Scientific Computing and Visualization PHYS 13, WSP 2010
Williams College January 8, 2009
Professor Frederick W. Strauch Exercise 3

Exercise 3

These computational exercises should be completed by **January 11** at **11:59AM**. Solutions should be turned in through the course website.

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1. **Sines and Cosines**

   We have now seen how to use the Taylor series method, which can be adapted to any function whose series is known. However, the exponential and trigonometric functions have identities which relate the value of the function to evaluations of the same function with reduced arguments. In particular, we have the identities

   $$\exp(x+y) = \exp x \exp y \tag{1}$$

   and

   $$\sin(x+y) = \sin x \cos y + \sin y \cos x \tag{2}$$
   $$\cos(x+y) = \cos x \cos y - \sin x \sin y. \tag{3}$$

   (These identities are in fact related by the Euler relation $e^{i\theta} = \cos\theta + i\sin\theta$.) Thus, scaling and squaring (S+S) methods can also be used to calculate the sines and cosines, provided we evaluate both at the same time.

   Modify the exponential functions `my_exps_v2.py` and the program `compare_exp.py` to calculate the sines and cosines (hence tangents) of a set of numbers, using the Taylor series and S+S methods, each with its own function. (You can also use complex values for x, if so desired).

2. **Inverse Functions by Root Finding**

   To find the inverse of some function $f(x)$, one can think of the problem as the following:

   Given a value of y, find the value of x such that $y - f(x) = 0$.

   (Note: Sometimes we might want to switch the labels x and y). Thus, we can evaluate inverse functions by a root-finding procedure. Write a program that finds the value of $\arctan(1) = \pi/4$ by a root-finding procedure, using either the bisection method or the Newton-Raphson method (where $\tan(x)$ can be obtained from `math` or from the sines and cosines above).

3. **Challenges**

   - Write a code that evaluates the n-th root of any number by a root-finding method.
   - Write a recursive bisection method.