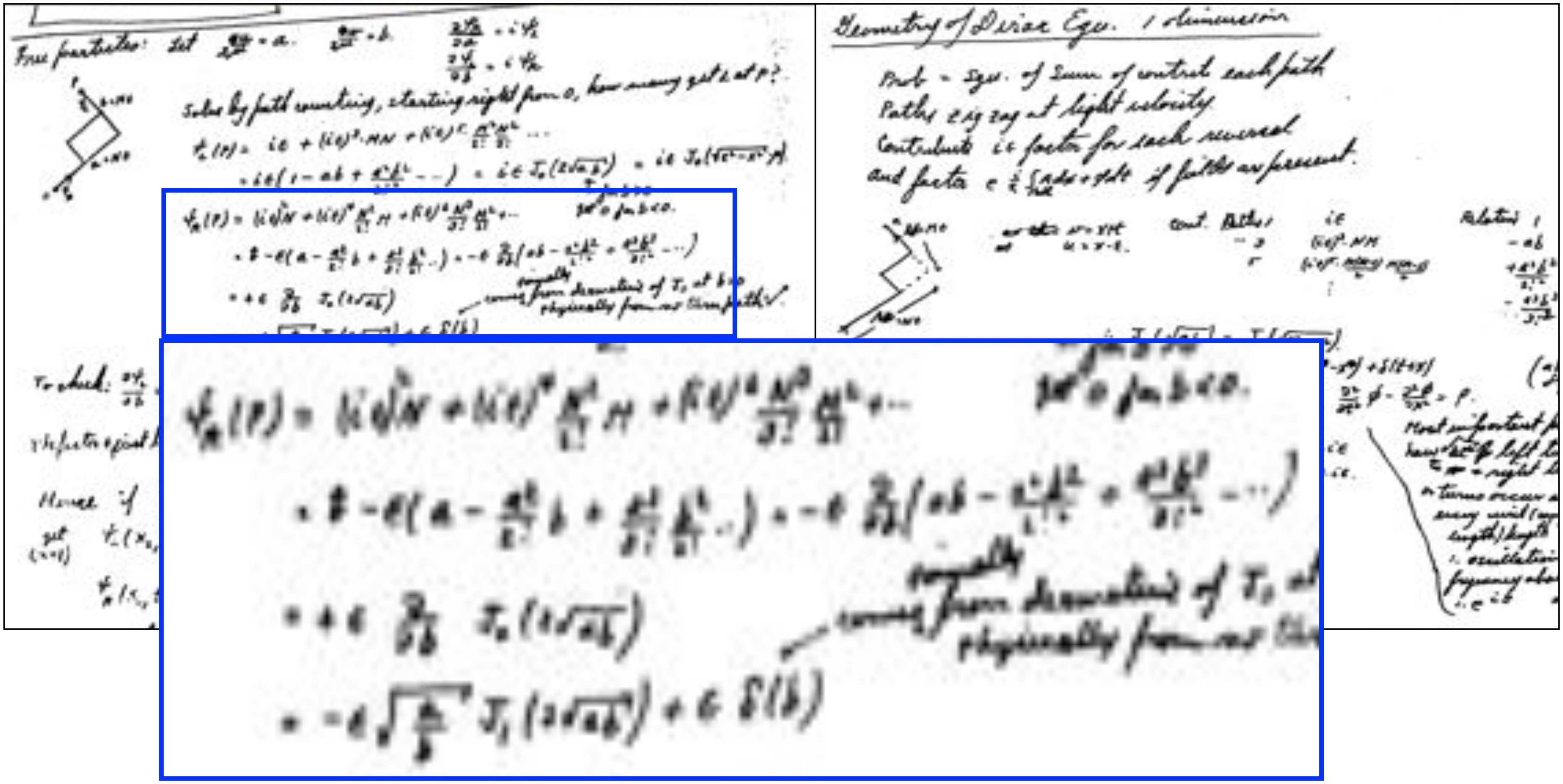


Figure 8 from “Feynman and the visualization of space-time processes”, Silvan S. Schweber, Rev. Mod. Phys. **58**, 499 (1986).

Feynman's Proof



Geometry of Dirac Eqn. 1 dimension

$$\psi_R(P) = (i\epsilon)N + (i\epsilon)^3\frac{N^2}{2!}M + (i\epsilon)^5\frac{N^3}{3!}\frac{M^2}{2!} + \cdots$$

$$= -\epsilon\left(a - \frac{a^2}{2!}b + \frac{a^3}{3!}\frac{b^2}{2!}\cdots\right) = -\epsilon\frac{\partial}{\partial b}\left(ab - \frac{a^2b^2}{2!\,2!} + \frac{a^3b^3}{3!\,3!} - \cdots\right)$$

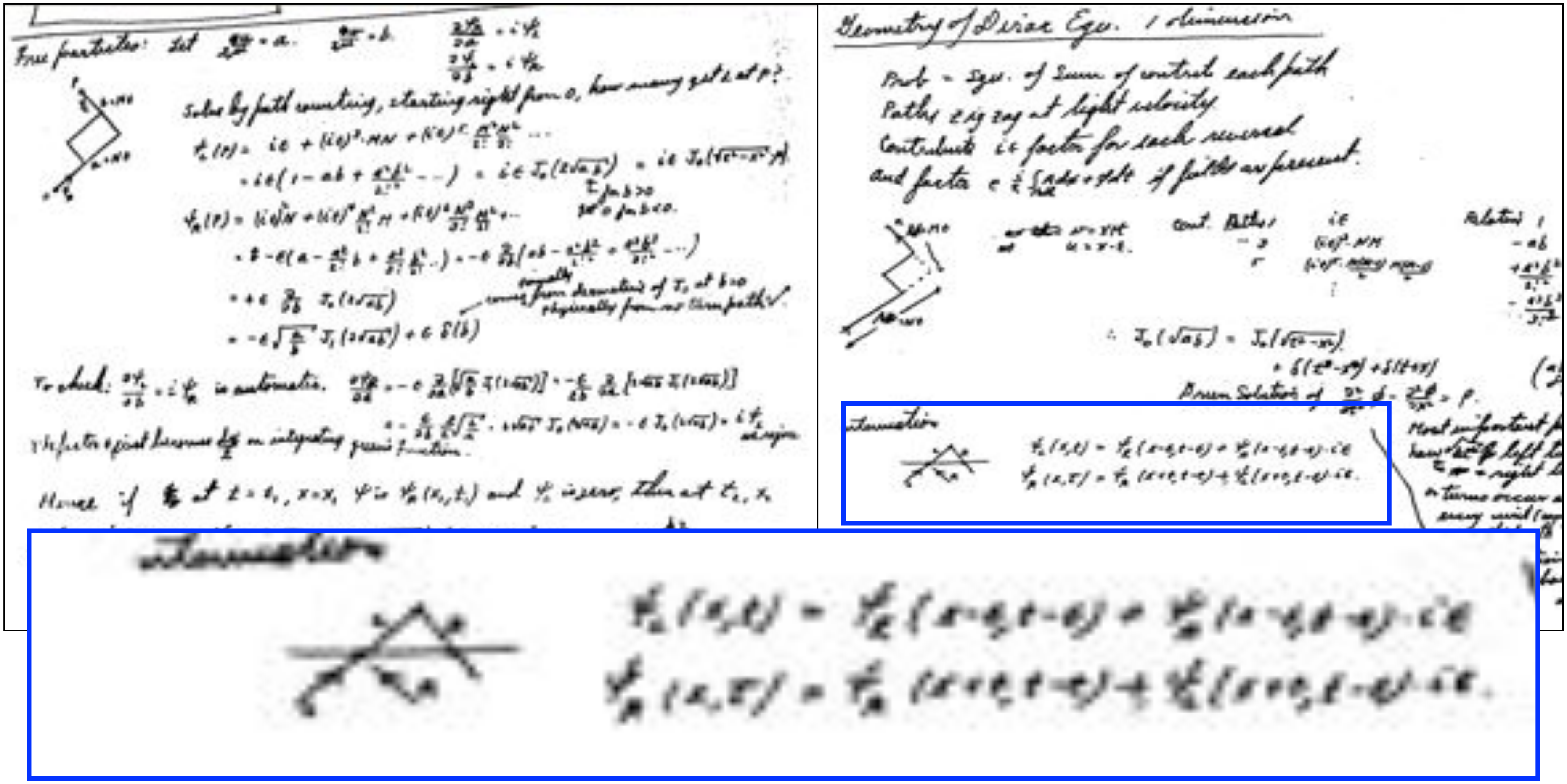
$$= +\epsilon\frac{\partial}{\partial b}J_0(2\sqrt{ab})$$

$$= -\epsilon\sqrt{\frac{a}{b}}\,J_1(2\sqrt{ab}) + \epsilon\,\delta(b)$$

Feynman knows his Bessel functions!

Feynman (1946)

Feynman’s Proof



$$\psi_L(x,t) = \psi_L(x-\epsilon, t-\epsilon) + \psi_R(x-\epsilon, t-\epsilon)\cdot i\epsilon$$
$$\psi_R(x,t) = \psi_R(x+\epsilon, t-\epsilon) + \psi_L(x+\epsilon, t-\epsilon)\cdot i\epsilon.$$

Feynman (1946)

This is the DTQW!

Relativistic Walks

- Can we use the Dirac equation to understand the DTQW?
- *Yes, if we have some method to generate wave-packet solutions*
- Look at wave-packet solutions of Dirac equation!
  - (Use Momentum Space!)
- Compare with wave-packet solutions of DTQW
  - (Use Momentum Space!)
- CTQW comes for free!

FWS, Phys. Rev. A **73**, 054302 (2006)

Dirac Wave-Packet

One-dimensional Dirac Hamiltonian:

$$H_{Dirac} = \sigma_z p + \sigma_x m = \begin{pmatrix} p & m \\ m & -p \end{pmatrix}, \qquad eig(H_{Dirac}) = \pm\sqrt{p^2 + m^2}$$

Dirac wave-packets

a = localization parameter

$$\Psi_\pm(x,t) = N\int_{-\infty}^{+\infty} e^{-(a\pm it)\sqrt{p^2+m^2}} e^{ipx} \left(I \pm \frac{H_{Dirac}}{\sqrt{p^2+m^2}} \right) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= N' \begin{pmatrix} {s_\pm}^{-1}(a \pm i(t+x))K_1(ms_\pm) \pm K_0(ms_\pm) \\ {s_\pm}^{-1}(a \pm i(t-x))K_1(ms_\pm) \pm K_0(ms_\pm) \end{pmatrix}$$

$$s_\pm = \sqrt{x^2 + (a \pm it)^2}$$

Modified Bessel fxn

DTQW Wave-Packet

DTQW unitary mapping

$$U = e^{-ik\sigma_z} e^{-i\theta\sigma_x} = \begin{pmatrix} e^{-ik}\cos\theta & -ie^{-ik}\sin\theta \\ -ie^{ik}\sin\theta & e^{ik}\cos\theta \end{pmatrix}, \quad eig(U) = e^{\pm i\omega(k)}$$

$$\cos\omega(k) = \cos\theta\cos k$$

DTQW “wave-packets”

α = localization parameter

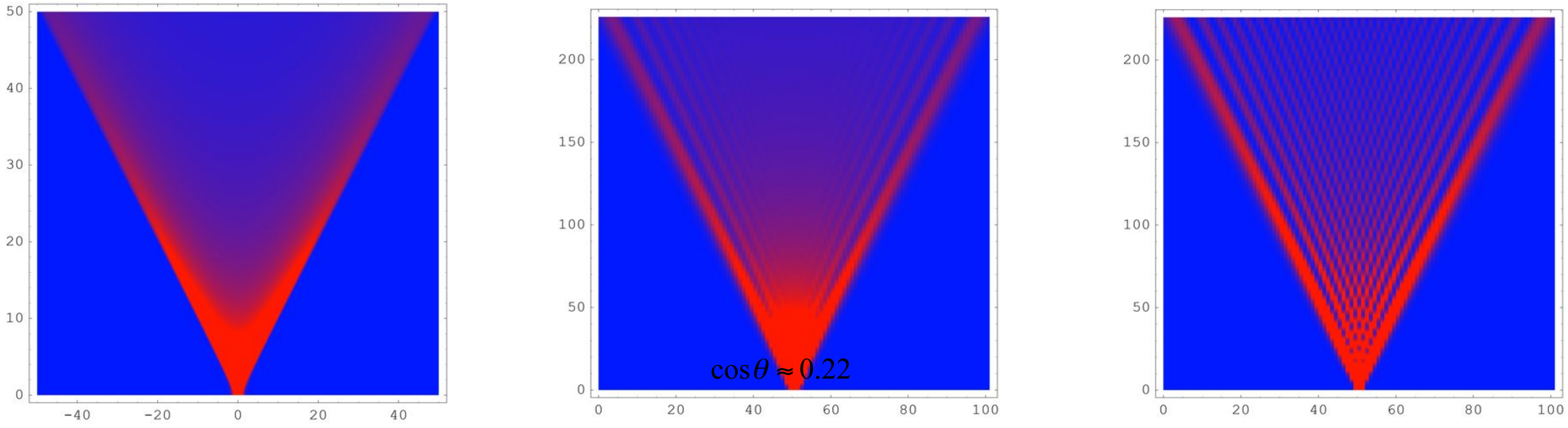
$$\psi_{\pm}(n,t) = N\int_{-\pi}^{+\pi} e^{-(\alpha \pm i\,t)\omega\,(k)} e^{ikn}(U - e^{\pm i\omega\,(k)})\begin{pmatrix}1\\1\end{pmatrix}dk$$

$$= N'\begin{pmatrix} I_n(\pm(t-1)-i\alpha) - e^{-i\theta}I_{n-1}(\pm t - i\alpha) \\ I_n(\pm(t-1)-i\alpha) - e^{-i\theta}I_{n+1}(\pm t - i\alpha) \end{pmatrix}$$

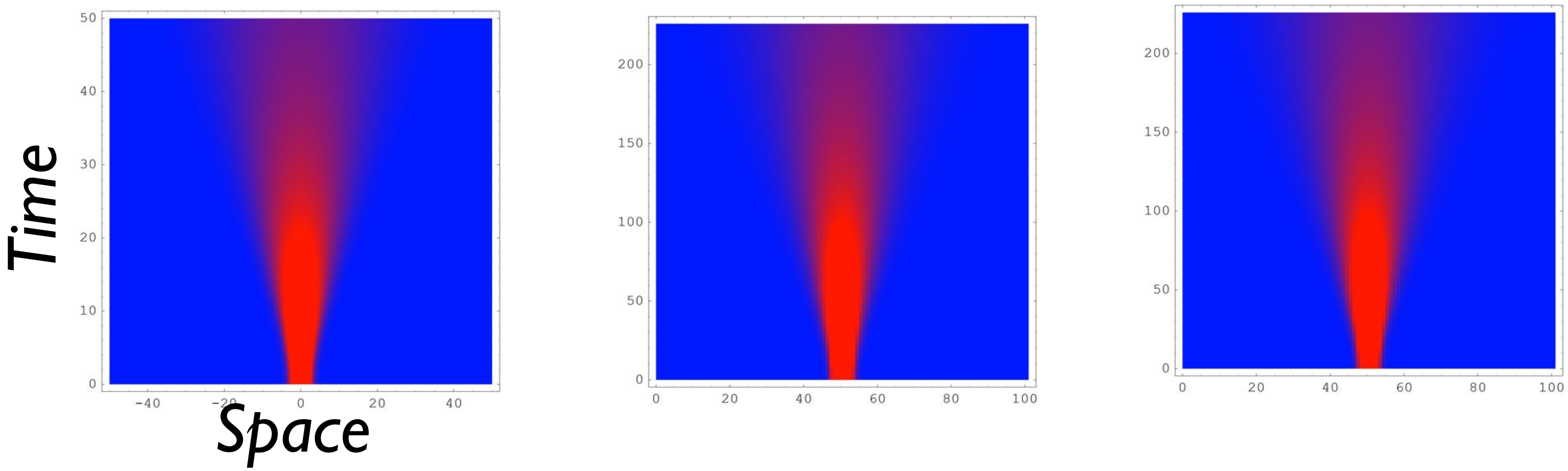
$$I_n(z) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} e^{-iz\omega\,(k)} e^{ikn}\,dk \approx i^n e^{-iz\pi/2} J_n(z\cos\theta), \cos\theta << 1$$

Dirac DTQW CTQW

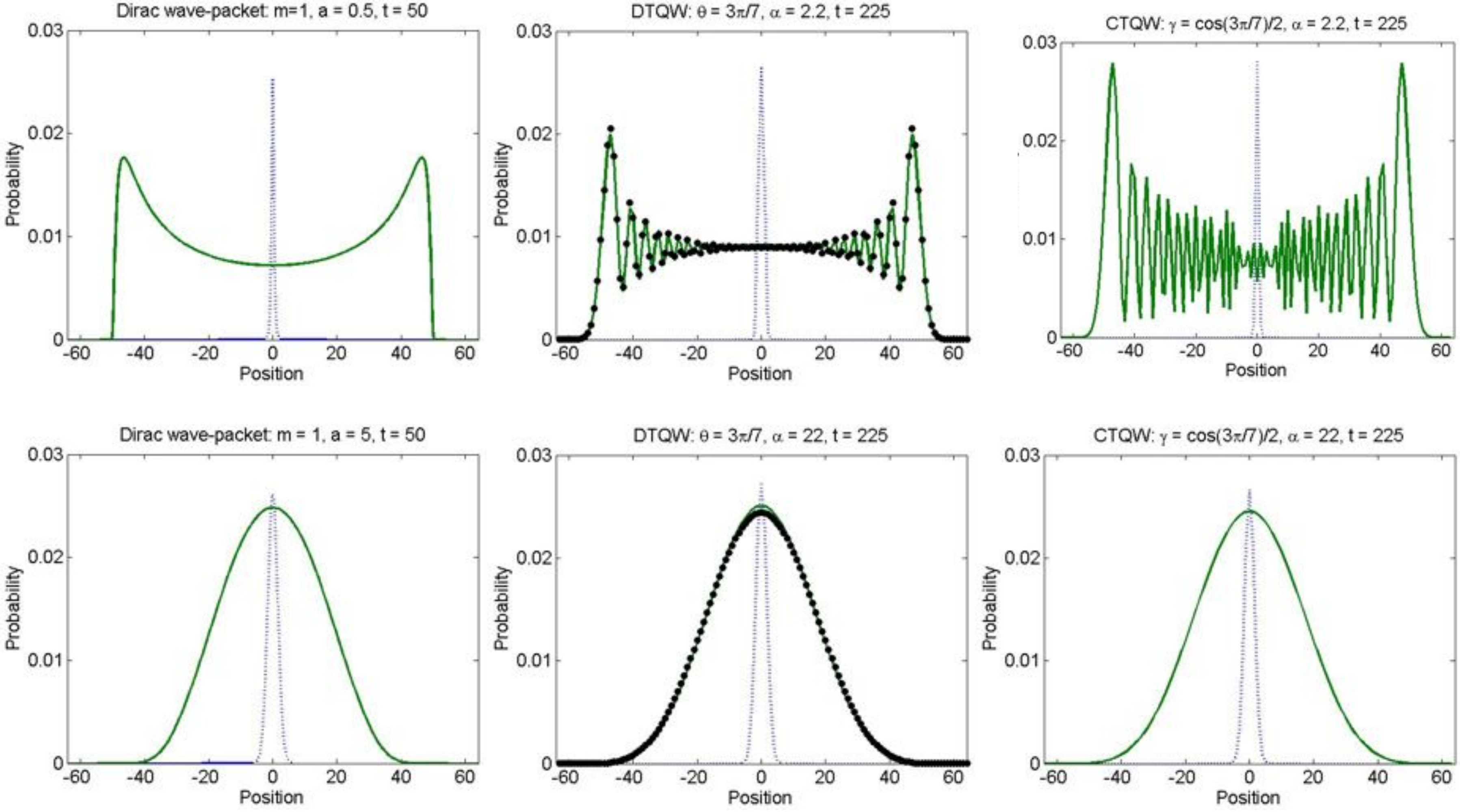
Relativistic Wave-Packet Spreading



Non-Relativistic Wave-Packet Spreading



Slices



Dirac wave-packet: m=1, a = 0.5, t = 50

DTQW: θ = 3π/7, α = 2.2, t = 225

CTQW: γ = cos(3π/7)/2, α = 2.2, t = 225

Dirac wave-packet: m = 1, a = 5, t = 50

DTQW: θ = 3π/7, α = 22, t = 225

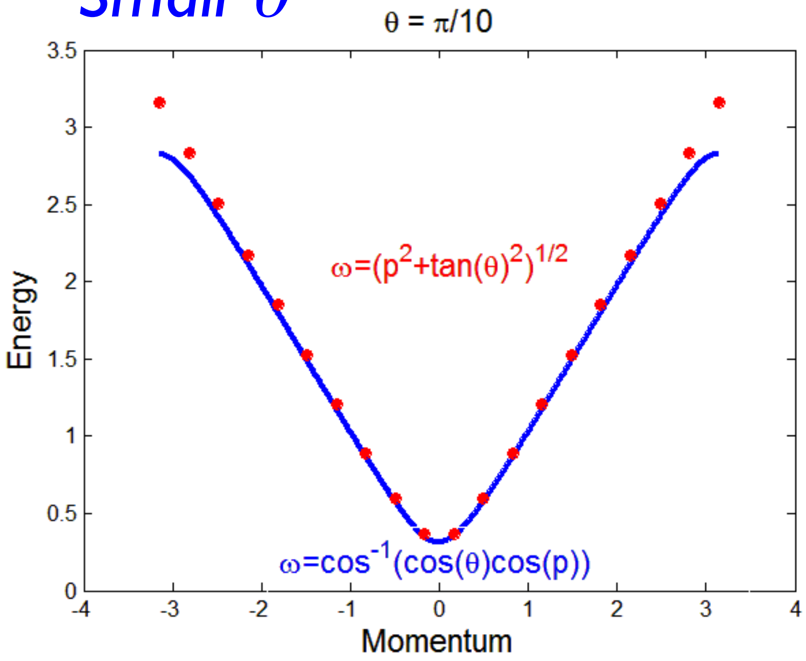
CTQW: γ = cos(3π/7)/2, α = 22, t = 225

Dynamics governed by dispersion relations

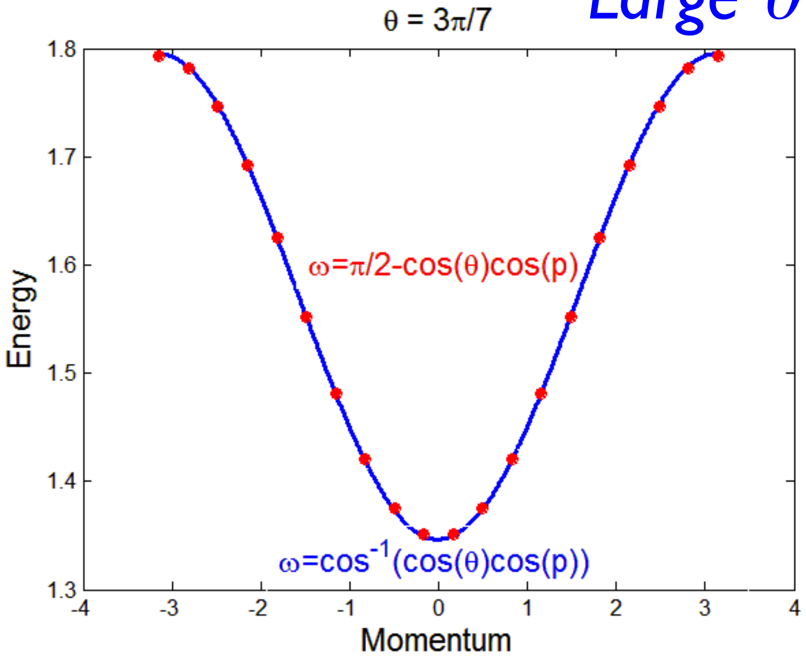
$$\omega_{DTQW} = \cos^{-1}(\cos\theta\cos p)$$

$$\omega_{Dirac} = \sqrt{p^2 + m^2} \qquad \omega_{CTQW} = 2\gamma(1 - \cos p)$$

Small θ



Large θ

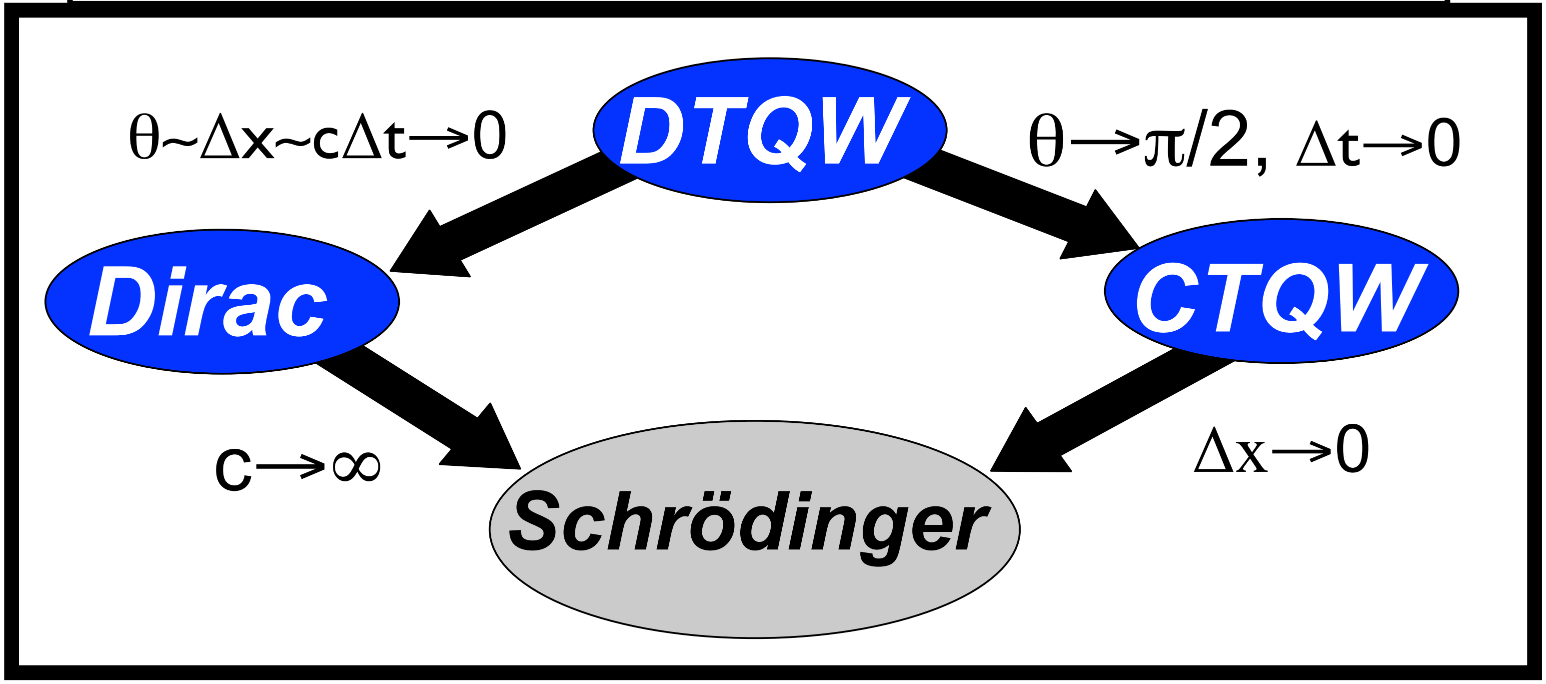


Each has $d\omega / dp < c$ = maximum speed

Connecting the Quantum Walks

Introduce arbitrary coin parameter θ, take various continuum limits!

$$\begin{aligned}\psi_R(n,\tau+1) &= \cos\theta\,\psi_R(n-1,\tau) - i\sin\theta\,\psi_L(n-1,\tau)\\ \psi_L(n,\tau+1) &= \cos\theta\,\psi_L(n+1,\tau) - i\sin\theta\,\psi_R(n+1,\tau)\end{aligned}$$



$\theta \sim \Delta x \sim c\Delta t \to 0$

DTQW

$\theta \to \pi/2,\ \Delta t \to 0$

Dirac

CTQW

$c \to \infty$

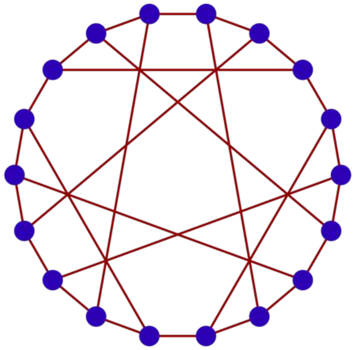
Schrödinger

$\Delta x \to 0$

FWS, Phys. Rev. A **74**, 030301 (R) (2006)
FWS, J. Math. Phys., **48**, 082102 (2007)

Classical Analogy: Simple Diffusion vs. Telegrapher's Equation

Connection for general graphs



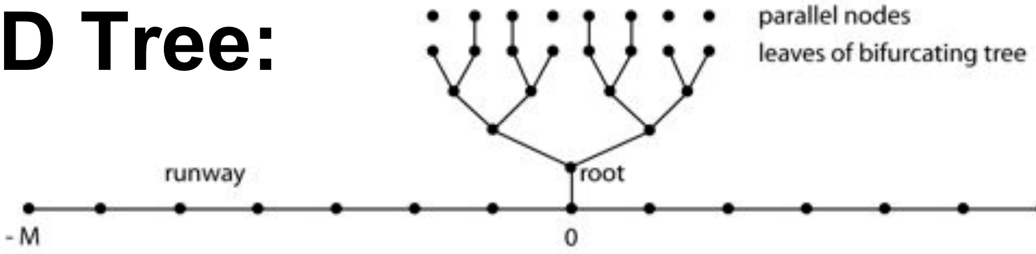
- Childs (2008):
  - –Uses Szedegy’s QW
  - –Hamiltonian simulation method
  - –QW “walks” on Hilbert space of dimension |G|^2 (each state ~ edge of graph).
- D’Allesandro (2008):
  - –Controls QW by varying coin-flips.
  - –Lie theoretical proof of universal control
  - –QW on graphs of size |G|, coin flips of length log |G|

Outline

- Quantum Walks
  - DTQW & CTQW
- Quantum Algorithms
  - Search, Mixing, Hitting, & Graph Traversal
- Connecting the quantum walks
  - Relativistic Limits & Other Approaches
- Applications: Present and Future
  - Algorithms, Implementations, Quantum State Transfer

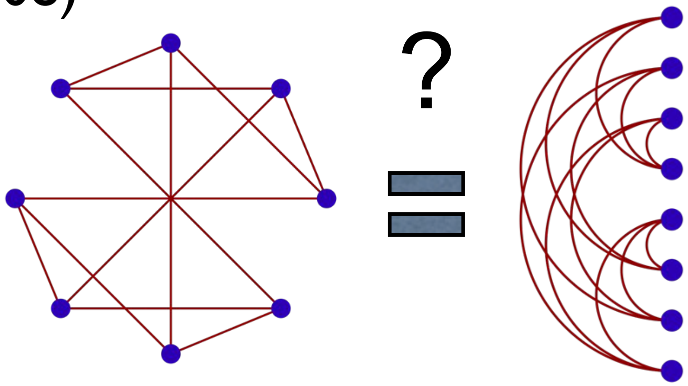
Algorithms

- **NAND Tree:**



Properties of graph revealed by scattering
Farhi, Goldstone, & Guttman (2008)

- **Graph Isomorphism**



Properties of graph revealed by multiple walkers (Wang + others)

- **Universality**

ALL Q Computations can be written as a CTQW
Childs (2008)

Encodings and Implementations

- **Quantum Computer**
  - Graph vertices encoded in quantum bits
  - Walks encoded by unitary logic gates
  - Most efficient encoding has $\log_2|G|$ qubits = **binary encoding**
- **Physical Network**
  - Graph vertices encoded in degrees of freedom (atom sites, electron sites, etc.)
  - Walks encoded by natural Hamiltonian
  - Most natural encoding has $|G|$ qubits = **unary encoding**

QW Implementations

(not an exhaustive list)

Proposal	**System**	**Walk**	**Encoding**
W. Dür *et al.* (2002)	Optical Lattices	1D DTQW	Unary
B. C. Sanders *et al.* (2002)	Cavity-QED	1D DTQW	Unary/Binary
A. Romito *et al.* (2005)	Superconductors	1D CTQW	Unary
C.A. Ryan et al. (2005)	NMR exp	2D DTQW	Binary
J. M. Taylor (2007)	Quantum Dots	NAND CTQW	Unary
F. W. Strauch (2008)	Superconductors	Hypercube	Unary
Karski et al. (2009)	Optical Lattices	1D DTQW	Unary
Zähringer et al. (2010)	Ion Traps	1D DTQW	Unary
Broome et al. (2010)	Photons	1D DTSQW	Unary

Most proposals encode the walk using a **unary** representation.
Does this really demonstrate quantum computational speedup?

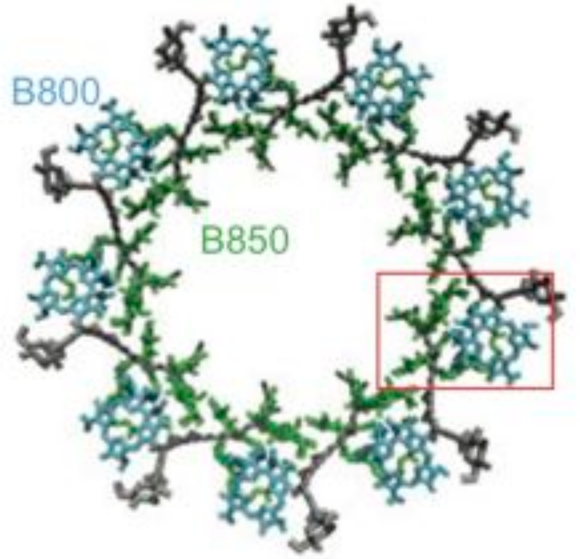
DTQW = Discrete-time quantum walk CTQW = Continuous-time quantum walk

Nature's Implementation?

Photosynthesis

FMO Complex

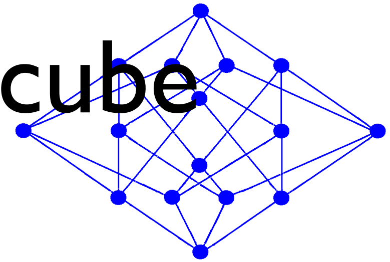
LH2 Complex



Unary QW with disorder & decoherence

M. Mohseni, P. Rebentrost, S. Lloyd, and A. Aspuru-Guzik,
J. Chem. Phys. **129**, 174106 (2008)

Decoherence in the Hypercube Quantum Walk



Two "natural" models of decoherence for the hypercube:

- **Subspace Model**
  - Each node is represented by a state of several qubits.
  - **Binary** encoding with **d** qubits
  - Dephasing occurs between subspaces of the hypercube.
  - Useful model for quantum computer implementation of quantum walks.

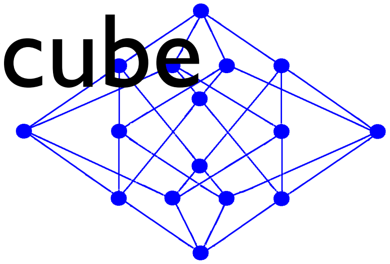
G. Alagic and A. Russell,
Phys. Rev. A **72**, 062304 (2005)

- **Vertex Model**
  - Each node is represented by the excited state of one qubit.
  - **Unary** encoding with $\mathbf{2^d}$ qubits
  - Dephasing occurs between vertices of the hypercube.
  - Useful model for quantum state transfer in qubit networks.

F. W. Strauch, Phys. Rev. A **79**, 032319 (2009)
arXiv: 0808.3403

See also A. P. Hines and P. C. E. Stamp, arXiv: 0711.1555

Decoherence in the Hypercube Quantum Walk



Lindblad Equation

$$\frac{d\rho}{dt} = -i[H,\rho] - \sum_j \lambda_j \left(L_j \rho L_j^+ - \frac{1}{2} L_j^+ L_j \rho - \frac{1}{2} \rho L_j^+ L_j \right)$$

Dephasing leads to decay of off-diagonal elements of ρ

Subspace Model ($x = x_1 x_2 x_3 x_4 x_5 \ldots$)

$$\frac{d\rho_{x,y}}{dt} = -i \sum_z (H_{xz} \rho_{z,y} - \rho_{x,z} H_{zy}) \boxed{- \lambda \sum_j (1 - \delta_{x_j y_j}) \rho_{x,y}}$$

Dephasing between different subspaces ($x_j \neq y_j$)

Vertex Model

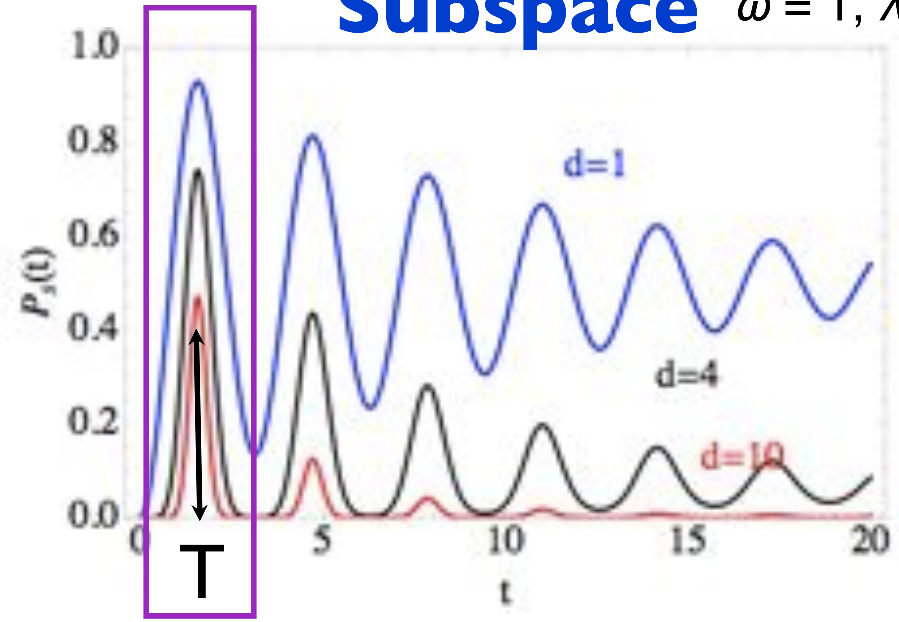
$$\frac{d\rho_{x,y}}{dt} = -i \sum_z (H_{xz} \rho_{z,y} - \rho_{x,z} H_{zy}) \boxed{- \lambda (1 - \delta_{xy}) \rho_{x,y}}$$

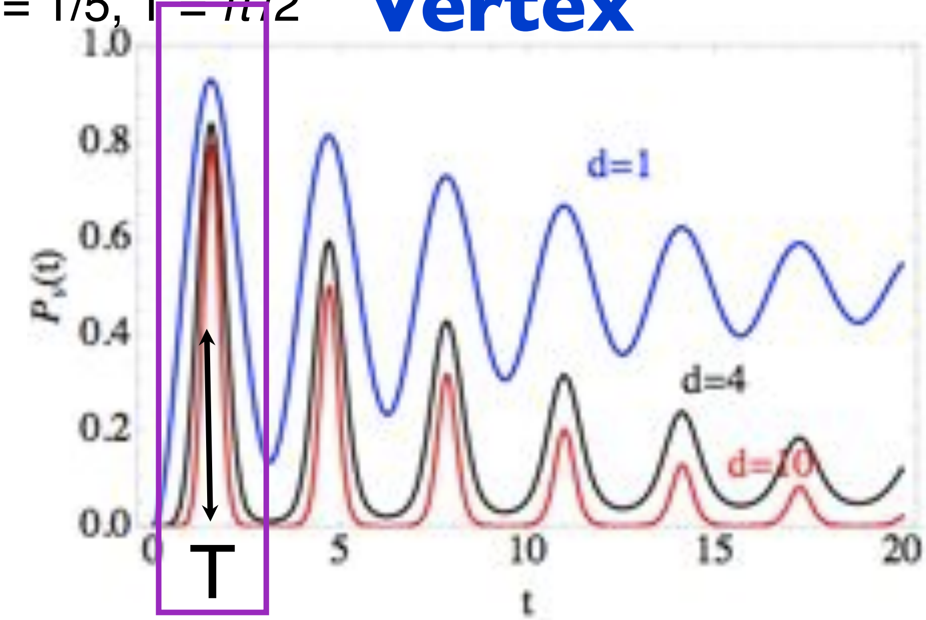
Dephasing between different vertices ($x \neq y$)

Comparison of Models

P(t) = Probability to transfer state from corner to corner.

Subspace $\omega = 1$, $\lambda = 1/5$, T = $\pi/2$ **Vertex**



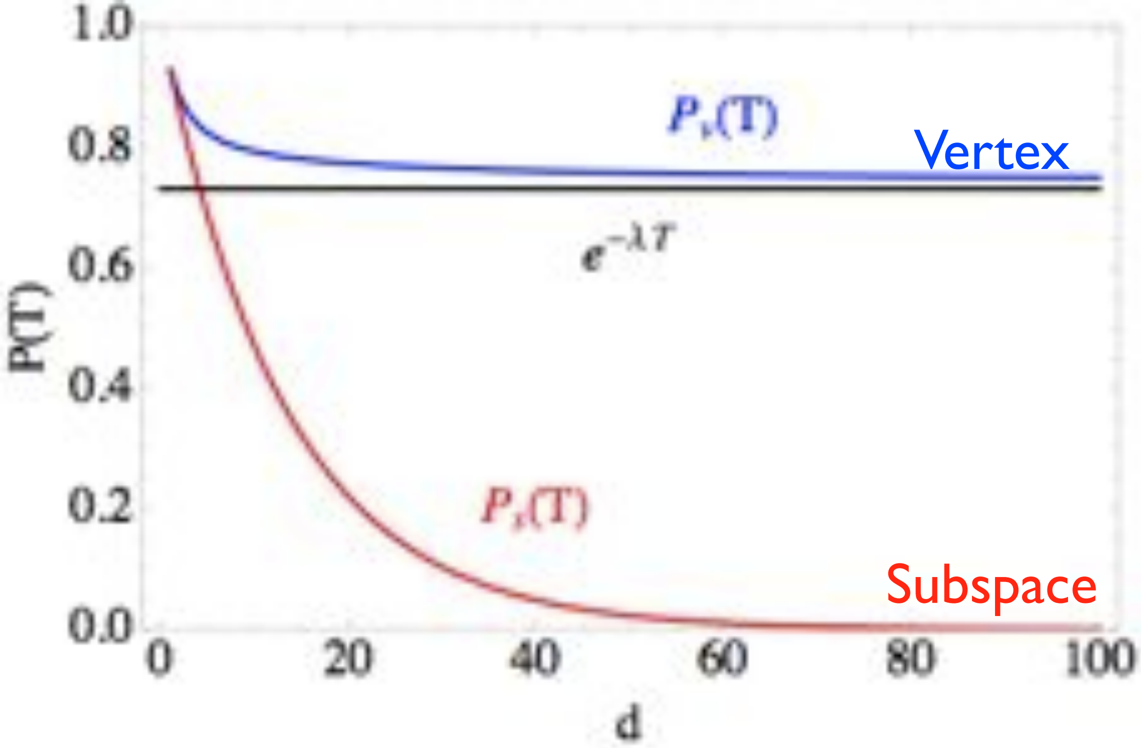


$$P_s(t) \approx 2^{-d}\left[1 - e^{-\lambda t/2}\cos(\omega t)\right]^d$$

Perturbative Results:
Subspace Model = Simple
Vertex Model = Not Simple

See F. W. Strauch, Phys. Rev. A **79**, 032319 (2009) for details

Hitting Probability



$$\boxed{P_v(T) > e^{-\lambda T}}$$

Decoherence in the vertex model has a hitting probability with a lower bound independent of the hypercube dimension ***d*** !

Quantum speedup robust for unary encoding.

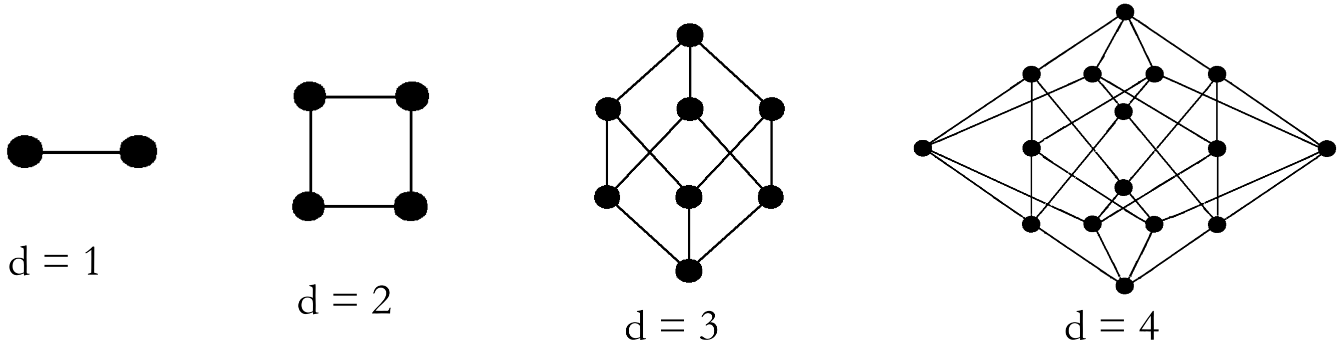
Quantum State Transfer

One can transfer the state of a single qubit from site A to site B using a set of permanently coupled qubits with Hamiltonian:

$$H = -\tfrac{1}{2}\sum_{j} \hbar\omega_{j}\sigma_{j}^{z} + \sum_{jk} \hbar\Omega_{jk}(\sigma_{j}^{+}\sigma_{k}^{-} + \sigma_{k}^{+}\sigma_{j}^{-})$$

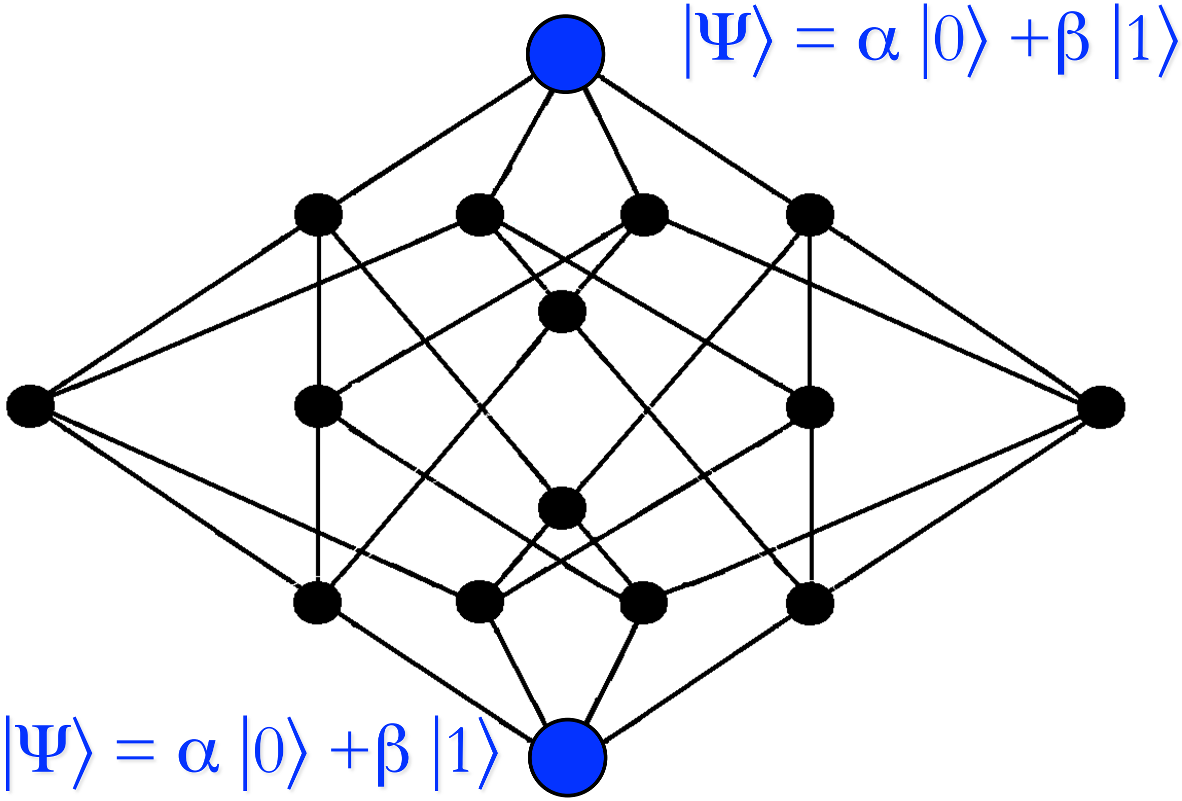
Dynamics of a single excitation (with $\omega = 0$) maps onto the *continuous-time quantum walk* with $H = \hbar\Omega$, where the coupling matrix Ω is proportional to the adjacency matrix of the coupling graph. Certain choices of couplings such as the **hypercube** lead to **perfect state transfer:**

Christandl et al. (2004)



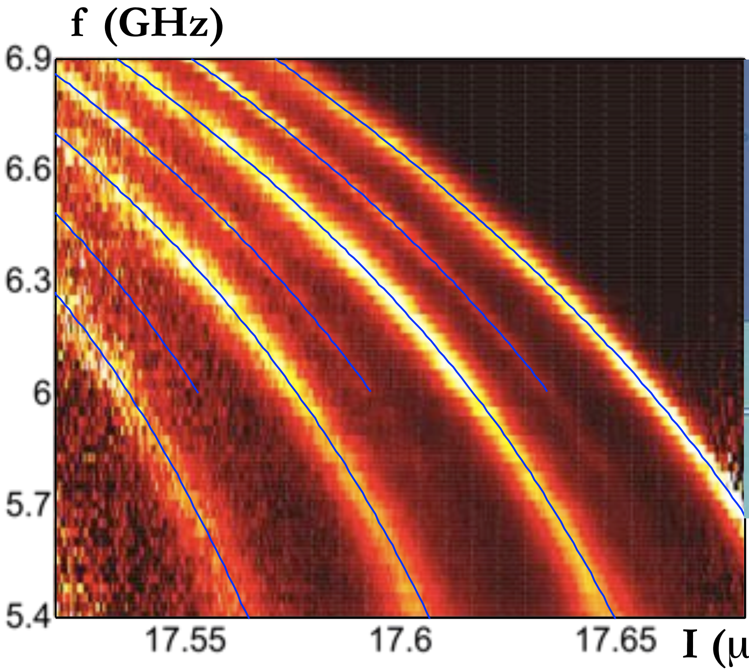
Hypercube State Transfer

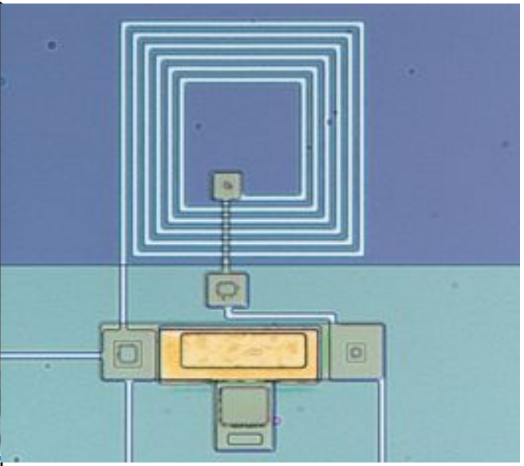
- Each vertex represents a qubit. Quantum states travel along **all paths simultaneously** in superposition with **full constructive interference,** yielding **perfect state transfer.**



Superconducting Circuits

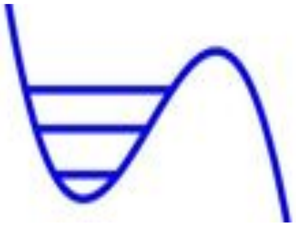
Artificial Atoms → Artificial Molecules → Artificial Solids



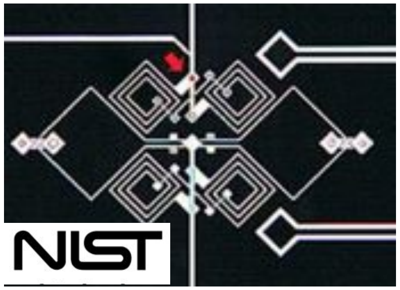


U Maryland

Phase Qubit

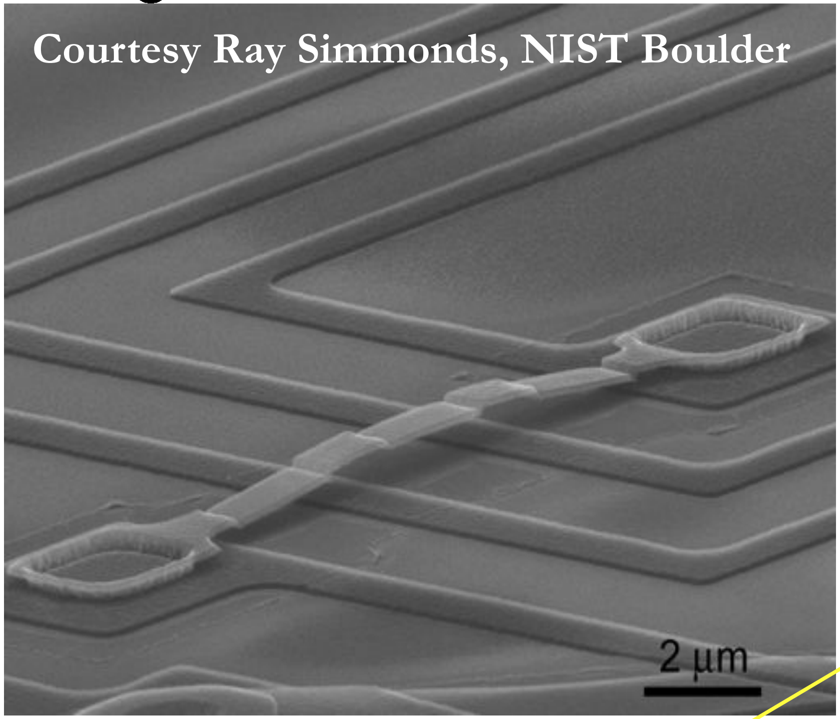


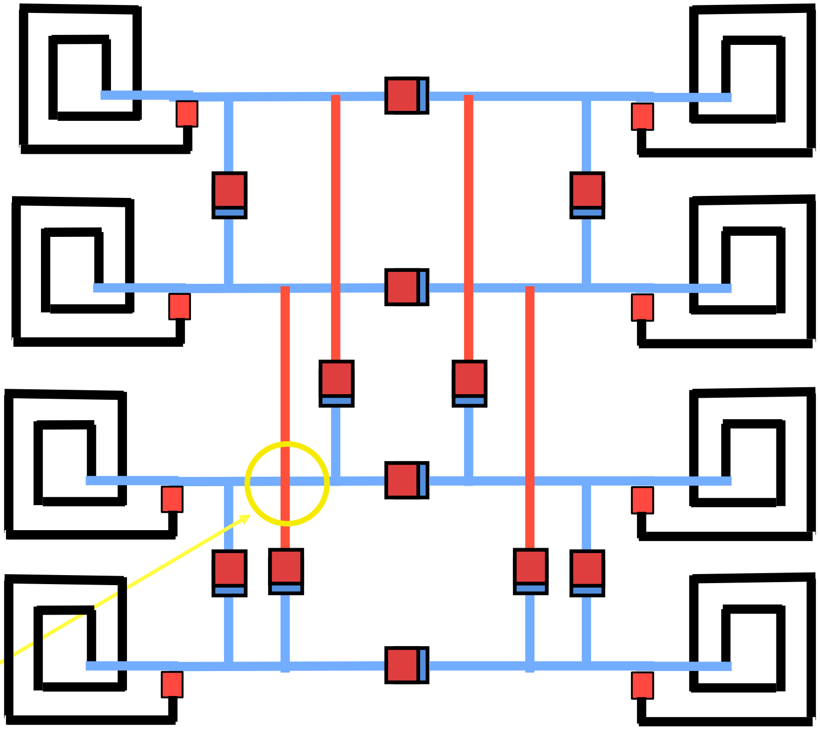
Qubits + Resonators

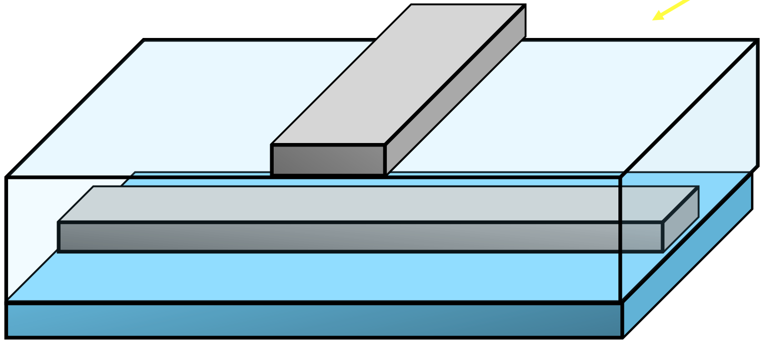


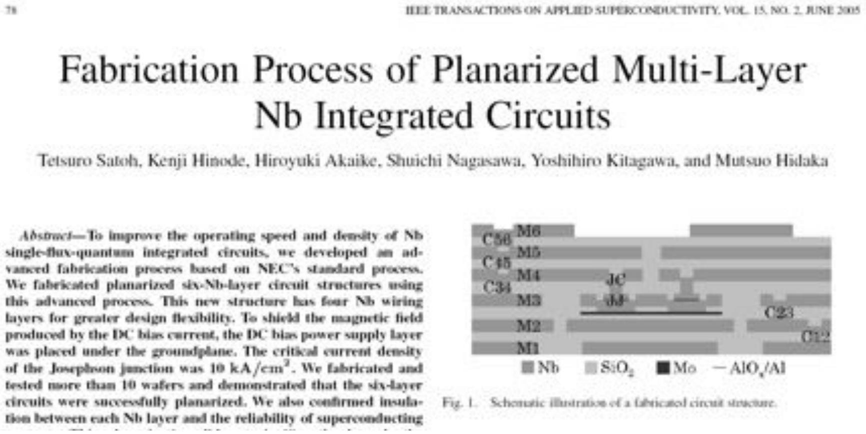
UC Santa Barbara

Phase Qubit Cube 1 2 3


Courtesy Ray Simmonds, NIST Boulder






78 IEEE TRANSACTIONS ON APPLIED SUPERCONDUCTIVITY, VOL. 15, NO. 2, JUNE 2005

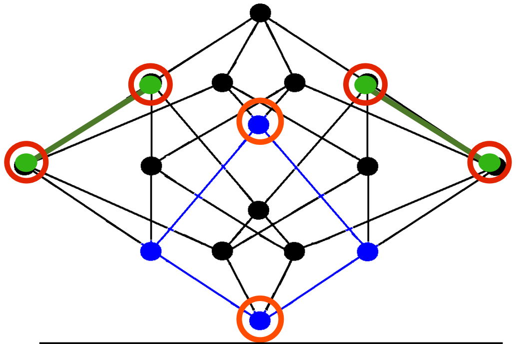
Fabrication Process of Planarized Multi-Layer Nb Integrated Circuits

Tetsuro Satoh, Kenji Hinode, Hiroyuki Akaike, Shuichi Nagasawa, Yoshihiro Kitagawa, and Mutsuo Hidaka

Abstract—To improve the operating speed and density of Nb single-flux-quantum integrated circuits, we developed an advanced fabrication process based on NEC's standard process. We fabricated planarized six-Nb-layer circuit structures using this advanced process. This new structure has four Nb wiring layers for greater design flexibility. To shield the magnetic field produced by the DC bias current, the DC bias power supply layer was placed under the groundplane. The critical current density of the Josephson junction was 10 kA/cm^2. We fabricated and tested more than 10 wafers and demonstrated that the six-layer circuits were successfully planarized. We also confirmed insulation between each Nb layer and the reliability of superconducting

Fig. 1. Schematic illustration of a fabricated circuit structure.

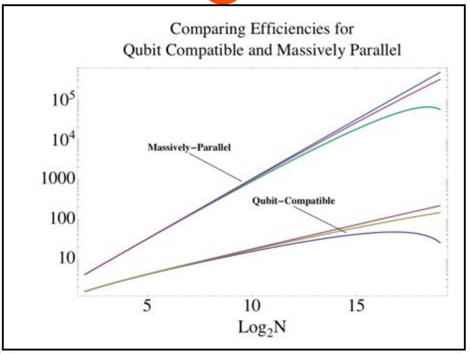
Perfect State Transfer with Phase Qubits

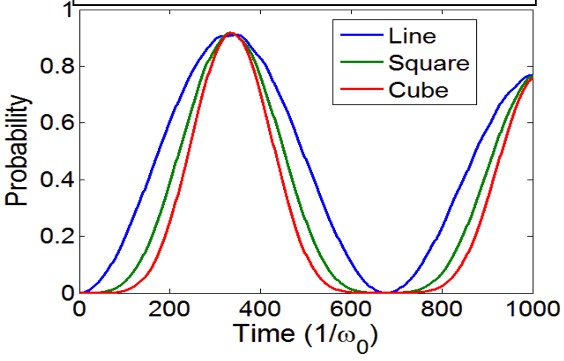


- **Programmable**: Any two nodes can communicate by programming the qubit frequencies in the network.
- **Parallel**: Multiple quantum states can be transferred at the same time.
- **Efficient**: Transfer time is independent of the distance between nodes!
- **High Fidelity:** F > 90% possible using existing technology, modest dimensions!

Requires Study of Disorder and Decoherencen

FWS and C.J. Williams,
Phys. Rev. B **78**, 094516 (2008)





C. Chudzicki and FWS,
Phys. Rev. Lett. **105**, 260501 (2010)

Conclusions

- Quantum Walks are an exciting testbed for quantum information processing
- Connections have emerged between and but each has their own advantages (CTQW for studying physical networks, DTQW for computer algorithms)
- Search algorithms well developed---new algorithms on the way?
- Implementations require study of encoding, decoherence, and disorder. Understanding robustness may be the key.