Arbitrary Control of Entanglement Between Two Superconducting Resonators

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with
Kurt Jacobs, UMass Boston
Ray Simmonds, NIST (Boulder)

Phys. Rev. Lett. **105**, 050501 (2010)

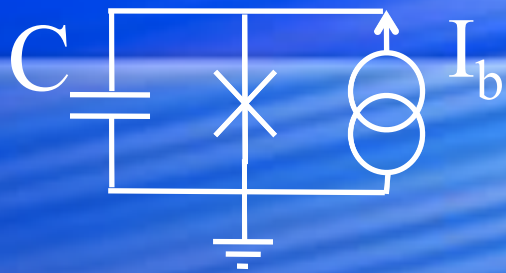
Research Corp., NSF

Outline

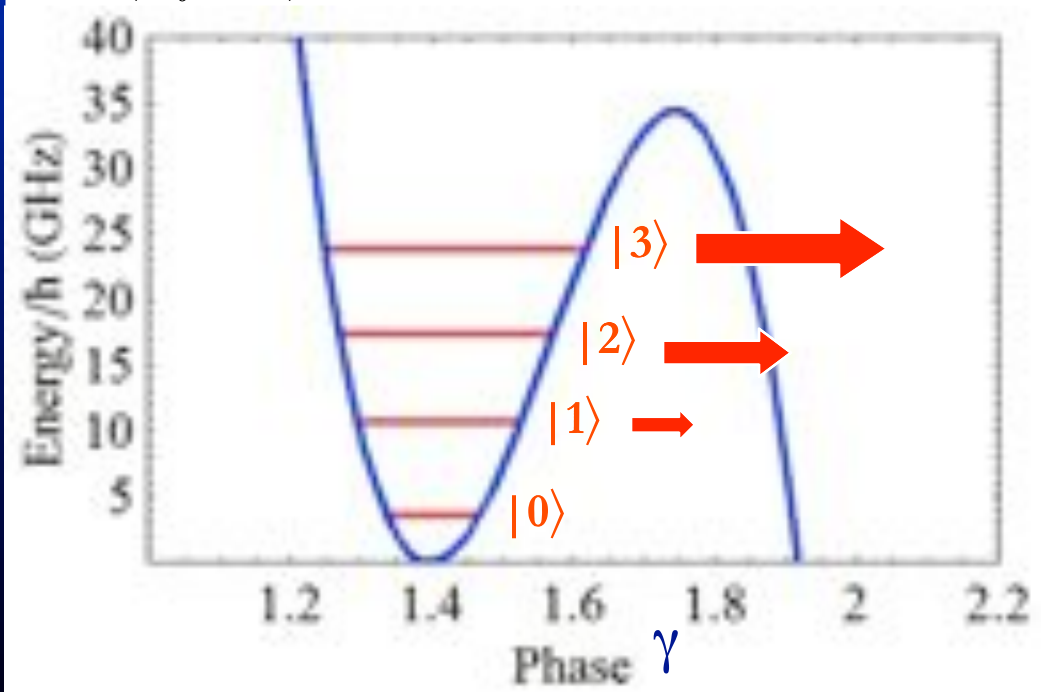
- Superconducting Qubits & Resonators
- NOON States
- State-synthesis Algorithm
  - JC Ladder => Fock-state Diagram
  - Stark-shifted Rabi Oscillations
  - NOON State Synthesis
- Quantum Routing of Entanglement on Oscillator Networks

Phase Qubit

Supercurrent oscillates like a pendulum!



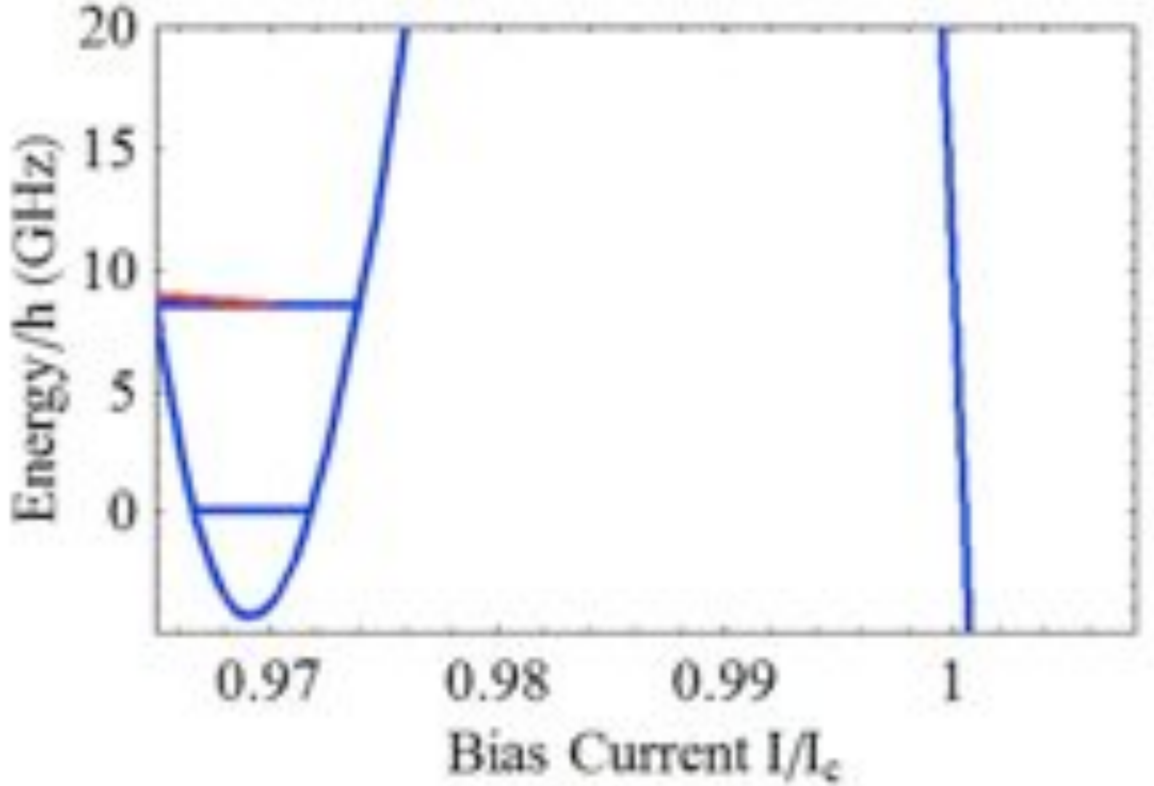
$$-\frac{\hbar^2}{2C\left(\Phi_0/2\pi\right)^2}\frac{d^2\Psi}{d\gamma^2}-\frac{\Phi_0}{2\pi}\left(I_c\cos\gamma+I_b\gamma\right)\Psi=E\Psi$$



- *Quantum oscillator* involving the superconducting current of *billions of Cooper pairs*.
- *Spectroscopic transitions* between energy levels can be probed by microwaves.
- States are *metastable*, will *tunnel* through barrier.

Tunable Oscillator

$$-\frac{\hbar^2}{2C(\Phi_0/2\pi)^2}\frac{d^2\Psi}{d\gamma^2}-\frac{\Phi_0}{2\pi}(I_c\cos\gamma+I_b\gamma)\Psi=E\Psi$$



Sweep of bias current allows experimental control of energy levels.

Tunable Oscillator

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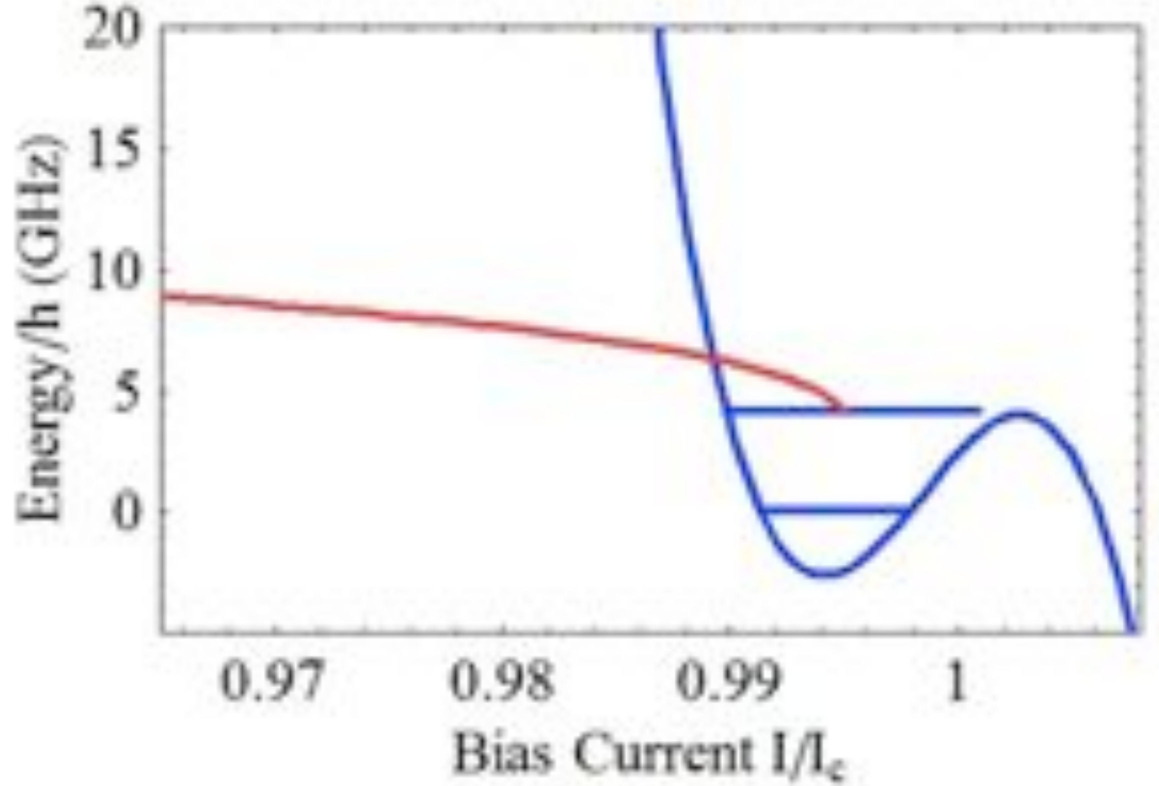

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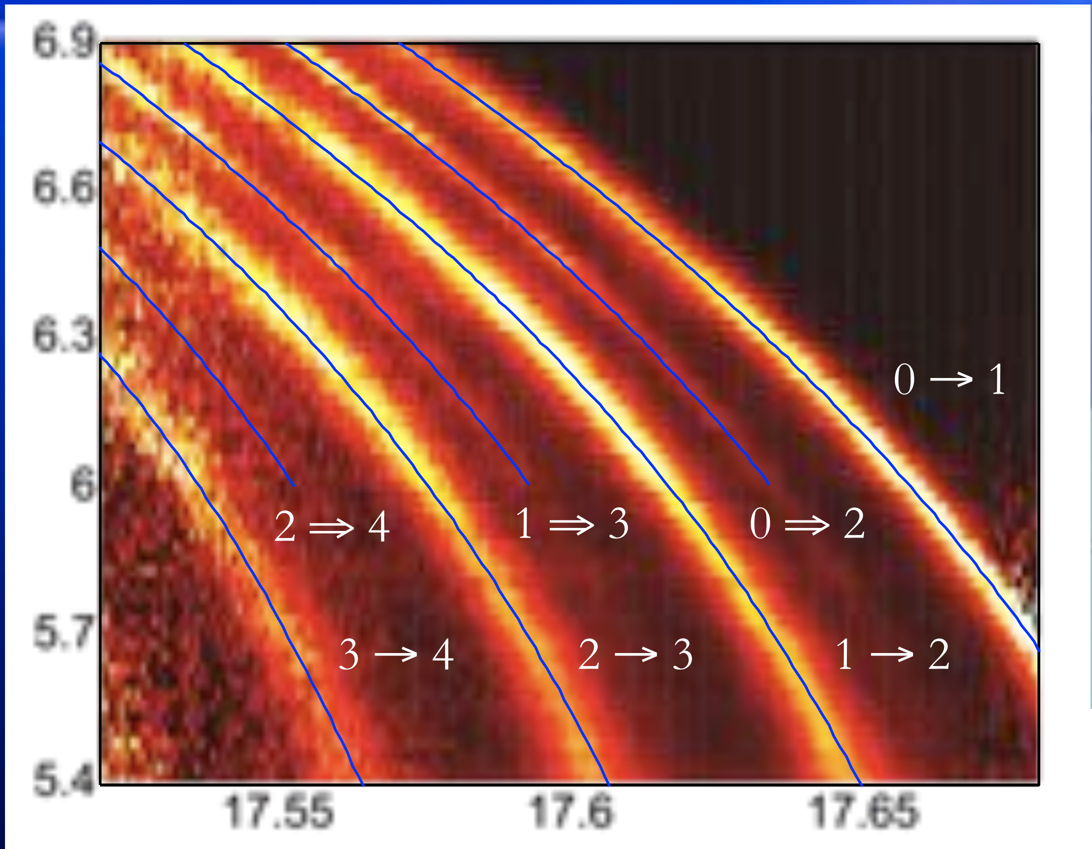
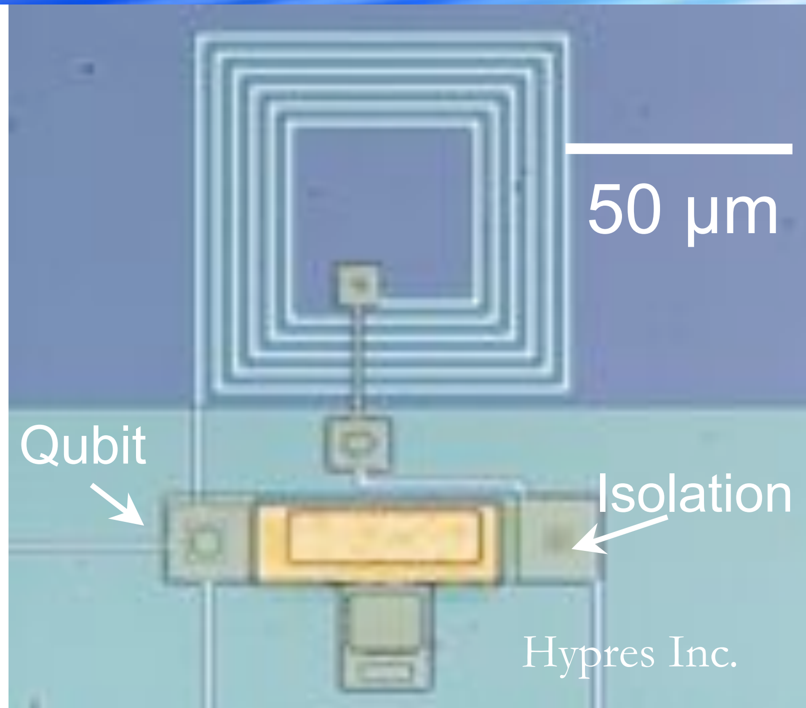
Multi-Photon Spectroscopy

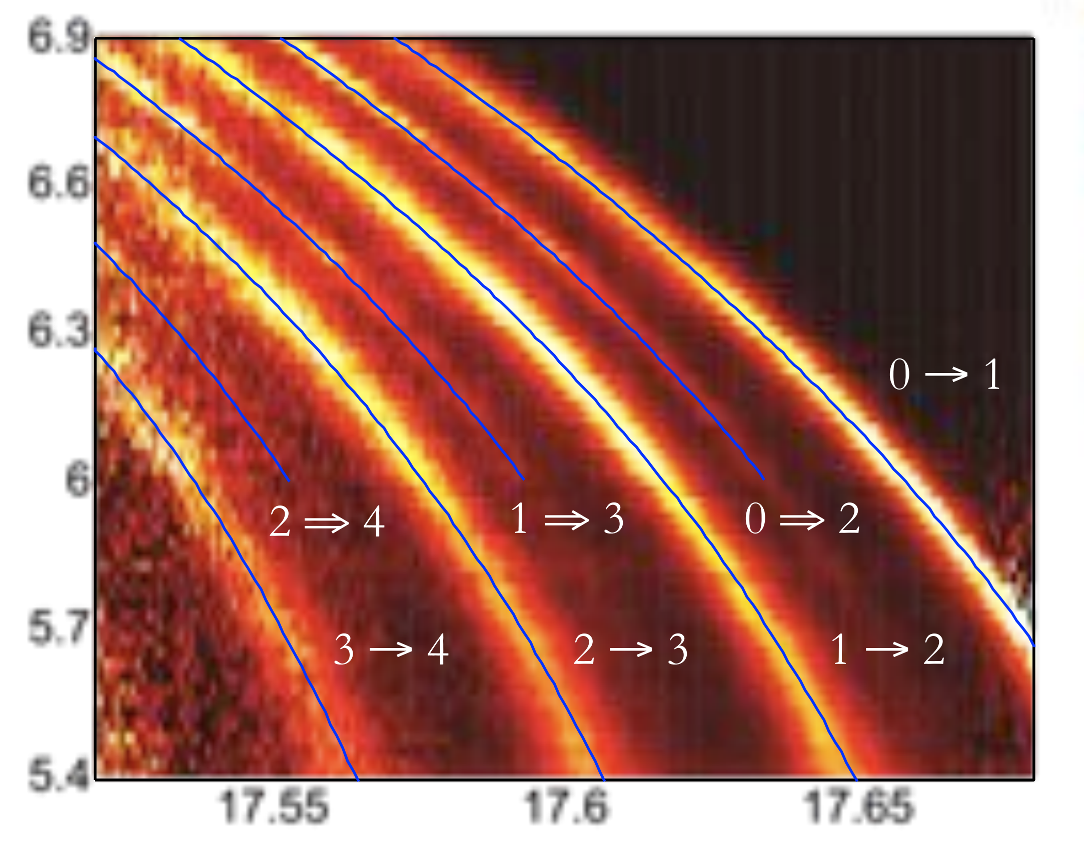


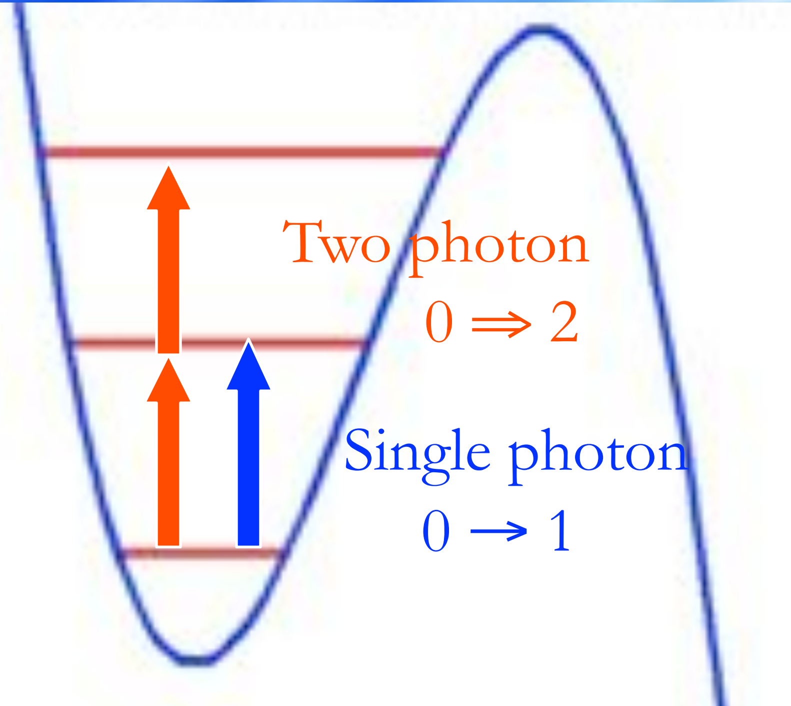


Sudeep Dutta et al. (Univ. Maryland)

Each microwave transition is an excitation of the junction with an increased tunneling rate. Bright indicates a large number of tunneling events, dark a small number of events.

Multi-Photon Spectroscopy





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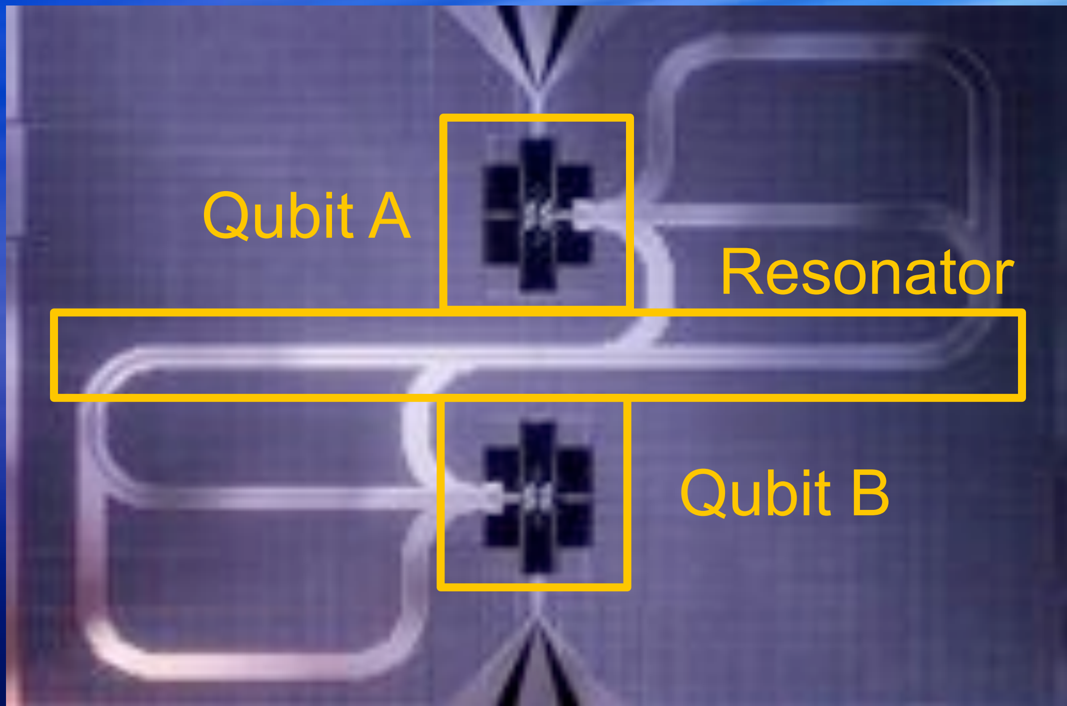
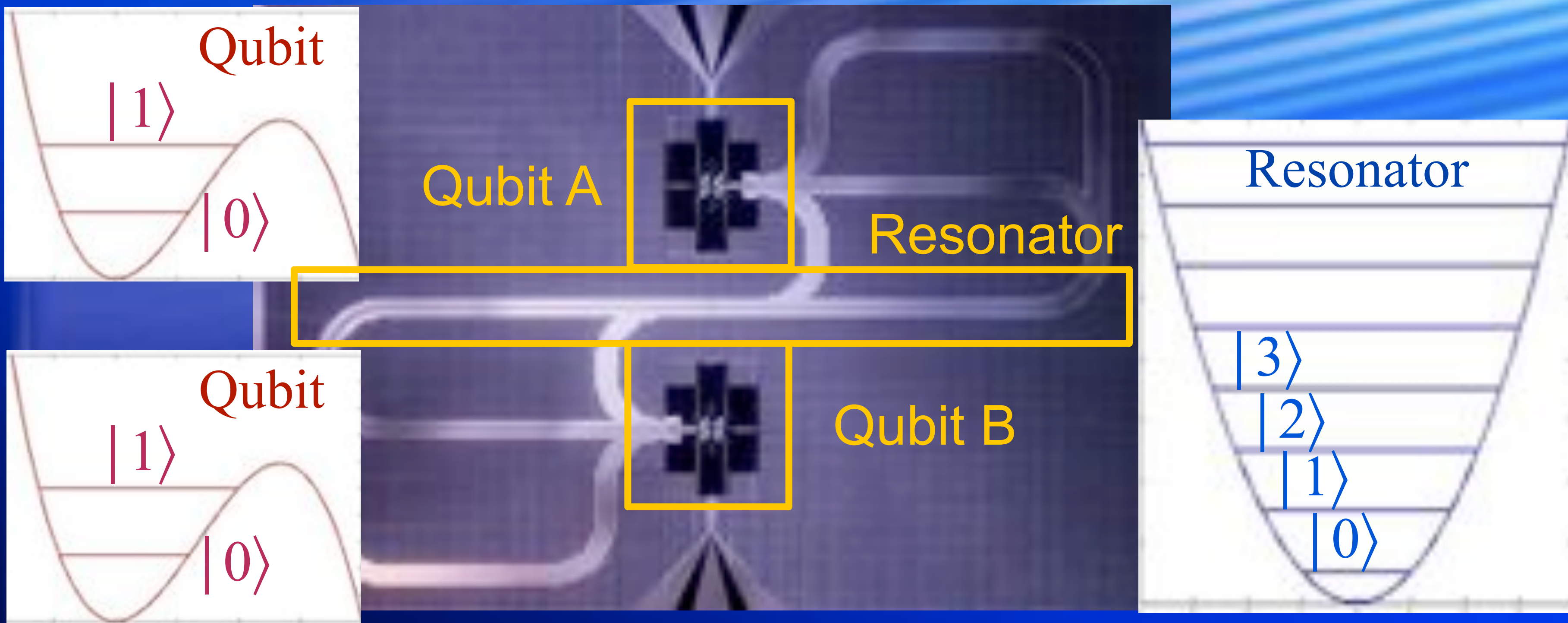
Coupling Qubits by Cavities



"Coherent quantum state storage and transfer between two phase qubits via a resonant cavity", M. Sillanpaa, J. I. Park, and R. W. Simmonds, *Nature* **449**, 438 (2007)

Coupling Qubits by Cavities



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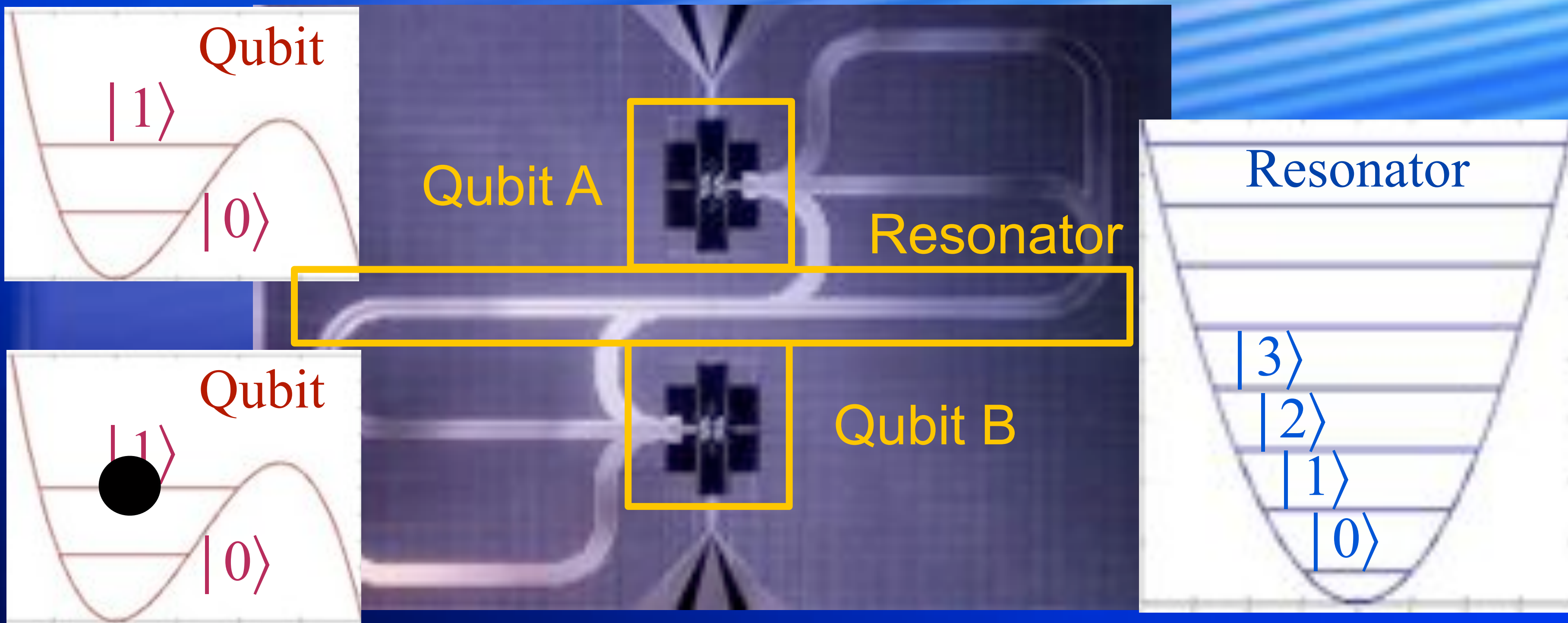

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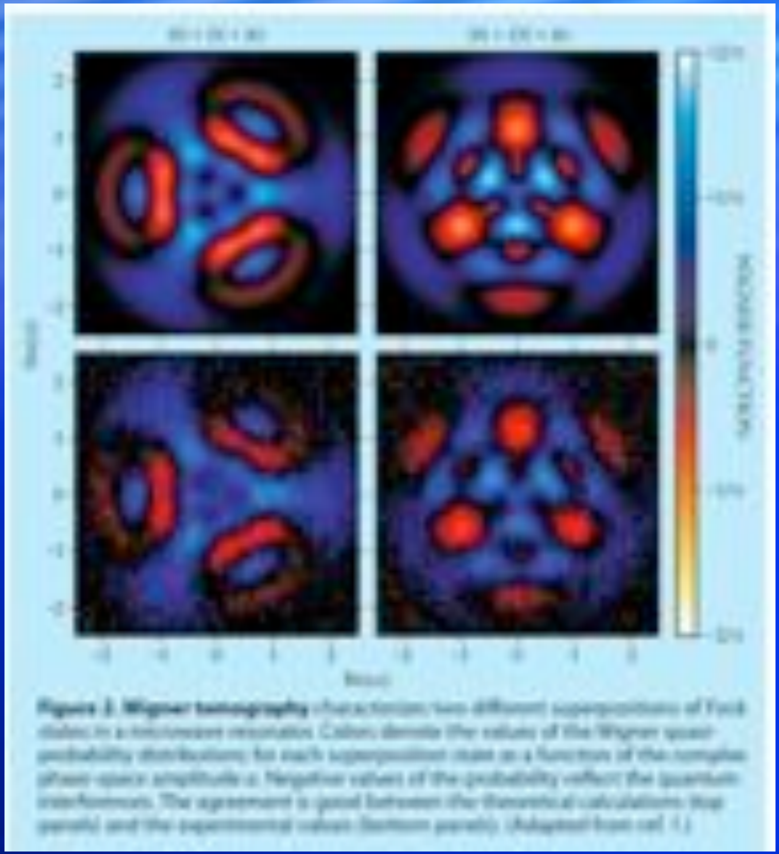
Arbitrary Control of Resonator

- Martinis Group, UC Santa Barbara



State-Synthesis Algorithm

VOLUME 76, NUMBER 7 PHYSICAL REVIEW LETTERS 12 FEBRUARY 1996

Arbitrary Control of a Quantum Electromagnetic Field

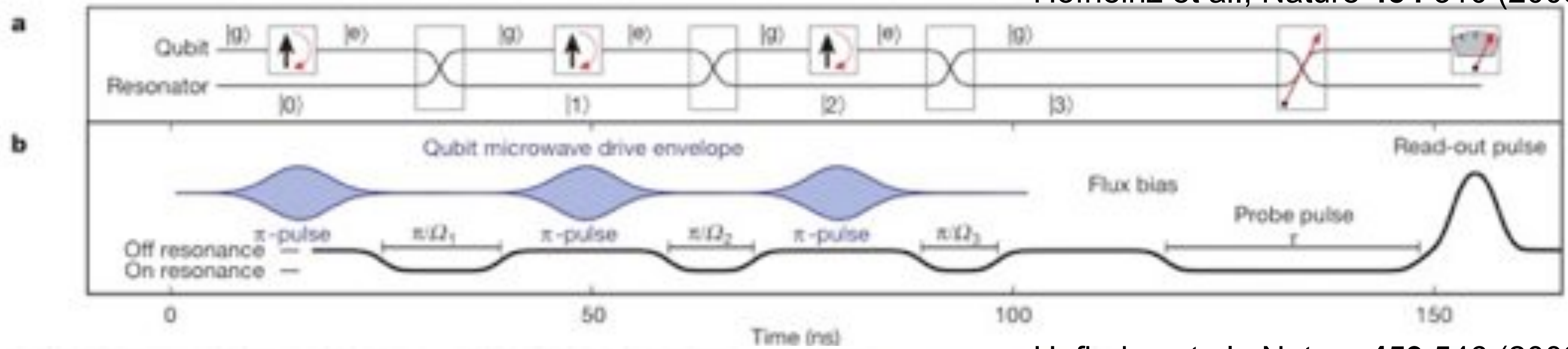
C. K. Law and J. H. Eberly
Rochester Theory Center for Optical Science and Engineering and Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 5 October 1995)

We present a cavity QED interaction which forces the ground state of a cavity field mode to evolve into an *arbitrary* quantum state at a prechosen time t^*. This method does not involve either atom-field state entanglement or the projections characteristic of quantum measurement.

- Law & Eberly developed a scheme to program an arbitrary state of a single harmonic oscillator mode by coupling to a two-level system.
- Climbing Jaynes-Cummings Ladder one rung at a time.

State-Synthesis Experiments

Hofheinz et al., Nature **454** 310 (2008)



Hofheinz et al., Nature **459** 546 (2009)

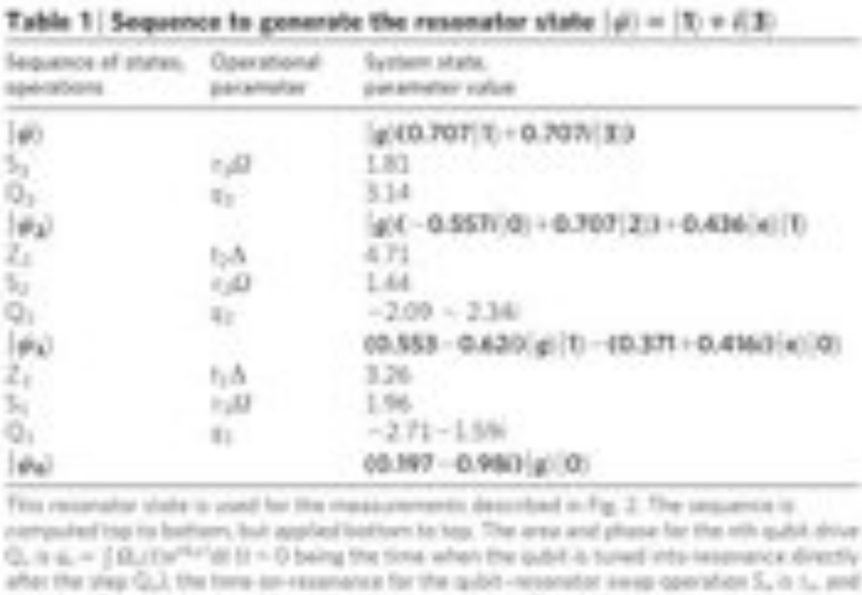
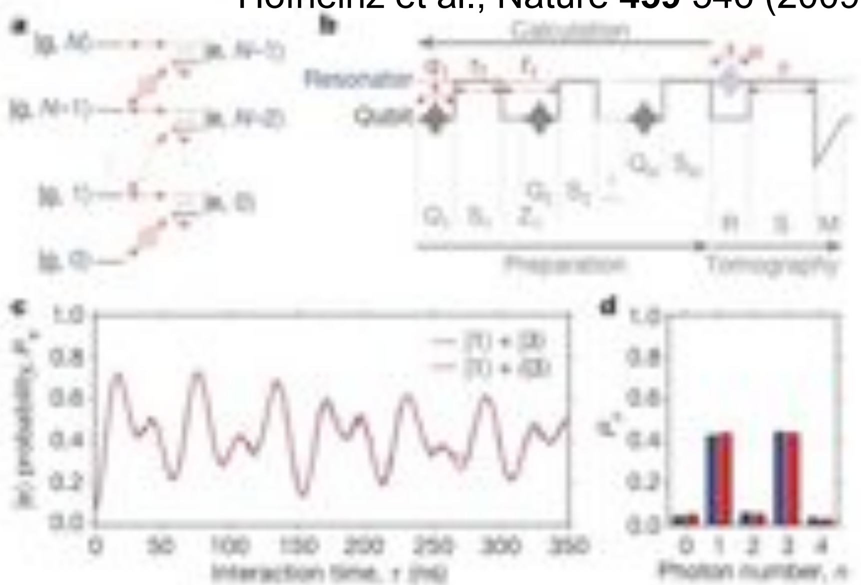


Table 1 | Sequence to generate the resonator state $|\psi\rangle = |1\rangle + i|3\rangle$

Sequence of states, operations	Operational parameter	System state, parameter value
$\lvert\psi\rangle$		$\lvert g\rangle(0.707\lvert 1\rangle + 0.707i\lvert 3\rangle)$
S_3	$\tau_3\Omega$	1.83
Q_3	a_3	3.14
$\lvert\psi_2\rangle$		$\lvert g\rangle(-0.557\lvert 0\rangle + 0.707\lvert 2\rangle) + 0.436\lvert e\rangle\lvert 1\rangle$
Z_2	$t_2\Delta$	4.71
S_2	$\tau_2\Omega$	1.44
Q_2	a_2	$-2.09 - 2.34i$
$\lvert\psi_1\rangle$		$(0.553 - 0.620i)\lvert g\rangle\lvert 1\rangle - (0.371 + 0.416i)\lvert e\rangle\lvert 0\rangle$
Z_1	$t_1\Delta$	3.26
S_1	$\tau_1\Omega$	1.96
Q_1	a_1	$-2.71 - 1.59i$
$\lvert\psi_0\rangle$		$(0.197 - 0.980i)\lvert g\rangle\lvert 0\rangle$

This resonator state is used for the measurements described in Fig. 2. The sequence is computed top to bottom, but applied bottom to top.



Tuesday, November 22, 2011

Big Question: Qubits or Resonators

- Is it better to use harmonic oscillators (resonators) or qubits for quantum computing?
- Common wisdom: ***It depends...***
  - Use qubit nonlinearity to encode information.
  - Use oscillator coherence to store information.
  - Use resonators to couple qubits.
  - Use ? to readout information.
  - **Use ? to process information.**

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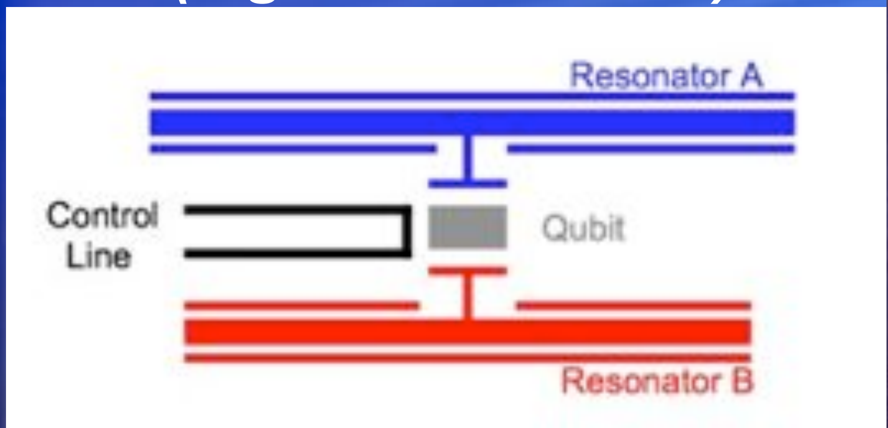
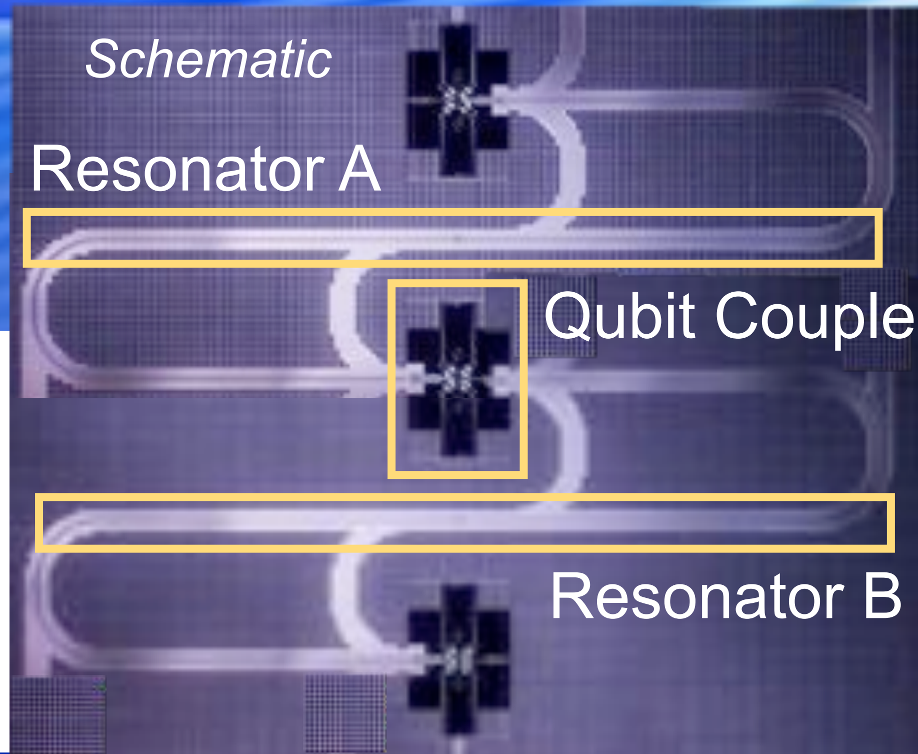
  - **Use ? to process information.**

Little Question:

What control sequence is required to generate an ***arbitrary entangled state?*** ***(e.g. NOON states)***





$$|\Psi\rangle = \sum_{n_a=0}^{N_a} \sum_{n_b=0}^{N_b} c_{n_a,n_b} |n_a\rangle \otimes |n_b\rangle. \quad |\Psi\rangle = \frac{1}{\sqrt{2}} \left(|N,0\rangle + |0,N\rangle\right).$$

High NOON and Beyond

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|N, 0\rangle + |0, N\rangle \right).$$

- Schrödinger Cat State
- Generalizes Bell/GHZ states
- Useful for metrology:

$$|\langle\Psi|e^{i\phi a^\dagger a}|\Psi\rangle| = |\cos(N\phi/2)|$$

- May have applications for quantum state preparation, entanglement transfer and distribution, ...

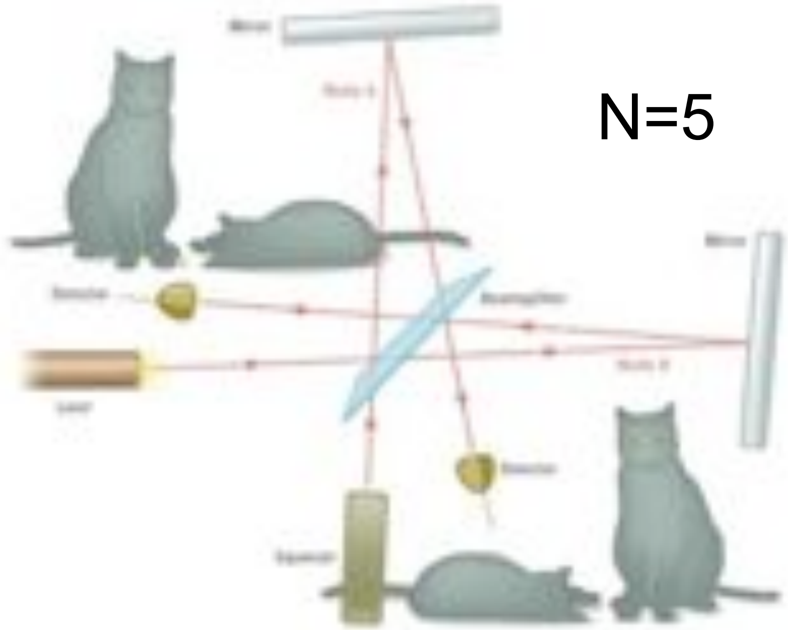
Recent NOON Linear Optics

High-NOON States by Mixing Quantum and Classical Light

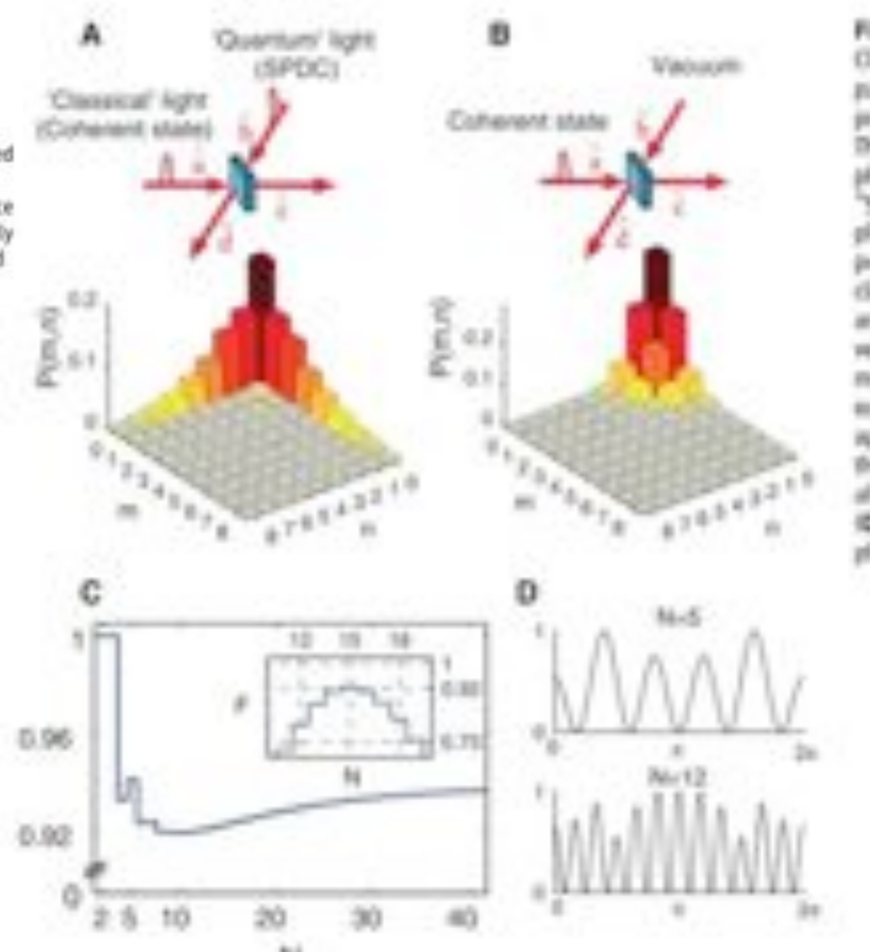
Science **328**, 879 (2010)

Itai Afek, Oron Ambar, Yaron Silberberg*

Precision measurements can be brought to their ultimate limit by harnessing the principles of quantum mechanics. In optics, multiphoton entangled states, known as NOON states, can be used to obtain high-precision phase measurements, becoming more and more advantageous as the number of photons grows. We generated "high-NOON" states (N = 5) by multiphoton interference of quantum down-converted light with a classical coherent state in an approach that is inherently scalable. Super-resolving phase measurements with up to five entangled photons were produced with a visibility higher than that obtainable using classical light only.

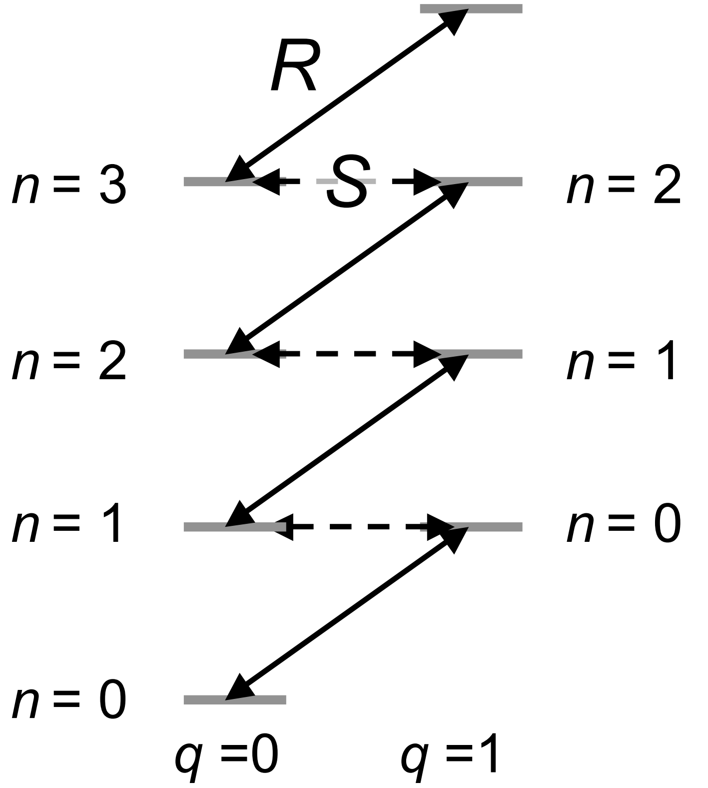


N=5



Jaynes-Cummings Ladder

$$H = \omega_q(t)|1\rangle\langle 1| + \tfrac{1}{2}\left(\Omega(t)|1\rangle\langle 0| + \Omega^*(t)|0\rangle\langle 1|\right) + \omega_a a^\dagger a + g_a\left(\sigma_+ a + \sigma_- a^\dagger\right)$$



Rabi pulses (R) drive qubit transitions (q=0 → 1)

Shift pulses (S) transfer quanta between qubit and oscillator.

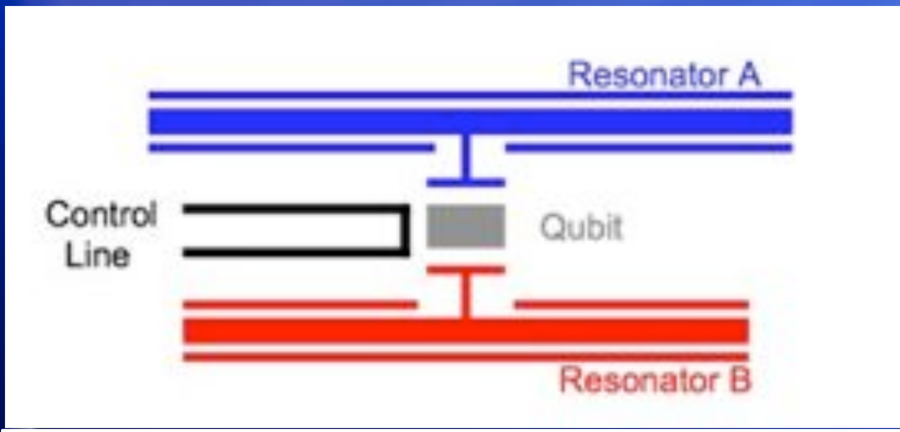
Used to generate arbitrary superpositions of Fock states: Hofheinz *et al.*, *Nature* **459**, 456 (2009)

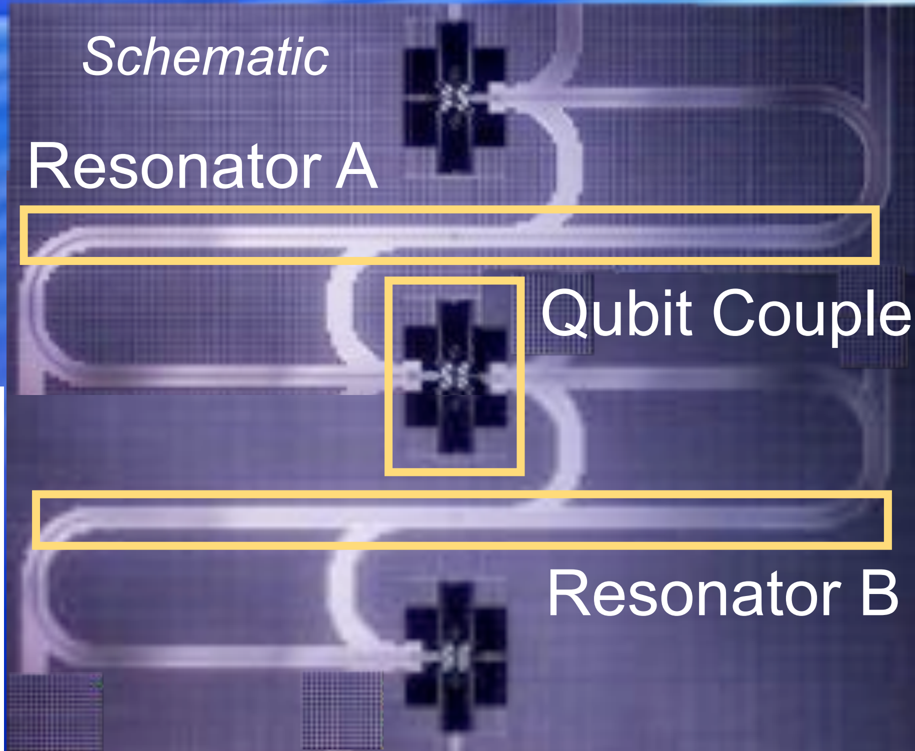
Two-Mode Jaynes Cummings

Minimal Scheme

one qubit
(tunable frequency)

two resonators
(different frequencies)





$$
\begin{aligned}
H \;=\; & \omega_q(t)|1\rangle\langle 1| + \tfrac{1}{2}\left(\Omega(t)|1\rangle\langle 0| + \Omega^*(t)|0\rangle\langle 1|\right) \\
& + \omega_a a^\dagger a + \omega_b b^\dagger b \\
& + g_a\left(\sigma_+ a + \sigma_- a^\dagger\right) + g_b\left(\sigma_+ b + \sigma_- b^\dagger\right).
\end{aligned}
$$

$\omega_a < \omega_q < \omega_b$

Two-Mode Jaynes-Cummings Diagram

$$\begin{aligned} H &= \omega_q(t)|1\rangle\langle 1| + \tfrac{1}{2}\left(\Omega(t)|1\rangle\langle 0| + \Omega^*(t)|0\rangle\langle 1|\right) \\ &+\omega_a a^\dagger a + \omega_b b^\dagger b \\ &+g_a\left(\sigma_+ a + \sigma_- a^\dagger\right) + g_b\left(\sigma_+ b + \sigma_- b^\dagger\right). \end{aligned}$$

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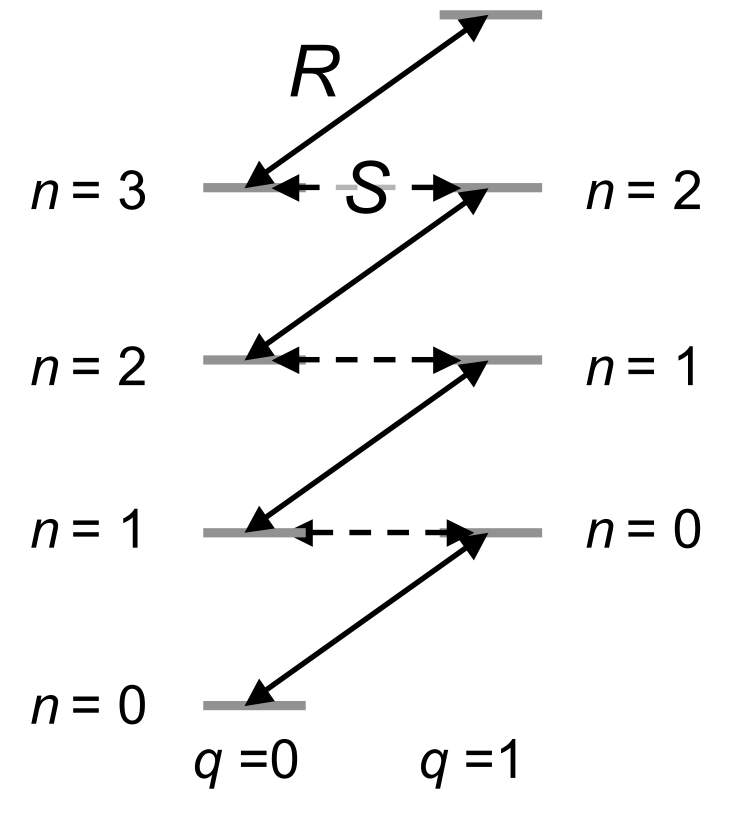

Rabi pulses (R) drive qubit transitions (q=0 → 1)

Shift pulses (A + B) transfer quanta between qubit and oscillators (a + b).

Need Fock-state-selective interaction!

Two-Mode Jaynes-Cummings Diagram

$$H = \omega_q(t)|1\rangle\langle 1| + \tfrac{1}{2}\left(\Omega(t)|1\rangle\langle 0| + \Omega^*(t)|0\rangle\langle 1|\right) + \omega_a a^\dagger a + \omega_b b^\dagger b + g_a\left(\sigma_+ a + \sigma_- a^\dagger\right) + g_b\left(\sigma_+ b + \sigma_- b^\dagger\right).$$

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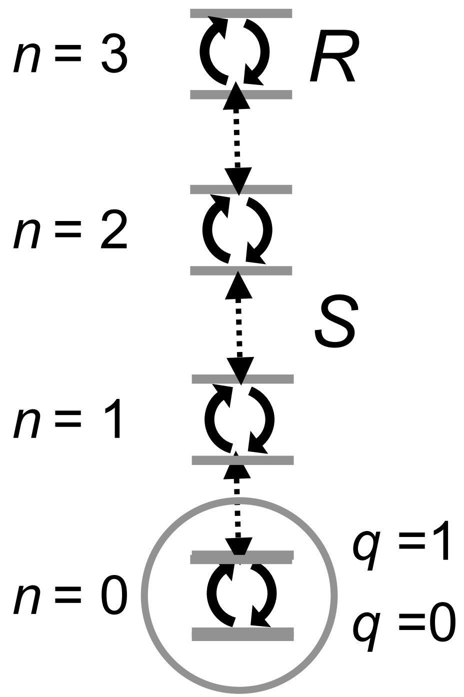

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\end{aligned}
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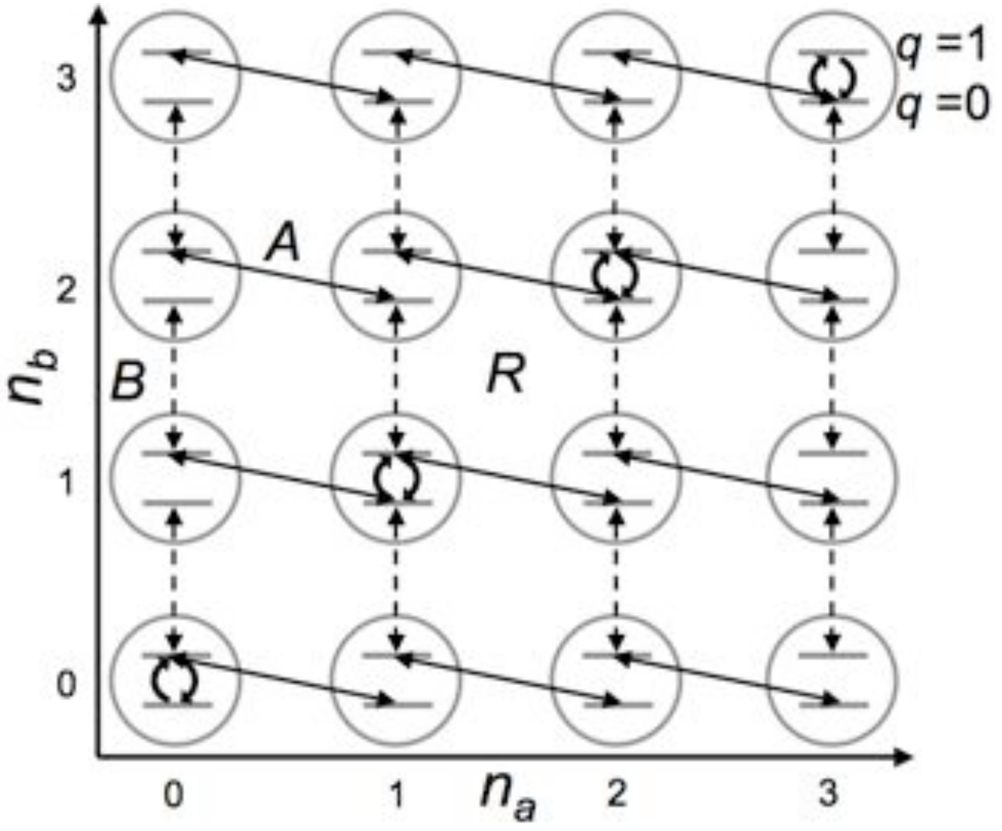

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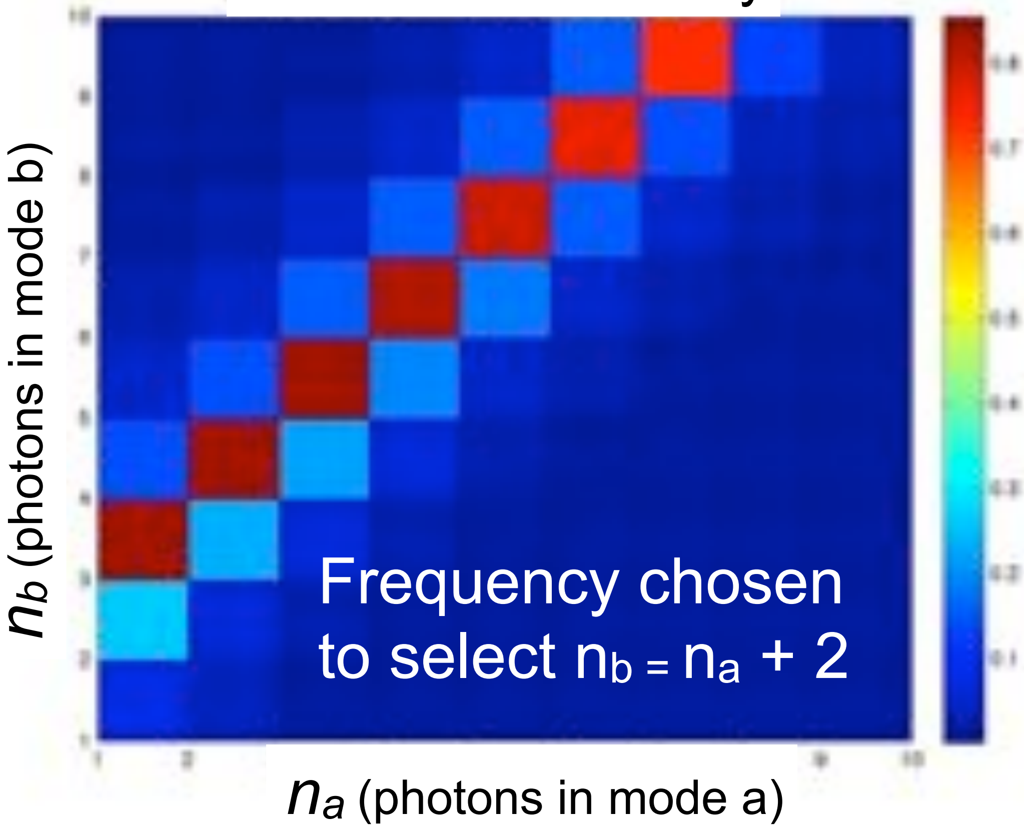
Stark-shifted Rabi Oscillations

$$\omega = \omega_q + \frac{g_a^2}{\omega_q - \omega_a}(2n_a + 1) + \frac{g_b^2}{\omega_q - \omega_b}(2n_b + 1)$$

Transition Probability



Special condition:

$g_a^2 / (\omega_q - \omega_a) = -g_b^2 / (\omega_q - \omega_b)$

Stark-shifted Rabi transitions yield Fock-state-selective qubit rotations!

cf. Schuster *et al.*, *Nature* **445**, 515 (2007)

State-Synthesis Algorithm

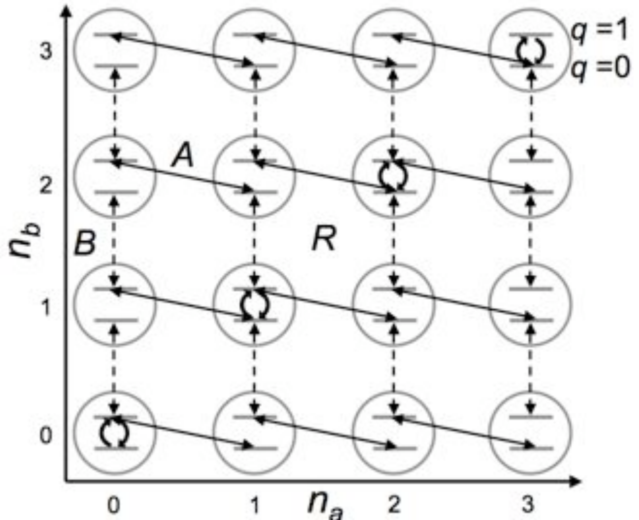
$$U = \left(\prod_{j=1}^{N_b} U_{b,j} \right) U_a$$

$$U_a = \prod_{j=1}^{N_a} A_j R_{a,j}, \text{ and } U_{b,j} = \prod_{k=1}^{N_a} B_{jk} R_{b,jk}.$$

$$A_j = \exp(-iH_a t_{a,j}), \text{ and} B_{jk} = \exp(-iH_b t_{b,jk}).$$

Interaction times + qubit rotations chosen to satisfy

$$|\Psi\rangle = U|0,0,0\rangle = |0\rangle \otimes \sum_{n_a=1}^{N_a} \sum_{n_b=1}^{N_b} c_{n_a,n_b} |n_a, n_b\rangle,$$

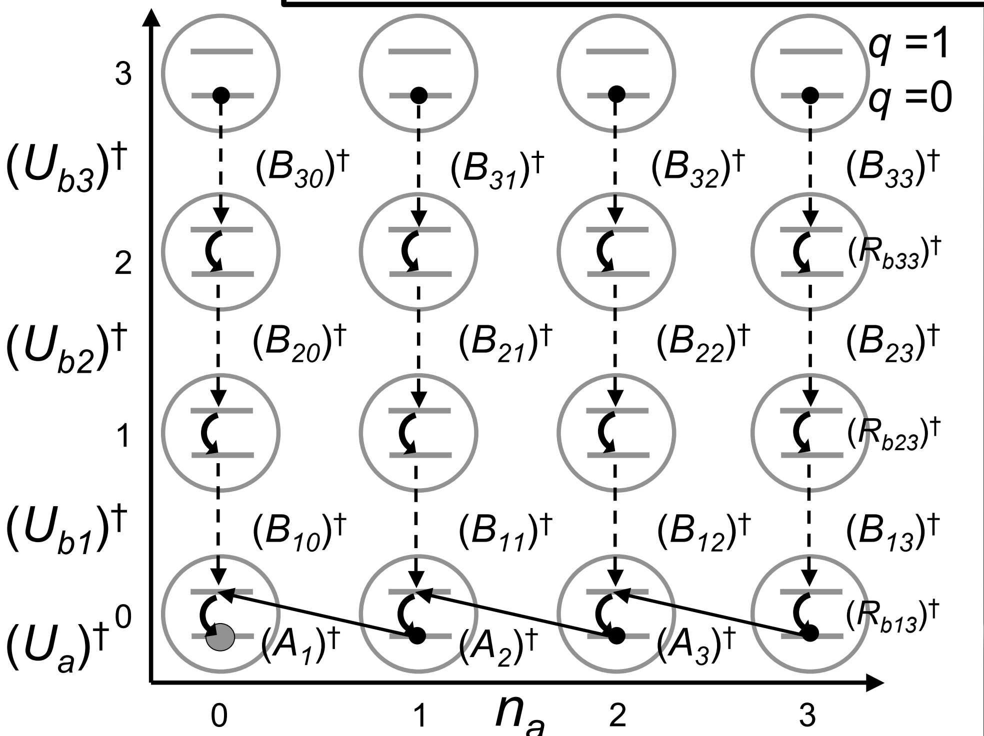


Solution requires solving for the inverse, clearing amplitudes row-by-row, column-by-column, performing two-state oscillations at each step.

Algorithm Details

$$U^\dagger|\Psi\rangle = U_a^\dagger \prod_{j=N_b}^{1} U_{b,j}^\dagger |\Psi\rangle = |0,0,0\rangle. \quad U_a = \prod_{j=1}^{N_a} A_j R_{a,j}, \, U_{b,j} = \prod_{k=1}^{N_a} B_{jk} R_{b,jk}.$$

$$T_{\max} = N_a(N_b+1)\frac{\pi}{\Omega} + \sum_{j=1}^{N_a} \frac{\pi}{g_a\sqrt{j}} + N_a \sum_{j=1}^{N_b} \frac{\pi}{g_b\sqrt{j}}.$$



Clear amplitudes row-by-row, column-by-column. (Stark shift is key!)

Time ~ N^2

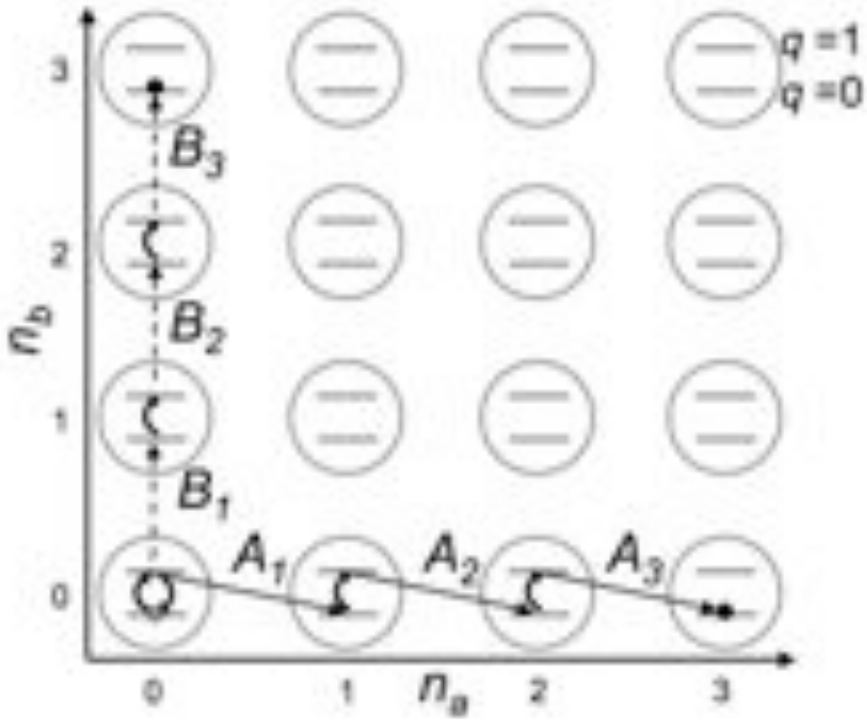
NOON State Example

High NOON state:
J.P. Dowling, Contemp. Phys. **49**, 125 (2008)

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|3,0\rangle + |0,3\rangle\right)$$

TABLE I: NOON State Synthesis Procedure

Step	Parameters	State
$R_{a,1}$	$\Omega t_{ge,1} = \pi/2, \omega_d = \omega_0$	$\|0,0,0\rangle - i\|1,0,0\rangle$
A_1	$g_a t_{a,1} = \pi$	$\|0,0,0\rangle - \|0,1,0\rangle$
$R_{a,2}$	$\Omega t_{ge,2} = \pi, \omega_d = \omega_1$	$\|0,0,0\rangle + i\|1,1,0\rangle$
A_2	$g_a t_{a,2} = \pi/\sqrt{2}$	$\|0,0,0\rangle + \|0,2,0\rangle$
$R_{a,3}$	$\Omega t_{ge,3} = \pi, \omega_d = \omega_2$	$\|0,0,0\rangle - i\|1,2,0\rangle$
A_3	$g_a t_{a,3} = \pi/\sqrt{3}$	$\|0,0,0\rangle - \|0,3,0\rangle$
$R_{b,1}$	$\Omega t_{gb,1} = \pi, \omega_d = \omega_0$	$-i\|1,0,0\rangle - \|0,3,0\rangle$
B_1	$g_b t_{b,1} = \pi$	$-\|0,0,1\rangle - \|0,3,0\rangle$
$R_{b,2}$	$\Omega t_{gb,2} = \pi, \omega_d = \omega_{-1}$	$i\|1,0,1\rangle - \|0,3,0\rangle$
B_2	$g_b t_{b,2} = \pi/\sqrt{2}$	$\|0,0,2\rangle - \|0,3,0\rangle$
$R_{b,3}$	$\Omega t_{gb,3} = \pi, \omega_d = \omega_{-2}$	$-i\|1,0,2\rangle - \|0,3,0\rangle$
B_3	$g_b t_{b,3} = \pi/\sqrt{3}$	$-\|0,0,3\rangle - \|0,3,0\rangle$

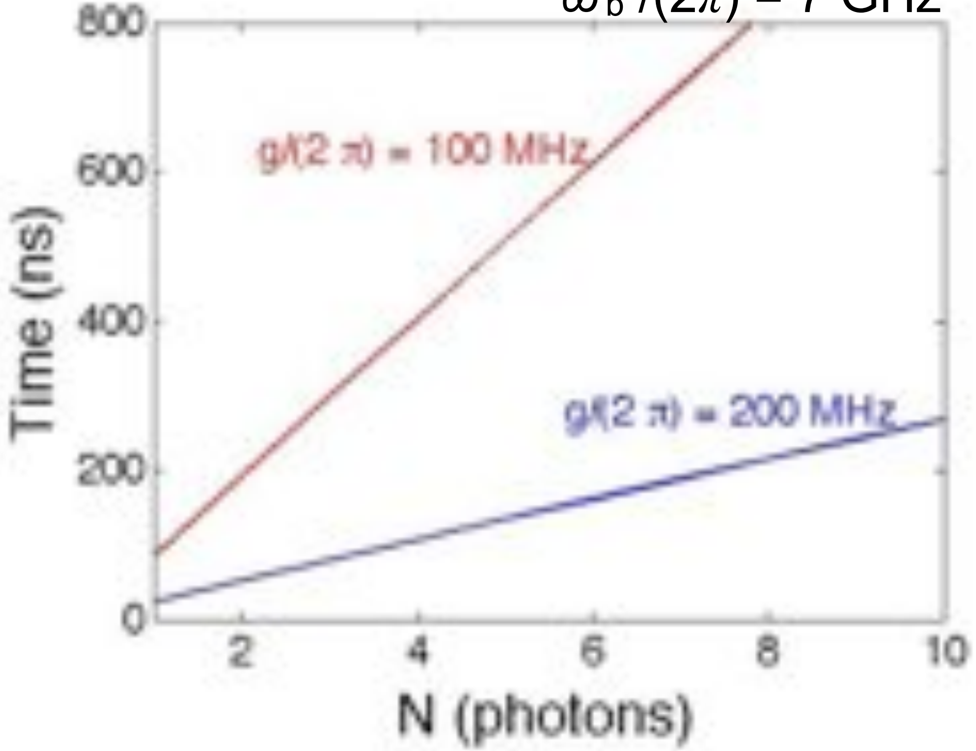


NOON State Synthesis

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|N,0\rangle + |0,N\rangle\right).$$

$$T_N = (2N - \tfrac{1}{2})\frac{\pi}{\Omega} + \frac{2\pi}{g}\sum_{j=1}^{N}\frac{1}{\sqrt{j}}$$

$\omega_a/(2\pi)$ = 6 GHz
$\omega_q/(2\pi)$ = 6.5 GHz
$\omega_b/(2\pi)$ = 7 GHz

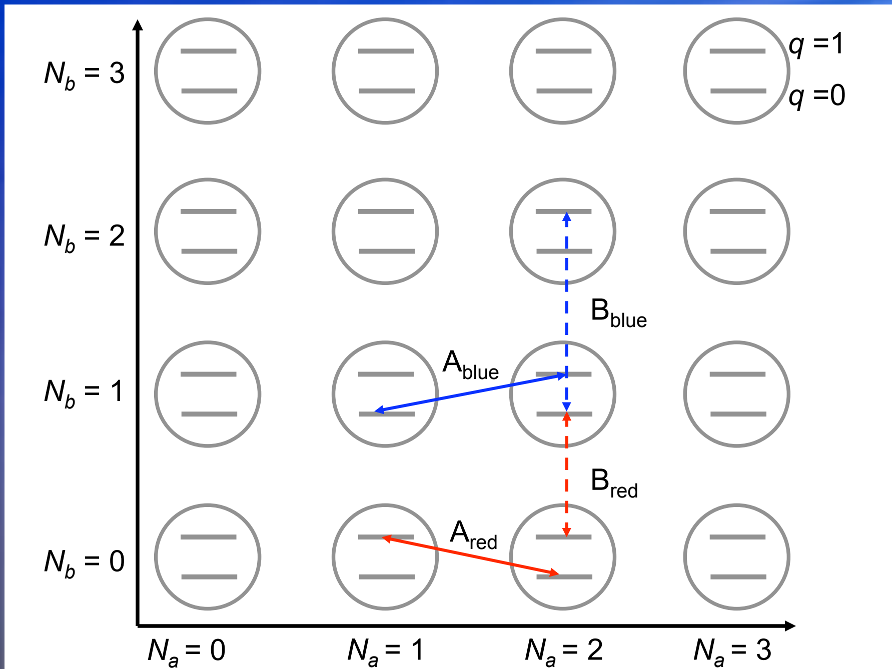


Main limitation is due to Rabi frequency, to remain in Stark-shifted regime (can be optimized using pulse shapes).

High NOON states are accessible using existing technology!

Other Methods: Sidebands

- New transitions, new paths



Red + Blue Sidebands for each qubit

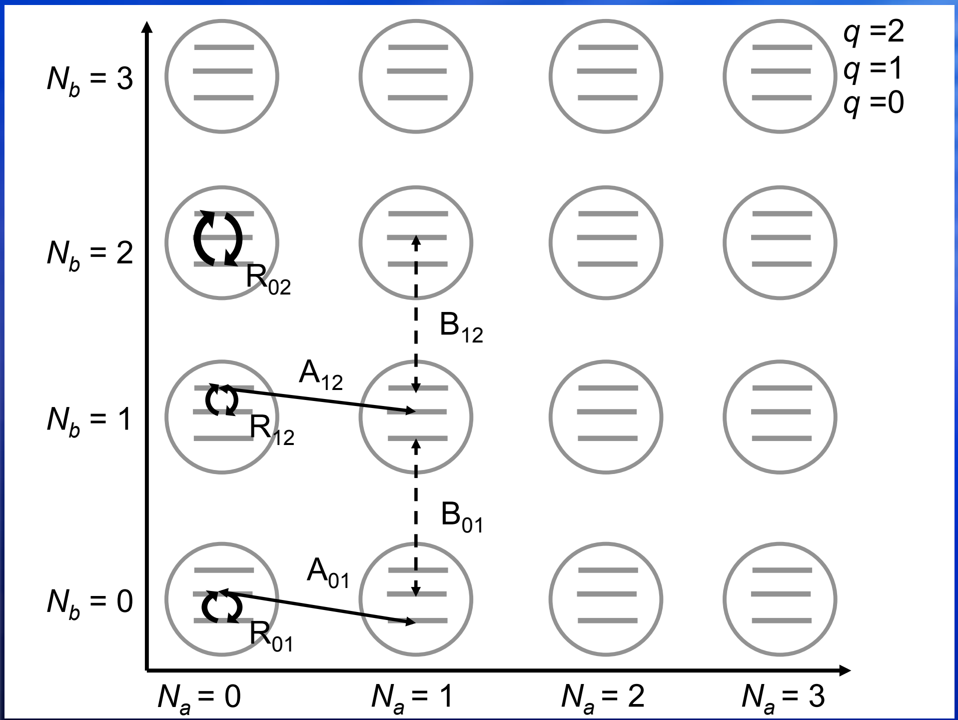
Flips qubit + photon

Requires two-photon process for transmons?

- Probably slow... $\Omega/2\pi \sim 10$ MHz

Other Methods: Higher Levels

- |0>, |1>, |2>, ... to mediate interactions



Two types of qutrit-resonator interactions.

Three types of Rabi oscillations.

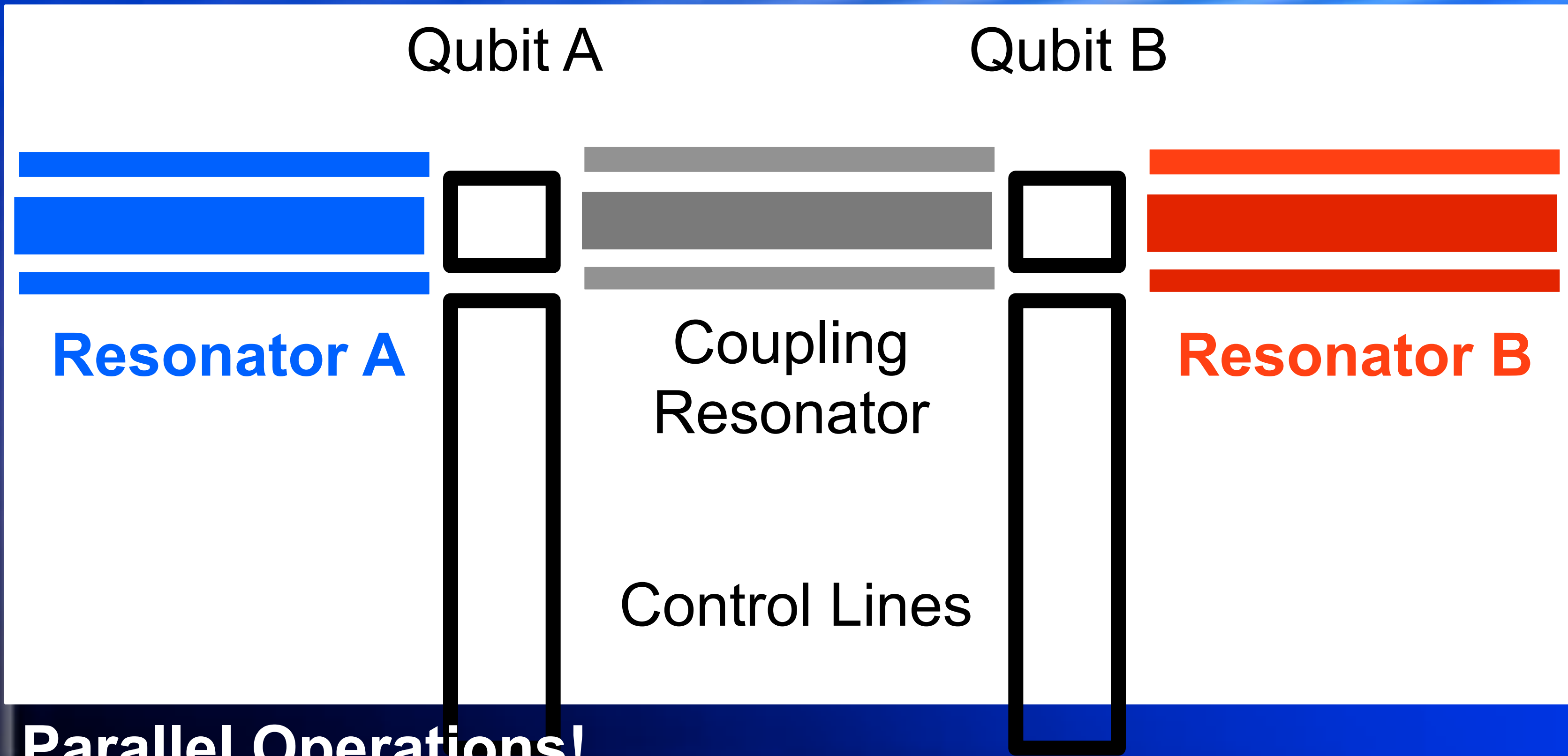
More complex Stark shifts.

- Less coherent? $T_{12} < T_{01}$

Another Scheme

Martinis & Cleland
Merkel & Wilhelm
arXiv: 1006.1336

Two qubits, three resonators.
Entanglement transferred from qubits to resonators.



Parallel Operations!
General State Synthesis?

Many Applications

- High-dimensional states provide more efficient means to distribute entanglement ***(quantum routing)***
- Higher-dimensional Bell inequalities (more settings, less locality?)
- State preparation of qubits by entangled resonators (ancillas, etc.)
- Quantum logic between resonators?
- Measuring entangled photon states?

- ***These and other issues under investigation***

Quantum Routing

- **Goal:** Use a network of elements (qubits or resonators) to transfer quantum information.
- **Programmable**---send information between any two nodes.
- **Parallel**---information between different pairs of nodes can be sent at the same time.
- Ideally suited for **entanglement distribution** between distinct registers for teleportation, error detection, ancilla preparation, and other steps toward fault tolerance.

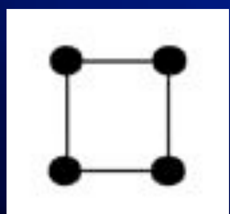
Quantum State Transfer

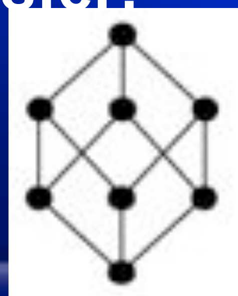
One can transfer the state of a single qubit from site A to site B using a set of permanently coupled qubits with Hamiltonian:

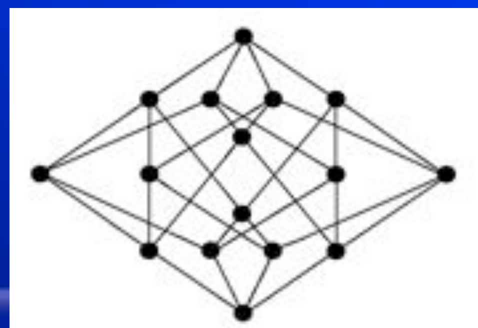
$$H = -\tfrac{1}{2}\sum_j \hbar\omega_j \sigma_j^z + \sum_{jk} \hbar\Omega_{jk}(\sigma_j^+\sigma_k^- + \sigma_k^+\sigma_j^-)$$

Dynamics of a single excitation (with $\omega = 0$) maps onto a tight-bonding model with $H = \hbar\Omega$, where the coupling matrix Ω is proportional to the adjacency matrix of the coupling graph. Certain coupling schemes such as the **hypercube** (M. Christandl *et al.*, Phys. Rev. Lett. **92**, 187902 (2004)) lead to **perfect state transfer:**


d = 1

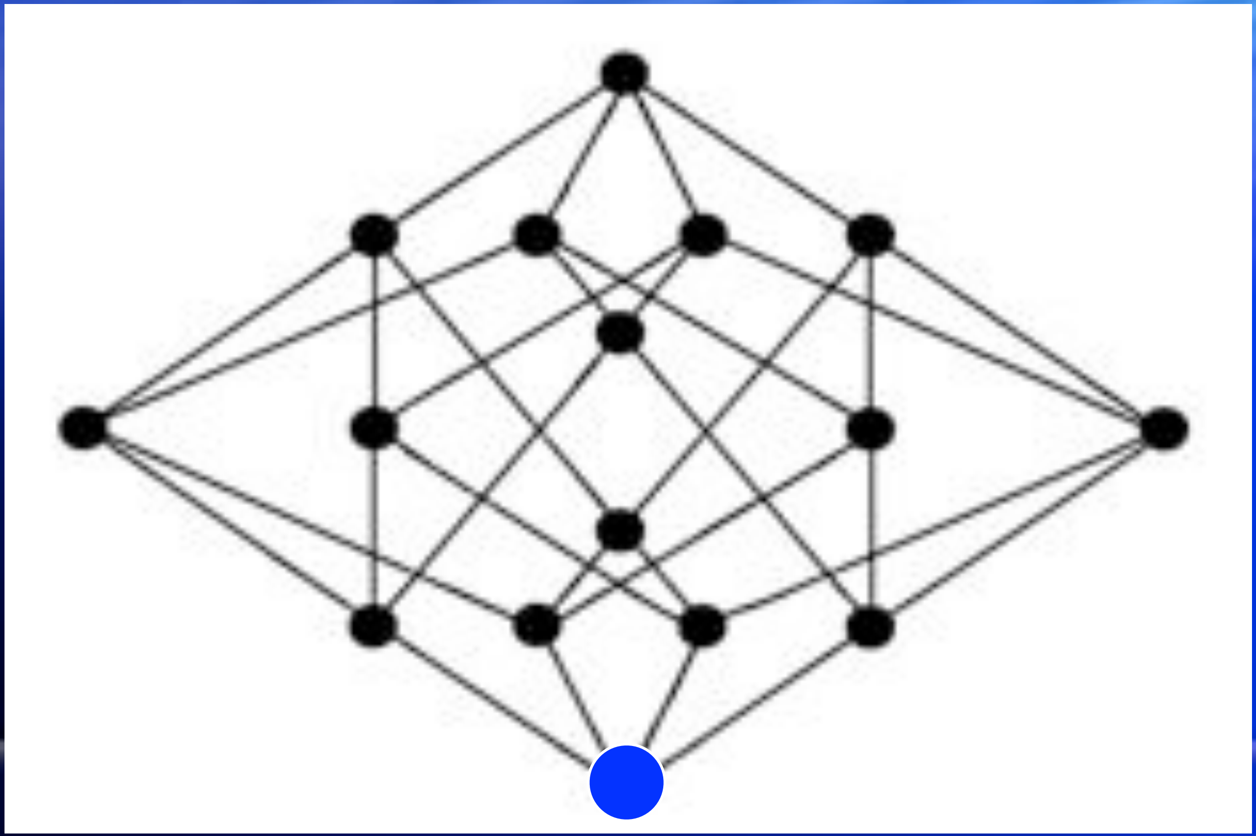

d = 2


d = 3


d = 4

Hypercube State Transfer

- Each vertex represents a qubit. Quantum states travel along **all paths simultaneously** in superposition with **full constructive interference,** yielding **perfect state transfer.**



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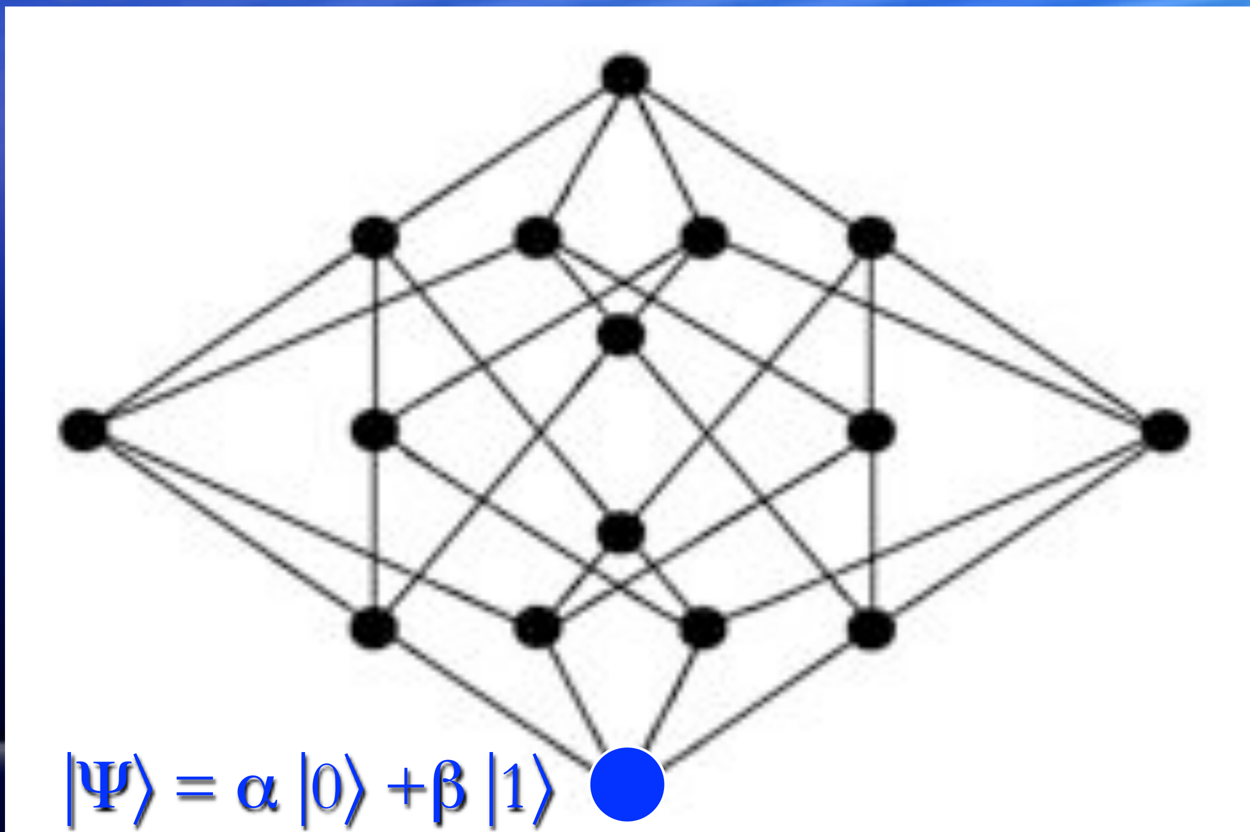

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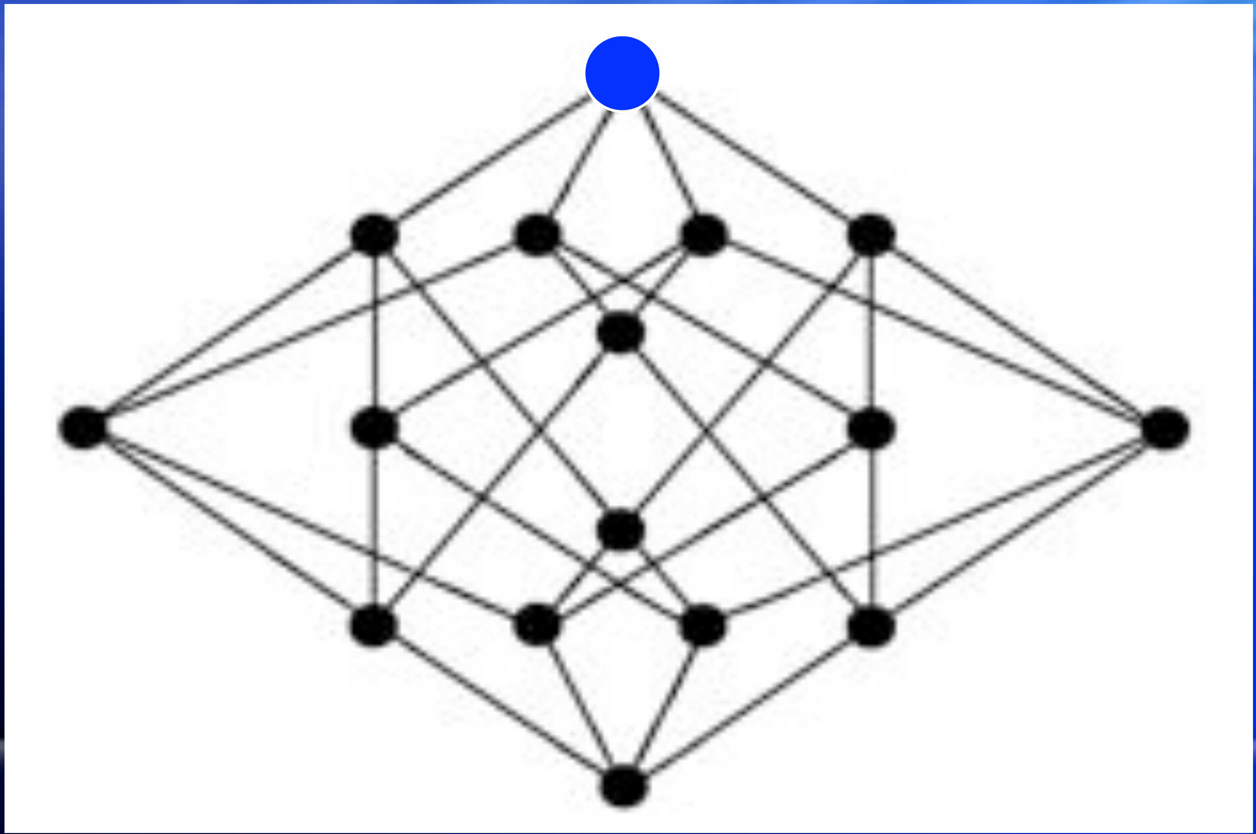

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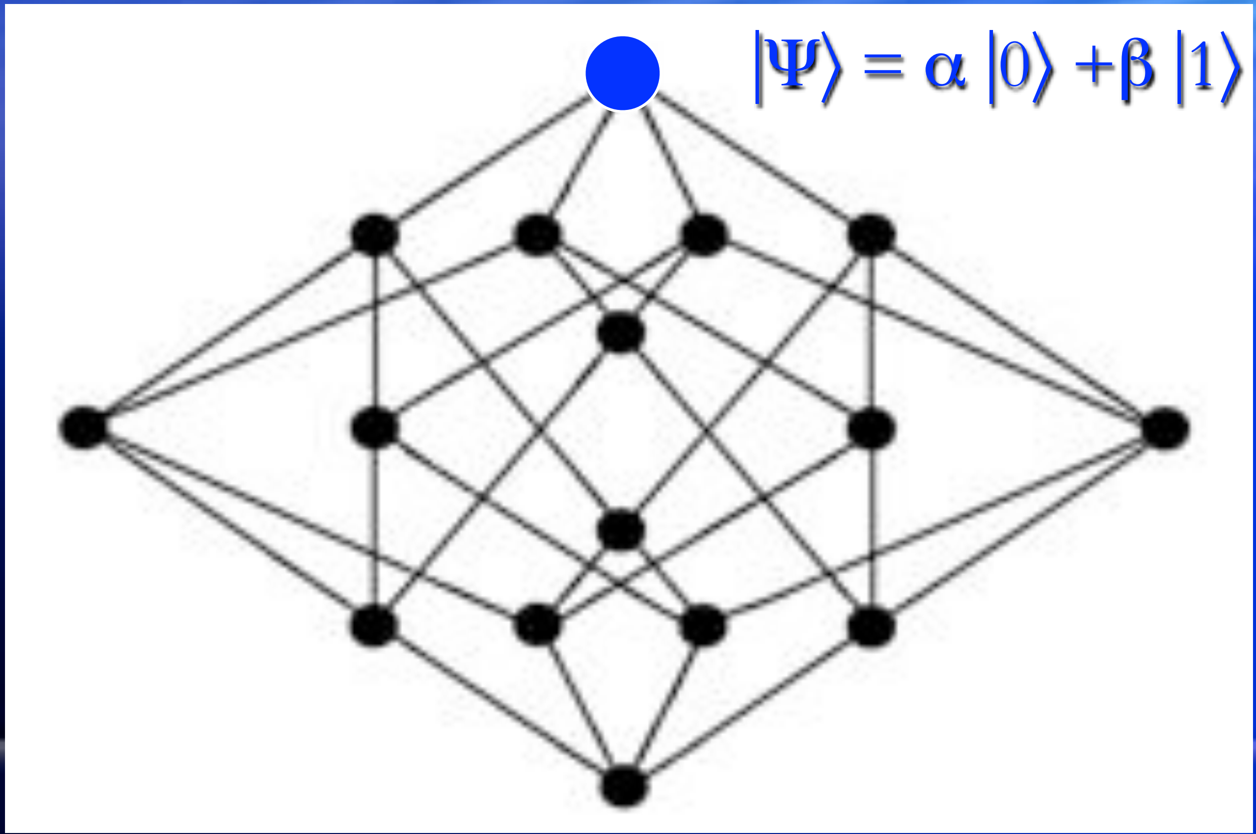
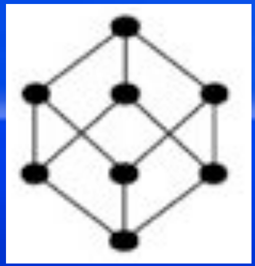
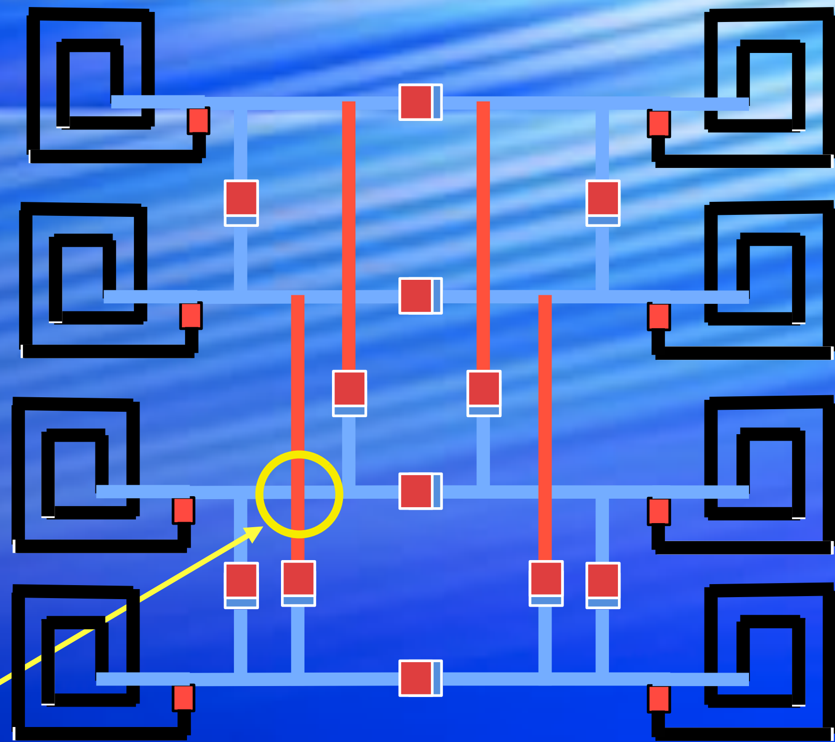


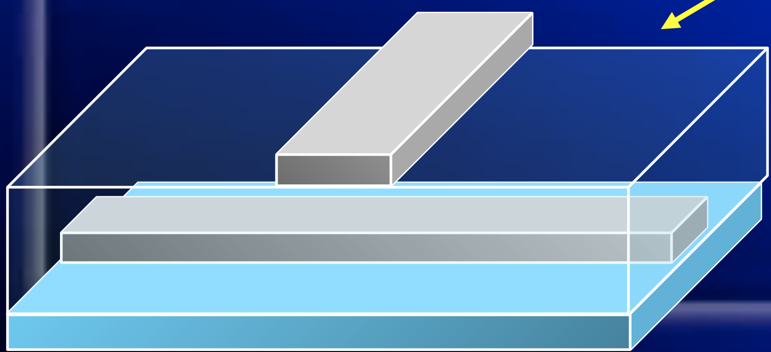


Phase Qubit Cube

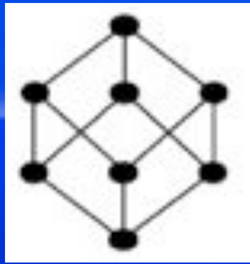
1 2 3

- *Circuits do not need to be simple two-dimensional layouts.*
- *Multi-layer interconnects allow many crossovers and complex couplings.*



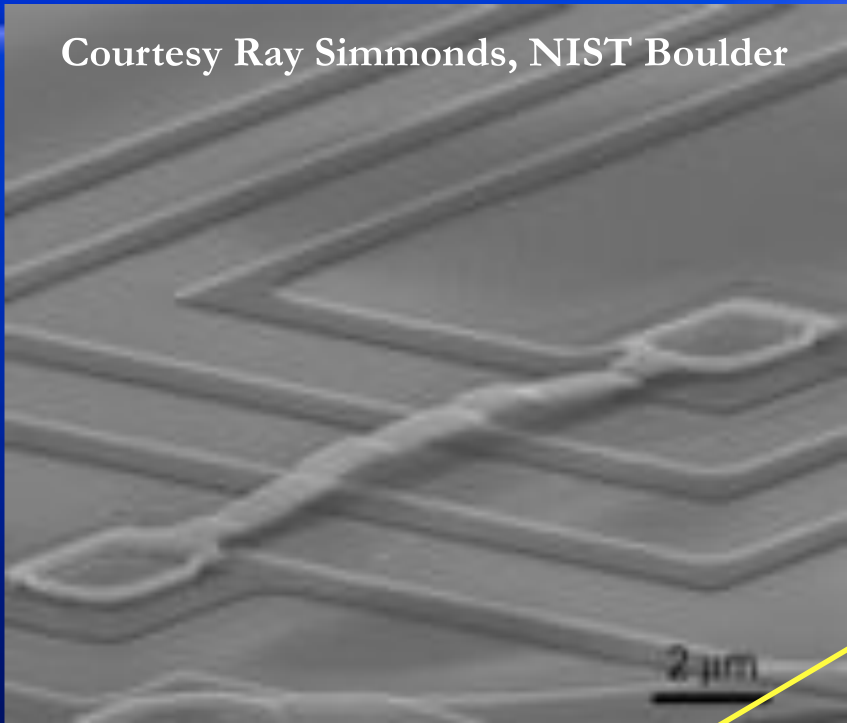


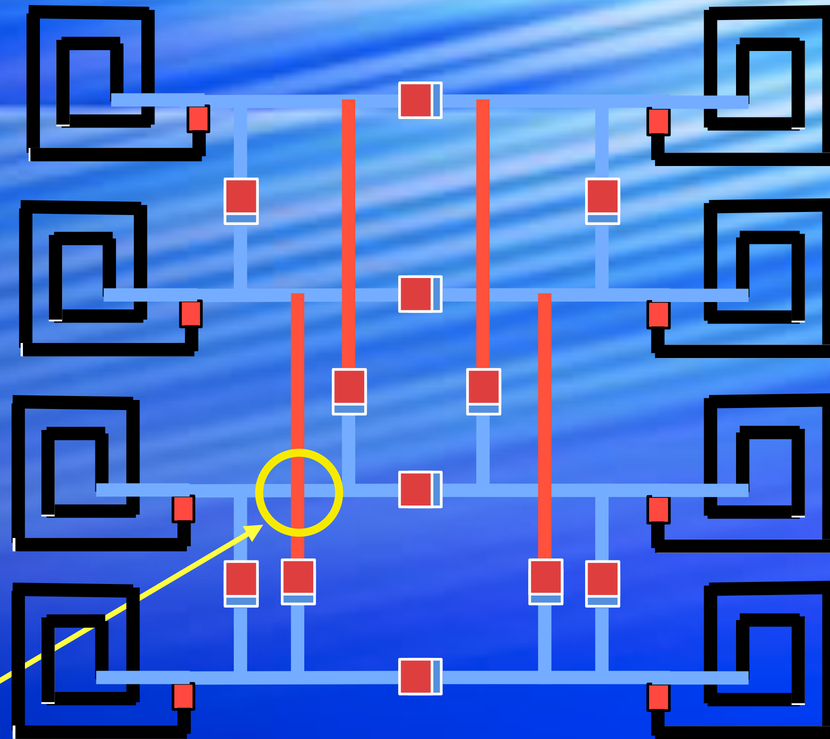
Fabrication Process of Planarized Multi-Layer Nb Integrated Circuits

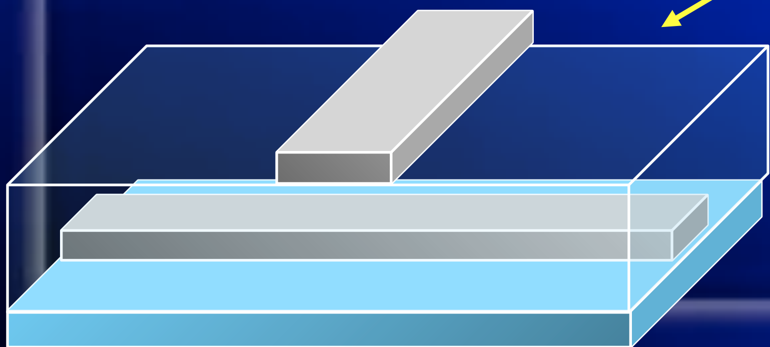


Phase Qubit Cube

1 2 3





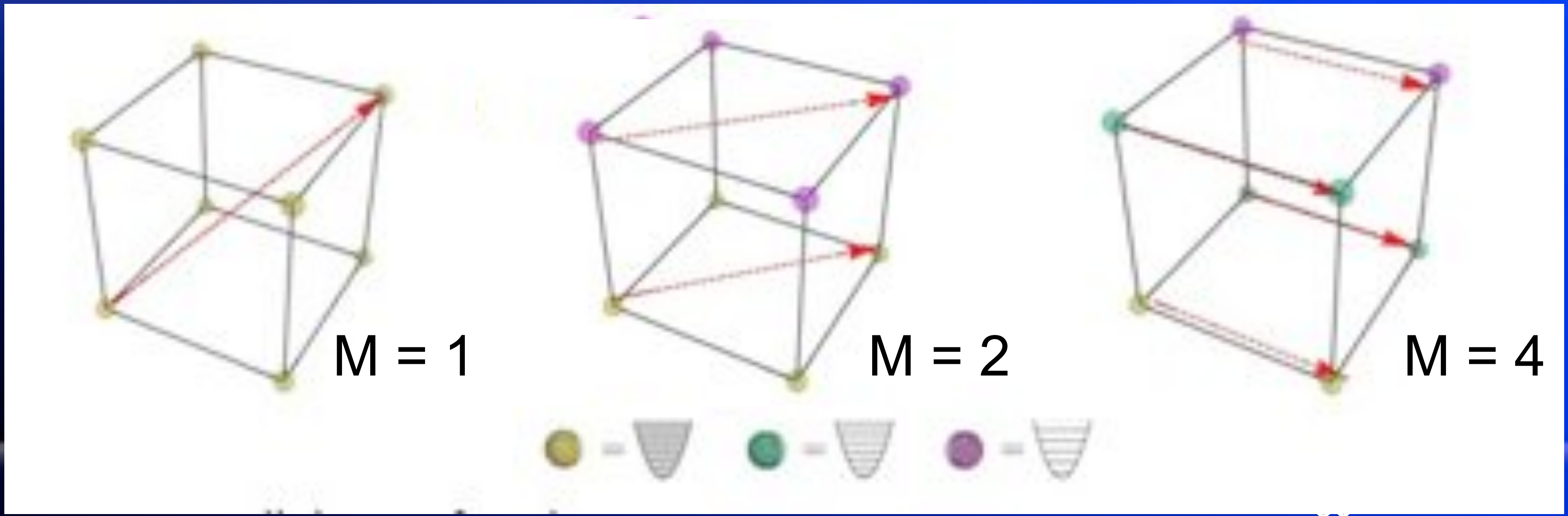


Fabrication Process of Planarized Multi-Layer Nb Integrated Circuits

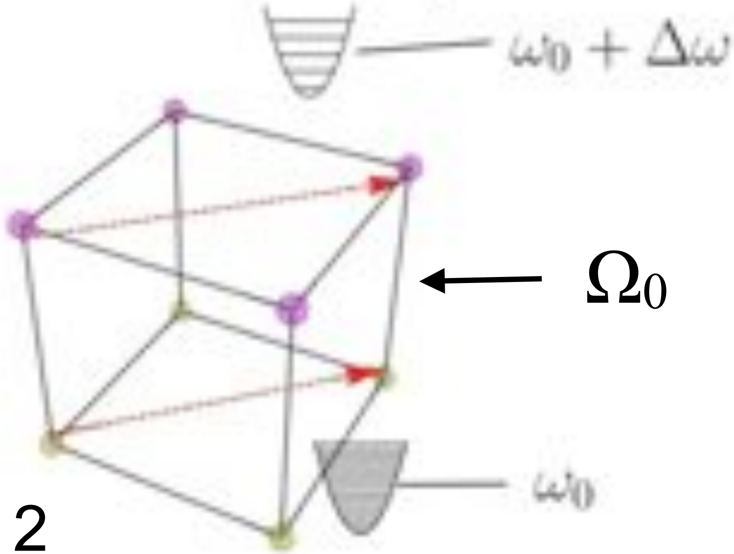
Entanglement Distribution by Parallel Quantum State Transfer

with Chris Chudzicki '10, Williams College

- Study tunable resonators: exactly solvable!
- Split cube into M subcubes by frequency detuning.
- Send quantum states in parallel.



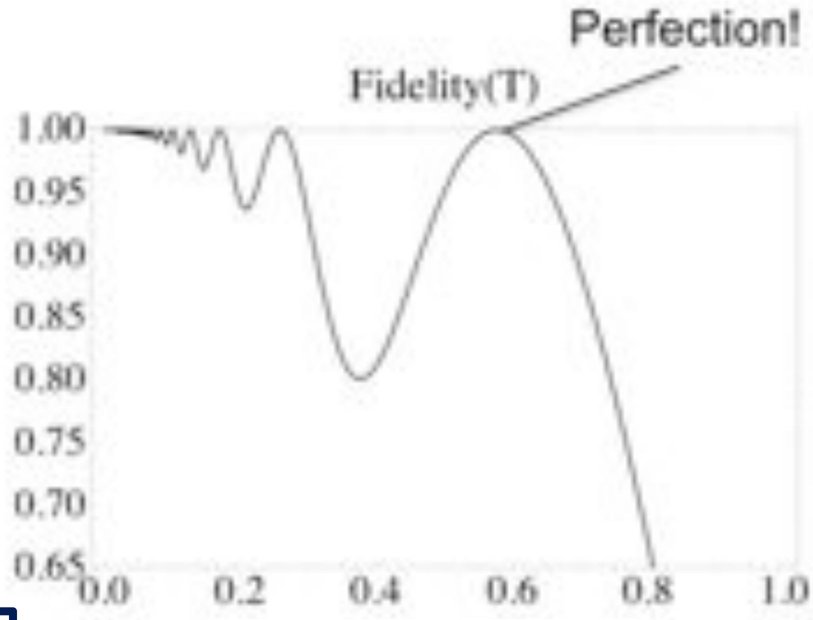
Parallel Transfer Fidelity



M = 2

$$F(T) \gtrsim 1 - \frac{3}{2}\,\log_2(M)\,\eta^2 \sin^2 \xi_T$$

$$\xi_T = \frac{\pi}{2}\sqrt{1+\eta^{-2}}$$



$$\eta = 2\Omega_0 / \Delta\omega$$

M = # of "messages"

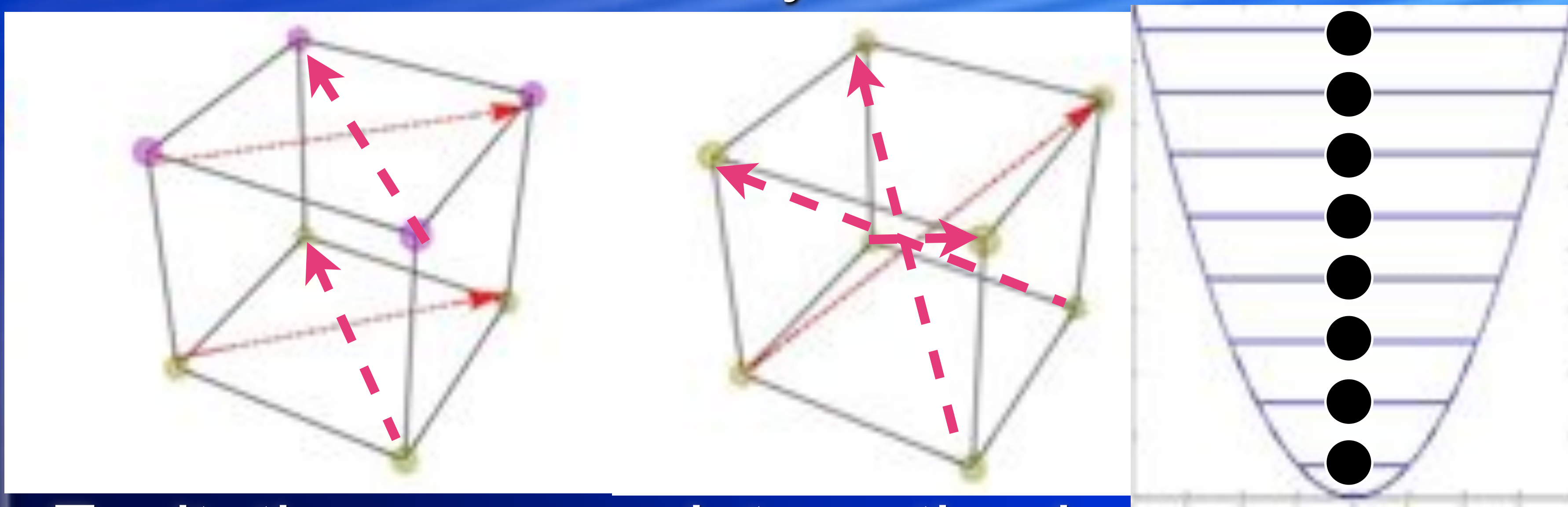
Entanglement Distribution Rate

- Distribute entanglement between every pair of nodes (N total nodes).
- Send states one at a time on each subcube.
- Keep resonators in a fixed bandwidth
- The entanglement distribution rate scales as:

$$\mathcal{R} \gtrsim \boldsymbol{N}^{\mathbf{0.415}} \left(1 - \frac{1}{2} \left(\frac{\Omega_0}{\omega_{\max} - \omega_{\min}} \right)^2 (\log_2 N)^3 \right)$$

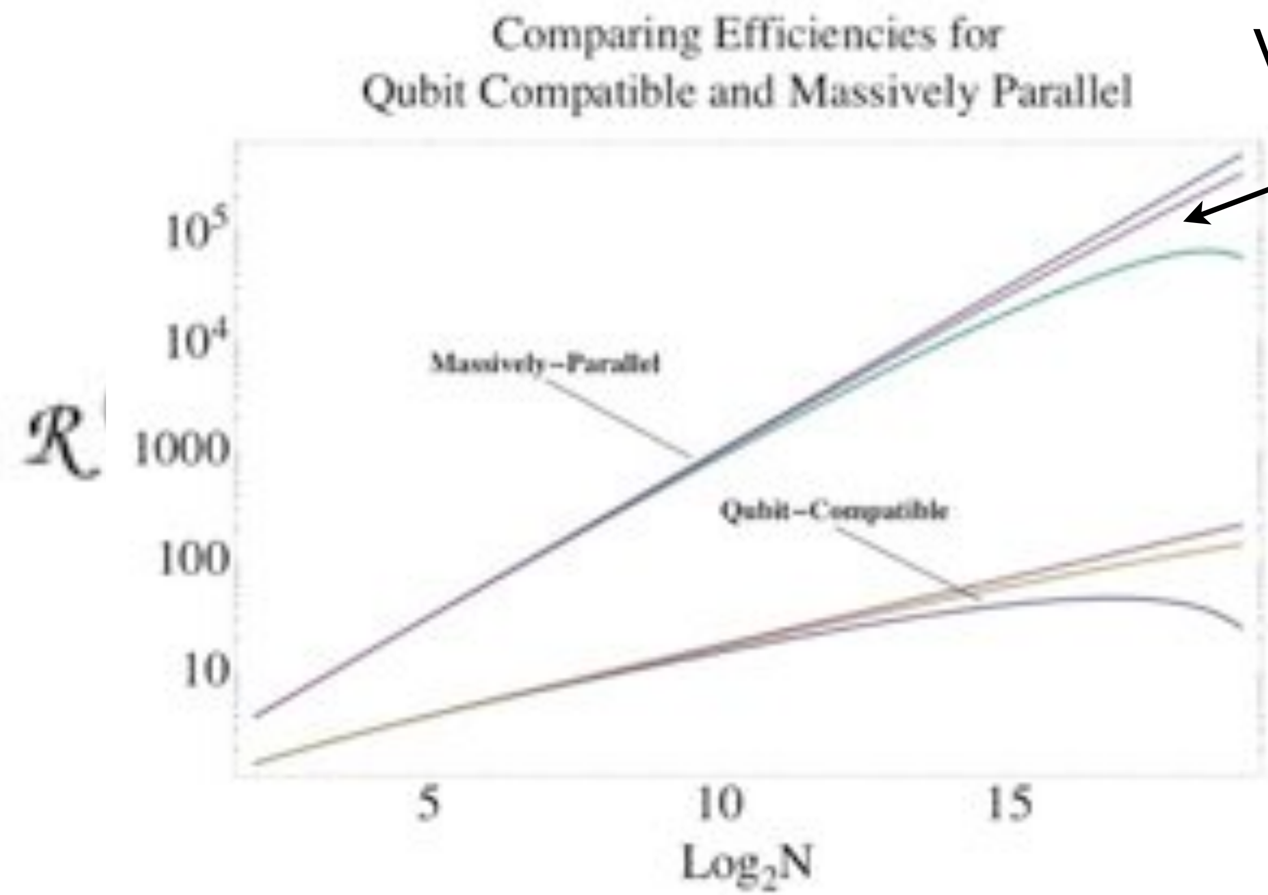
Massively Parallel Distribution

- Send oscillators states between all corners simultaneously!



- Excitations are noninteracting bosons: multiple photons just pass through each other.

Massively Parallel Rate



Various Bandwidths

$$\mathcal{R}^{(QC)} \propto N^{0.415}$$

$$\mathcal{R}^{(MP)} \propto N$$

(N registers)

Conclusions

- Resonators are a powerful tool for quantum information: let's use them!
- Developed state-synthesis algorithm for arbitrary entangled states
  - (NOON states very efficient)
- Quantum Routing with oscillator networks
  - Massively Parallel using multiple excitations

Phys. Rev. Lett. **105**, 050501 (2010)

ArXiv: 1008.1806