Arbitrary Control of Entanglement Between Two Superconducting Resonators

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Research Corp., NSF
Outline

- Superconducting Qubits & Resonators
- NOON States
- State-synthesis Algorithm
  - JC Ladder => Fock-state Diagram
  - Stark-shifted Rabi Oscillations
  - NOON State Synthesis
- Quantum Routing of Entanglement on Oscillator Networks
Supercurrent oscillates like a pendulum!

\[ -\frac{\hbar^2}{2C(\Phi_0/2\pi)^2} \frac{d^2\Psi}{d\gamma^2} - \frac{\Phi_0}{2\pi}(I_c \cos\gamma + I_b\gamma)\Psi = E\Psi \]

- Quantum oscillator involving the superconducting current of billions of Cooper pairs.
- Spectroscopic transitions between energy levels can be probed by microwaves.
- States are metastable, will tunnel through barrier.
Tunable Oscillator

\[-\frac{\hbar^2}{2C\left(\Phi_0 / 2\pi \right)^2} \frac{d^2\Psi}{d\gamma^2} - \frac{\Phi_0}{2\pi} \left( I_c \cos \gamma + I_b \gamma \right) \Psi = E \Psi\]

Sweep of bias current allows experimental control of energy levels.
Tunable Oscillator

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Coupling Qubits by Cavities

Arbitrary Control of Resonator

- Martinis Group, UC Santa Barbara
Law & Eberly developed a scheme to program an arbitrary state of a single harmonic oscillator mode by coupling to a two-level system. Climbing Jaynes-Cummings Ladder one rung at a time.
State-Synthesis Experiments

Hofheinz et al., Nature 454 310 (2008)

Hofheinz et al., Nature 459 546 (2009)
Big Question: Qubits or Resonators

- Is it better to use harmonic oscillators (resonators) or qubits for quantum computing?

- Common wisdom: *It depends*...
  - Use qubit nonlinearity to encode information.
  - Use oscillator coherence to store information.
  - Use resonators to couple qubits.
  - Use ? to readout information.
  - Use ? to process information.
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Little Question:

What control sequence is required to generate an arbitrary entangled state? (e.g. NOON states)

\[
|\Psi\rangle = \sum_{n_a=0}^{N_a} \sum_{n_b=0}^{N_b} c_{n_a,n_b} |n_a\rangle \otimes |n_b\rangle. \quad |\Psi\rangle = \frac{1}{\sqrt{2}} (|N, 0\rangle + |0, N\rangle).
\]

\[\text{Schematic}\]

Resonator A

Qubit Coupler

Resonator B

\[\text{Control Line}\]

\[\text{Qubit}\]
High NOON and Beyond

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} (|N, 0\rangle + |0, N\rangle). \]

- Schrödinger Cat State
- Generalizes Bell/GHZ states
- Useful for metrology:

\[ |\langle \Psi | e^{i\phi a^\dagger} a |\Psi\rangle| = |\cos(N\phi/2)| \]

- May have applications for quantum state preparation, entanglement transfer and distribution, ...
Recent NOON Linear Optics

High-NOON States by Mixing Quantum and Classical Light
Itai Afek, Oron Ambar, Yaron Silberberg

Precision measurements can be brought to their ultimate limit by harnessing the principles of quantum mechanics. In optics, multiphoton entangled states, known as NOON states, can be used to obtain high-precision phase measurements, becoming more and more advantageous as the number of photons grows. We generated "high-NOON" states (N = 5) by multiphoton interference of quantum down-converted light with a classical coherent state in an approach that is inherently scalable. Super-resolving phase measurements with up to five entangled photons were produced with a visibility higher than that obtainable using classical light only.

N=5

Science 328, 879 (2010)
Jaynes-Cummings Ladder

\[ H = \omega_q(t)|1\rangle\langle 1| + \frac{1}{2} (\Omega(t)|1\rangle\langle 0| + \Omega^*(t)|0\rangle\langle 1|) + \omega_a a^\dagger a + g_a (\sigma^+ a + \sigma^- a^\dagger) \]

Rabi pulses (R) drive qubit transitions (q=0 → 1)

Shift pulses (S) transfer quanta between qubit and oscillator.

Used to generate arbitrary superpositions of Fock states: Hofheinz et al., Nature 459, 456 (2009)
Two-Mode Jaynes Cummings

Minimal Scheme
one qubit
(tunable frequency)
two resonators
(different frequencies)

\[
H = \omega_q(t)|1\rangle\langle 1| + \frac{1}{2} (\Omega(t)|1\rangle\langle 0| + \Omega^*(t)|0\rangle\langle 1|) \\
+ \omega_a a^\dagger a + \omega_b b^\dagger b \\
+ g_a (\sigma_+ a + \sigma_- a^\dagger) + g_b (\sigma_+ b + \sigma_- b^\dagger) .
\]

\[ \omega_a < \omega_q < \omega_b \]
Two-Mode Jaynes-Cummings Diagram

\[ H = \omega_q(t)|1\rangle\langle 1| + \frac{1}{2} (\Omega(t)|1\rangle\langle 0| + \Omega^*(t)|0\rangle\langle 1|) + \omega_a a^\dagger a + \omega_b b^\dagger b + g_a (\sigma_+ a + \sigma_- a^\dagger) + g_b (\sigma_+ b + \sigma_- b^\dagger). \]

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Need Fock-state-selective interaction!
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Stark-shifted Rabi Oscillations

\[ \omega = \omega_q + \frac{g_a^2}{\omega_q - \omega_a}(2n_a + 1) + \frac{g_b^2}{\omega_q - \omega_b}(2n_b + 1) \]

**Transition Probability**

- Frequency chosen to select \( n_b = n_a + 2 \)

**Special condition:**

\[ \frac{g_a^2}{\omega_q - \omega_a} = -\frac{g_b^2}{\omega_q - \omega_b} \]

**Stark-shifted Rabi transitions yield Fock-state-selective qubit rotations!**

State-Synthesis Algorithm

\[
U = \left( \prod_{j=1}^{N_b} U_{b,j} \right) U_a
\]

\[
U_a = \prod_{j=1}^{N_a} A_j R_{a,j}, \text{ and } U_{b,j} = \prod_{k=1}^{N_b} B_{j,k} R_{b,j,k}.
\]

Interaction times + qubit rotations chosen to satisfy

\[
|\Psi\rangle = U |0, 0, 0\rangle = |0\rangle \otimes \sum_{n_a=1}^{N_a} \sum_{n_b=1}^{N_b} c_{n_a,n_b} |n_a, n_b\rangle,
\]

Solution requires solving for the inverse, clearing amplitudes row-by-row, column-by-column, performing two-state oscillations at each step.
Algorithm Details

\[ U^\dagger |\Psi\rangle = U_a^\dagger \prod_{j=N_b}^{1} U_{b,j}^\dagger |\Psi\rangle = |0, 0, 0\rangle. \]
\[ U_a = \prod_{j=1}^{N_a} A_j R_{a,j}, \quad U_{b,j} = \prod_{k=1}^{N_b} B_{jk} R_{b,jk}. \]

\[ T_{\text{max}} = N_a (N_b + 1) \frac{\pi}{\Omega} + \sum_{j=1}^{N_a} \frac{\pi}{g_a \sqrt{j}} + N_a \sum_{j=1}^{N_b} \frac{\pi}{g_b \sqrt{j}}. \]

Clear amplitudes row-by-row, column-by-column. (Stark shift is key!)

Time \sim N^2
NOON State Example

**High NOON state:**

\[
|\Psi\rangle = \frac{1}{\sqrt{2}} (|3,0\rangle + |0,3\rangle)
\]

### Table 1: NOON State Synthesis Procedure

<table>
<thead>
<tr>
<th>Step Parameters</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_{n,1} )</td>
<td>(\Omega_{m,n,1} = \pi/2, \omega_d = \omega_0)</td>
</tr>
<tr>
<td>(A_1)</td>
<td>(g_n f_{a,1} = \pi)</td>
</tr>
<tr>
<td>(R_{n,2})</td>
<td>(\Omega_{m,n,2} = \pi, \omega_d = \omega_1)</td>
</tr>
<tr>
<td>(A_2)</td>
<td>(g_n f_{a,2} = \pi/\sqrt{2})</td>
</tr>
<tr>
<td>(R_{n,3})</td>
<td>(\Omega_{m,n,3} = \pi, \omega_d = \omega_2)</td>
</tr>
<tr>
<td>(A_3)</td>
<td>(g_n f_{a,3} = \pi/\sqrt{3})</td>
</tr>
<tr>
<td>(R_{n,1})</td>
<td>(\Omega_{m,n,1} = \pi, \omega_d = \omega_0)</td>
</tr>
<tr>
<td>(B_1)</td>
<td>(g_n f_{b,1} = \pi)</td>
</tr>
<tr>
<td>(R_{n,2})</td>
<td>(\Omega_{m,n,2} = \pi, \omega_2 = \omega_1)</td>
</tr>
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<td>(g_n f_{b,2} = \pi/\sqrt{2})</td>
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</tr>
<tr>
<td>(B_3)</td>
<td>(g_n f_{b,3} = \pi/\sqrt{3})</td>
</tr>
</tbody>
</table>
NOON State Synthesis

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} (|N, 0\rangle + |0, N\rangle) \]

\[ T_N = (2N - \frac{1}{2}) \frac{\pi}{\Omega} + \frac{2\pi}{g} \sum_{j=1}^{N} \frac{1}{\sqrt{j}} \]

Main limitation is due to Rabi frequency, to remain in Stark-shifted regime (can be optimized using pulse shapes).

\[ \omega_a/(2\pi) = 6 \text{ GHz} \]
\[ \omega_q/(2\pi) = 6.5 \text{ GHz} \]
\[ \omega_b/(2\pi) = 7 \text{ GHz} \]

High NOON states are accessible using existing technology!
Other Methods: Sidebands

- New transitions, new paths

\[ \Omega/2\pi \approx 10 \text{ MHz} \]

Red + Blue Sidebands for each qubit

Flips qubit + photon

Requires two-photon process for transmons?
Other Methods: Higher Levels

- $|0>, |1>, |2>, ...$ to mediate interactions

Two types of qutrit-resonator interactions.

Three types of Rabi oscillations.

More complex Stark shifts.

- Less coherent? $T_{12} < T_{01}$
Another Scheme

Two qubits, three resonators. Entanglement transferred from qubits to resonators.

Parallel Operations!
General State Synthesis?

Martinis & Cleland
Merkel & Wilhelm
arXiv: 1006.1336
Many Applications

- High-dimensional states provide more efficient means to distribute entanglement (*quantum routing*)
- Higher-dimensional Bell inequalities (more settings, less locality?)
- State preparation of qubits by entangled resonators (ancilllas, etc.)
- Quantum logic between resonators?
- Measuring entangled photon states?

*These and other issues under investigation*
Quantum Routing

- **Goal:** Use a network of elements (qubits or resonators) to transfer quantum information.

- **Programmable**—send information between any two nodes.

- **Parallel**—information between different pairs of nodes can be sent at the same time.

- Ideally suited for **entanglement distribution** between distinct registers for teleportation, error detection, ancilla preparation, and other steps toward fault tolerance.
Quantum State Transfer

One can transfer the state of a single qubit from site A to site B using a set of permanently coupled qubits with Hamiltonian:

\[
H = -\frac{1}{2} \sum_j \hbar \omega_j \sigma_j^z + \sum_{jk} \hbar \Omega_{jk} (\sigma_j^+ \sigma_k^- + \sigma_k^+ \sigma_j^-)
\]

Dynamics of a single excitation (with \( \omega = 0 \)) maps onto a tight-bonding model with \( H = \hbar \Omega \), where the coupling matrix \( \Omega \) is proportional to the adjacency matrix of the coupling graph. Certain coupling schemes such as the hypercube (M. Christandl et al., Phys. Rev. Lett. 92, 187902 (2004)) lead to perfect state transfer:
Hypercube State Transfer

- Each vertex represents a qubit. Quantum states travel along all paths simultaneously in superposition with full constructive interference, yielding perfect state transfer.
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Hypercube State Transfer

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\[ |\Psi\rangle = \alpha |0\rangle + \beta |1\rangle \]
• Circuits do not need to be simple two-dimensional layouts.

• Multi-layer interconnects allow many crossovers and complex couplings.
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• Multi-layer interconnects allow many crossovers and complex couplings.

Courtesy Ray Simmonds, NIST Boulder
Entanglement Distribution by Parallel Quantum State Transfer

with Chris Chudzicki ‘10, Williams College

- Study tunable resonators: exactly solvable!
- Split cube into M subcubes by frequency detuning.
- Send quantum states in parallel.

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Parallel Transfer Fidelity

\[ \eta = \frac{2\Omega_0}{\Delta\omega} \]

\[ M = \# \text{ of “messages”} \]

\[ \xi_T = \frac{\pi}{2} \sqrt{1 + \eta^{-2}} \]

\[ F(T) \geq 1 - \frac{3}{2} \log_2(M) \eta^2 \sin^2 \xi_T \]

\[ M = 2 \]

\[ \Omega_0 \]
Entanglement Distribution Rate

- Distribute entanglement between every pair of nodes (N total nodes).
- Send states one at a time on each subcube.
- Keep resonators in a fixed bandwidth.
- The entanglement distribution rate scales as:

\[
R \gtrsim N^{0.415} \left( 1 - \frac{1}{2} \left( \frac{\Omega_0}{\omega_{\text{max}} - \omega_{\text{min}}} \right)^2 \left( \log_2 N \right)^3 \right)
\]
Massively Parallel Distribution

- Send oscillators states between all corners simultaneously!

- Excitations are noninteracting bosons: multiple photons just pass through each other.
Massively Parallel Rate

Comparing Efficiencies for Qubit Compatible and Massively Parallel

Various Bandwidths

\[ R_{(QC)} \propto N^{0.415} \]
\[ R_{(MP)} \propto N \]

(N registers)
Conclusions

- Resonators are a powerful tool for quantum information: let’s use them!
- Developed state-synthesis algorithm for arbitrary entangled states
  - (NOON states very efficient)
- Quantum Routing with oscillator networks
  - Massively Parallel using multiple excitations

ArXiv: 1008.1806