Physics 142 Literature Assignment #4
Quantum teleportation

This week's paper is a Scientific American article by Anton Zeilinger about the first "quantum teleportation" experiments. These experiments teleported the quantum state (polarization) of a photon.

1 Entanglement and collapsing the wavefunction

The "singlet" state is a prototypical entangled state of two spins.

$$|\Psi\rangle_{\text{singlet}} = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \right) \tag{1}$$

If we measure the state of spin # 1 (say, with a Stern-Gerlach apparatus) we have a 50% chance of measuring spin up, and a 50% chance of measuring spin down.

However, if we measure the state of the first spin as spin up, then we project $|\Psi\rangle_{\text{singlet}}$ into $|\uparrow\rangle_1 |\downarrow\rangle_2$. This means that we now have a 100% chance of measuring spin #2 in the spin down state.

Here is where things start to get really weird.... suppose you have prepared a pair of silver atoms in the state $|\Psi\rangle_{\text{singlet}}$ Maybe you're on the East coast and you ship one of these spins to a friend on the West coast, 4800 km away. You ship it *very carefully*, perhaps trapped in a vacuum chamber so no gas molecules bump into it (and you've carefully shielded it from any magnetic fields that might change the direction of the spin, etc....) long story short, it is still entangled with the spin you kept in your possession.

Now, suppose your friend, with their Stern-Gerlach apparatus, measures the west coast spin to be spin up, projecting the wavefunction. You make a measurement a microsecond ($= 10^{-6}$ s) later on the "east coast spin"—what do you measure?

What's weird about this is.... it takes light 0.016 s to travel across the country. So how did *your* spin know that its wavefunction had collapsed???

Some things to think about: when you measure your spin, is there anything that would tell you whether or not your friend had made a measurement already? Could the procedure just described be used to send any kind of message between you and your friend?....What if *you* measured first, is there any physical difference in the possible outcomes?

This is really deep, and I'm interested to hear what you think about it! I should add that these types of experiments have been done—probably not with spins—, but (within experimental error), they perfectly match the predictions of quantum mechanics—if you measure spin up, your friend will always measure spin down, and vice versa.

2 Brief introduction to polarization

Zeilinger's experiment used photons instead of spins—one of the advantages of this is that photons are fairly insensitive to environmental noise, which might destroy the entanglement. Instead of spin up and spin down, the photon has a polarization. The polarization of the photon has to do with the direction of the electric field of the photon.

An electromagnetic wave has oscillating electric and magnetic fields. The electric field points perpendicular to the direction of propagation. So, if the light propagates, let's say, in the z-direction, its electric field could point anywhere in the xy-plane.

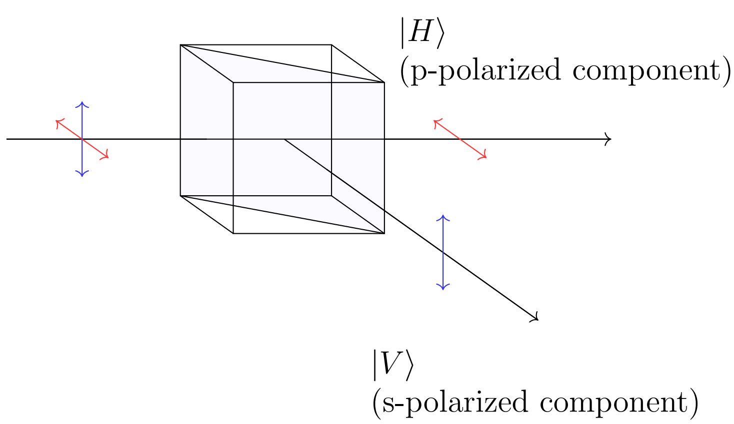
There are a number of handy optical elements that can be used to control and measure the polarization of light. A common one is a polarizer, which absorbs light of one polarization, letting the other polarization through. Check out the following video demonstrating the concept of polarization.

`https://www.youtube.com/watch?v=E9qpbt0v5Hw`

Here is a slightly surprising demo using three polarizers (it comes at the end of the video): `https://www.youtube.com/watch?v=MhhHPOxTUy8`

Can you explain why adding the middle polarizer [at certain angles] causes light to be transmitted? (Is the transmitted light as intense as the light source, or has it been attenuated?)

Another useful polarization optic is the *polarizing beamsplitter* cube. A PBS reflects light of "s" polarization and transmits light with "p"-polarization, as drawn below.[1] If we imagine the beamsplitter is set up in the lab, we could refer to the reflected polarization as "vertical" and the transmitted as "horizontal".



Classically, we can describe an electromagnetic wave in terms of its oscillating electric field;

[1]Here, s- and p- are relative to the "plane of incidence", which is the plane formed by the propagation direction of the incoming beam and the surface normal of the reflecting interface. "p" stands for *p*arallel to the plane of incidence, and "s" stands form...erm.... actually, it stands for *s*enkrecht, which means perpendicular in German.

if we allow arbitrary polarization in the xy plane, this could be written

$$\vec{E}(z,t) = E_0 \cos(kz - \omega t) \cos(\alpha)\, \hat{\imath} + E_0 \cos(kz - \omega t) \sin(\alpha)\, \hat{\jmath} \tag{2}$$

This beam is "linearly" polarized, and its polarization makes an angle α with the x-axis.

As usual, the intensity of the light is proportional to the square of the amplitude of the electric field: $I = c\epsilon_0 E_0^2/2$

→ Suppose we send a laser-beam horizontally in the lab, polarized as described above, where we've defined the y-axis as the vertical axis. If the light shines on the polarizing beamsplitter cube, what's the intensity of the reflected light? The transmitted light?[2]

The classical description is valid if we have many photons in the beam (true, for example, of a laser beam). However, we can do this experiment with a single photon. The single photon also has a polarization: we describe the state of a photon polarized at an angle α as

$$|\Psi\rangle = \cos(\alpha)\,|H\rangle + \sin(\alpha)\,|V\rangle \tag{3}$$

Here, $|H\rangle$ refers to horizontal polarization and $|V\rangle$ to vertical polarization. Notice that this is normalized: $\cos^2(\alpha) + \sin^2(\alpha) = 1$. If we equip the two output ports of the beamsplitter cube with single-photon detectors, we can implement a measurement of the photon's polarization:

If the photon is reflected, we measure $|V\rangle$; if it's transmitted, $|H\rangle$. If we send in a single photon in this state, what is the probability of measuring $|V\rangle$? Of measuring $|H\rangle$?

Another really useful polarization optic is a *half-wave plate*, which can be used to change the angle of linear polarization.

3 Teleportation protocol

This is the idealized teleportation protocol:

1. An entangled photon pair $\frac{1}{\sqrt{2}}(|H\rangle_A |V\rangle_B - |V\rangle_A |H\rangle_B)$ is created and shared between Alice and Bob—we'll label those two photons A and B, respectively.

2. Alice has another photon (labeled D), which can be in an arbitrary quantum state $|\Psi\rangle_D = \cos\alpha\,|H\rangle_D + \sin\alpha\,|V\rangle_D$. This is the state that will be teleported.

3. She performs a special measurement called a "Bell measurement" on her pair (photons A and D), which affects Bob's photon B because it was entangled with A.

[2] Sanity check: total input intensity should equal total output intensity; in this context, since the beam isn't changing its size, and we're assuming the beamsplitter is loss-less, this is required by conservation of energy.

4. *Depending on the outcome of Alice's Bell measurement*, she then sends instructions to Bob for how he should transform his photon so that its state matches the *original* state of photon D. Once Bob follows her instructions, he should have $|\Psi\rangle_B = \cos\alpha\,|H\rangle_B + \sin\alpha\,|V\rangle_B$.

At this point, you might be thinking....if Alice has to send Bob some information anyway, why doesn't she just *tell* him the original state $|\Psi\rangle_D$? This is fine if Alice actually *knows* the quantum state of photon D. But the cool thing is the teleportation protocol works even if Alice *doesn't* know the state of D. Furthermore, there's a "no-cloning" theorem (proved by Professor Emeritus Bill Wooters!) that says we can't simply make a copy of photon D in the same quantum state.

4 Bell state measurement

The Bell state measurement (which is tricky to implement in practice) is a *joint* measurement on two photons. You can think of it as checking if photons A and D are in one of the four following "Bell states":

$$\left|\Phi^+\right\rangle_{AD} = \frac{1}{\sqrt{2}}(|H\rangle_A\,|H\rangle_D + |V\rangle_A\,|V\rangle_D) \tag{4}$$

$$\left|\Phi^-\right\rangle_{AD} = \frac{1}{\sqrt{2}}(|H\rangle_A\,|H\rangle_D - |V\rangle_A\,|V\rangle_D) \tag{5}$$

$$\left|\Psi^+\right\rangle_{AD} = \frac{1}{\sqrt{2}}(|H\rangle_A\,|V\rangle_D + |V\rangle_A\,|H\rangle_D) \tag{6}$$

$$\left|\Psi^-\right\rangle_{AD} = \frac{1}{\sqrt{2}}(|H\rangle_A\,|V\rangle_D - |V\rangle_A\,|H\rangle_D) \tag{7}$$

Based on the protocol, the initial state for the three photons is:

$$|\Psi\rangle_D\,\frac{1}{\sqrt{2}}(|H\rangle_A\,|V\rangle_B - |V\rangle_A\,|H\rangle_B) \tag{8}$$

This is a bit tedious, but this is the key idea behind the protocol (and I think you'll understand it best if you do the algebra yourself[3]). The initial state can be written as a superposition of terms where Alice's photons are in each of the four Bell states. See if you

[3] You may be able to *guess* the correct solution by looking for a pattern here...But make sure you understand how the algebra works out

can fill in the blanks (the answers are all $\sin\alpha$, $\cos\alpha$, $-\sin\alpha$, or $-\cos\alpha$).

$$\begin{aligned}
|\Psi\rangle_D \frac{1}{\sqrt{2}}(|H\rangle_A |V\rangle_B - |V\rangle_A |H\rangle_B) = \frac{1}{2}[&\left|\Phi^+\right\rangle_{AD} (-\sin\alpha\,|H\rangle_B + \cos\alpha\,|V\rangle_B) \\
&+ \left|\Phi^-\right\rangle_{AD} (\sin\alpha\,|H\rangle_B + \cos\alpha\,|V\rangle_B) \\
&+ \left|\Psi^+\right\rangle_{AD} (-\cos\alpha\,|H\rangle_B + \sin\alpha\,|V\rangle_B) \\
&+ \left|\Psi^-\right\rangle_{AD} (\ldots|H\rangle_B + \ldots|V\rangle_B)]
\end{aligned} \tag{9}$$

→ Once you've figured out the missing terms in the equation, show me your work and check with me that you have the correct answer.

Using equation 9, fill out the following table

Outcome of Alice's measurement	Resulting state of Bob's photon	Probability
$\|\Phi^+\rangle_{AD}$	$-\sin\alpha\,\|H\rangle_B + \cos\alpha\,\|V\rangle_B$	$\frac{1}{4}$
$\|\Phi^-\rangle_{AD}$		
$\|\Psi^+\rangle_{AD}$		
$\|\Psi^-\rangle_{AD}$		

→ Also check with me that you filled out this table correctly.

Why does the protocol require Alice to send some additional information to Bob in order for him to fully "receive" the teleported state?

Read the section of the article called "Building a teleporter." Notice that, in the original experiment, Zeilinger and co. didn't implement a full Bell measurement—they simply measured whether or not they got *one* of the Bell states, specifically $|\Psi^-\rangle_{AD}$ (see the figure labeled "beamsplitter" Why the combination of the beamsplitter and the detectors selects this particular state is a bit technical, but I can provide more information if you are interested.).

More questions:

- Even assuming that teleportation could work for large objects like people, could you use it to travel faster than the speed of light?
- It seems like Bob ends up with a perfect copy of photon D at the end of the teleportation protocol. Why does this not violate the no-cloning theorem?

Further reading: D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, "Experimental quantum teleportation," *Nature* **390**, 575 (1997)