Physics 142 Literature assignment # 3
Optical Clocks and Relativity

1 Getting started...

This week's paper:

C. W. Chou, D. B. Hume, T. Rosenband, and D. J. Wineland, "Optical clocks and relativity," *Science* **329**, 1630 (2010) `https://www.science.org/doi/10.1126/science.1192720`.

1. As usual, start by reading the title, abstract, and introduction, and locate the "in this paper" sentence/paragraph. In your own words, what is this paper about?

2 Background

How do you build a clock? Essentially, you need some process that is *periodic*, i.e. that repeats itself identically—think, for example, of an old-fashioned pendulum clock. In class this week, we have learned that atoms give off light at characteristic frequencies as they transition from higher to lower energy internal states. The frequencies of these transitions can be used for timekeeping—in other words, each period of the emitted light corresponds to a "tick" in the clock.

In fact, since 1967, the S.I. second has been defined as exactly 9 192 631 770 times one period of oscillation corresponding to a specific transition in the cesium-133 atom.

In this week's paper, the "optical" clocks use a transition in a trapped aluminum ion. These are actually *more* accurate than the clocks used to define the second. (This might seem a bit strange, but since the authors are comparing two clocks of the same type to each other, it works out). Here, the term *optical* refers to the frequency of the atomic transition, which is much higher than the cesium ground-state hyperfine transition used in the current definition of the second.

The ion is held in an *ion trap* so that its motion is minimized to avoid Doppler shifts. Rather than measuring the frequency of light *emitted* by the ion, the researchers compare the frequency of a reference laser against the transition frequency by checking if the ion can absorb light from the laser.

As it turns out, the aluminum ion has a very good transition to use as a "clock"—one of the required properties is that the transition have a narrow "natural linewidth" (more on this below). Atomic transition frequencies can be slightly affected by the presence of electric and magnetic fields, so a good choice of transition is one that is comparatively *insensitive* to these.

Unfortunately, the aluminum ions don't have good atomic transitions for laser-cooling or for checking whether the ion absorbed light on the "clock" transition. The clock design circumvents this by using a second ion that is trapped along with the aluminum ion. This partner ion has good properties for laser cooling and readout; the Coulomb (electric) force between the two ions allows the aluminum ion to be cooled as the partner ion is cooled, and it also allows information about the aluminum ion's state to be "imprinted" on the partner.[1] These techniques are borrowed from quantum computing experiments, so this clock is sometimes called a "quantum logic clock."

3 Time-dilation due to motion

A Paul trap (or "quadrupole ion trap") is used to trap the ions.[2] There is a theorem in electrostatics (called the Earnshaw theorem) that says that static electric fields alone can't trap a charged particle in empty space—you can't make the fields *all* point inwards in the vicinity at a point, there will always be some directions where they point outwards.[3] Paul traps circumvent this by using time-varying potentials. The potential at any moment in time is a saddle potential, but if it either rotates or oscillates at the right frequency, it can trap a charged particle.

→ This is intuitive once you see it in action—check out the following videos:

(Rotating saddle potential) `https://www.youtube.com/watch?v=XTJznUkAmIY&t=8s`

(Oscillating saddle potential) `https://www.youtube.com/watch?v=wlmJ5goHd3g`

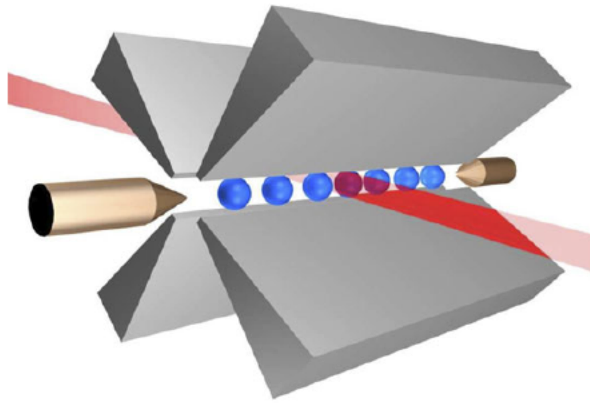
The videos all show trapping in two dimensions; the ion is trapped in the third dimension as well, using DC electric fields.

This is what the geometry looks like: the saddle potential is formed by the four blade-like electrodes, and the end-caps provide the DC electric field. This picture shows a row of ions in the trap, with one interacting with a laser beam. (In the experiment we're reading about, however, I'm pretty sure they work with just one aluminum and one beryllium ion).

[1] For a summary of some recent improvements to Aluminum ion clocks, see `https://physics.aps.org/articles/v12/79`.

[2] Prof. Charlie Doret uses Paul traps for his research.

[3] Why doesn't the Earnshaw theorem apply to the Penning trap?



(Image reproduced from A. M. Eltony et al., "Technologies for trapped-ion quantum information systems," Quantum Information Processing (2016))[4]

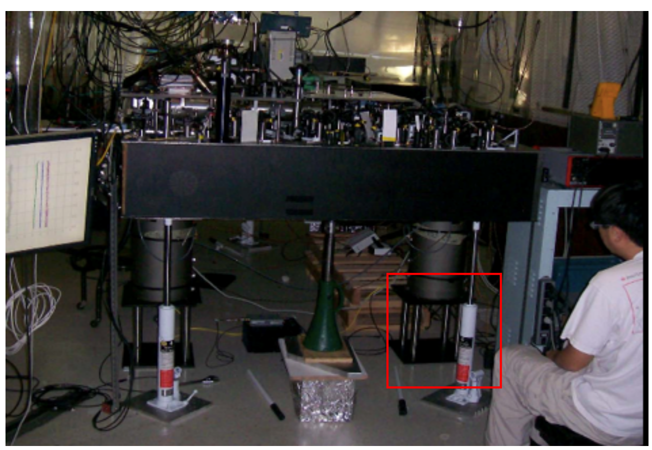
In theory, the ion can be perfectly still at the center of the trap. But, you may have noticed that the "ions" in the videos are not actually perfectly still. If the ion sits off-center, it is still trapped, but it undergoes small oscillations, driven by the oscillating electric field.

Normally, oscillations like this would be bad for clock performance, but they are introduced on purpose in the first part of the experiment.

4 Gravitational time dilation

This one is really easy (at least conceptually)—they just changed the height of one of the clocks.

Here's a picture of the clock that has been raised 33 cm—note that optical tables weigh ∼about a ton.



[4] http://dx.doi.org/10.1007/s11128-016-1298-8, distributed under a Creative Commons license: Attribution-Noncommercial-Share Alike http://creativecommons.org/licenses/by-nc-sa/4.0/

5 Figures and data

Locate the figures 2 and 3, showing the two time-dilation effects measured here. Take a moment to read the captions and examine them carefully.

2. For each figure, explain in your own words what the two parts of the figure show. What quantities are plotted in the graphs?

In figure 2, the x-axis uses v_{rms}. "rms" stands for "root mean squared", which means exactly that—take the average of the square of a quantity and then take the square root.

In this case, the velocity of the ion is oscillating sinusoidally, i.e.

$$v(t) = v_m \cos(\omega t), \tag{1}$$

where v_m is the maximum velocity, and $\omega = 2\pi f_{\text{osc}}$.

Recall from calculus that you can take the average of a function $f(x)$ for $x_1 < x < x_2$ using an integral:

$$\langle f(x) \rangle = \frac{\int_{x_1}^{x_2} f(x) dx}{x_2 - x_1} \tag{2}$$

When we average periodic functions, like $v(t)$ above, we always average over an integer number of periods (another way of looking at this is that we average over a large number of periods, so that the contribution of one partial period is negligible). Notice that the average of $\cos^2$ or $\sin^2$ over many periods is 1/2.

Clearly, $\langle v(t) \rangle = 0$. However, $v(t)^2$ is always greater than or equal to zero and has a non-negative average.

3. Show that

$$\langle (v(t))^2 \rangle = \frac{v_m^2}{2} \tag{3}$$

The rms velocity is just the square root of this average: $v_{\text{rms}} = \sqrt{\langle (v(t))^2 \rangle} = v_m/\sqrt{2}$.

Getting back to the time-dilation figure...

We are comparing two clocks—one that is at rest in the lab, the other is moving (the ion is a clock).

Let δt_0 be the time between two ticks of the clock in its rest frame. Then the time measured in the lab (between two ticks of that clock) is $\delta t_{\text{lab}} = \gamma \delta t_0$.

Now, γ is changing with time, because the moving clock is moving back and forth... but on average,

$$\langle \delta t_{\text{lab}} \rangle = \langle \gamma \delta t_0 \rangle = \langle \gamma \rangle \delta t_0 \tag{4}$$

(I could take the δt_0 out of the average in the last step because δt_0, the time between ticks in the clock's rest frame, is always the same!).

It follows that the frequency of the moving clock, as measured in the lab, is

$$f_{\text{lab}} = \frac{1}{\langle \delta t_{\text{lab}} \rangle} = \frac{1}{\langle \gamma \rangle \delta t_0} = \frac{f_0}{\langle \gamma \rangle}, \tag{5}$$

where f_0 is the frequency of the clock in its rest frame. How to get the average value of γ? (First—sanity check—since $\gamma \geq 1$, $\langle \gamma \rangle$ is also always greater than 1. Does the time dilation make sense as described?).

Check out the values of v_{rms} in figure 2, and convince yourself that at any given time, $v \ll c$.

We can therefore do a Taylor approximation *before* computing the average of γ.

4. Use the above to derive and explain the shape of the curve in Figure 2.

Further questions:

5. Why do you think the first yellow band thinner than the second yellow band in figure 3?
6. Read (or at least skim) the paper in its entirety. What systematic effect(s) do they discuss, and how did they address them?

6 Relative uncertainties, and converting uncertainty in frequency to uncertainty in period

Random question, which is a warm-up to the actual question:

Suppose you have two clocks: one has frequency f_0, and the other has frequency $f_1 = f_0 + \delta f = f_0(1 + \epsilon)$ where $\epsilon \equiv \delta f / f_0$.

Presumably, the second clock has a slightly *shorter* period than the first: The first clock's period is $T_0 = 1/f_0$, and the second clock's period is $T_1 = 1/f_1$, and I posit that $T_1 = T_0 - \delta T$. Given f_0 and δf, what is δT?

7. The first full paragraph on p. 1631 describes the accuracy of the clocks used in the paper and provides their systematic uncertainties as *relative* uncertainties (i.e., for center frequency f_0 and uncertainty δf, the relative uncertainty is $\delta f / f_0$). A NIST press release in 2008[5] described the aluminum clock as so accurate it would neither gain nor lose a second in more than a billion years. How do the reported systematic uncertainties compare with the description in the press release (i.e., is the statement in the press release justified)?

[5] https://www.nist.gov/news-events/news/2008/03/nist-quantum-logic-clock-rivals-mercury-ion-worlds-most-accurate-clock-0