Physics 142 Literature Assignment #2
Deuteron-to-Proton Mass Ratio

This week's paper:

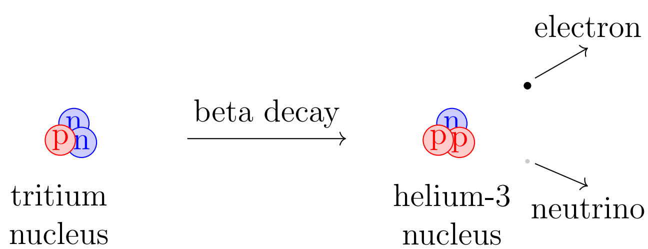
D. J. Fink and E. G. Myers, "Deuteron-to-proton mass ratio from the cyclotron frequency ratio of H_2^+ to D^+ with H_2^+ in a resolved vibrational state," *Phys. Rev. Lett.* **124**, 0123001 (2020).

https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.124.013001.

1 Background

We saw last week that there are three flavors of neutrino with different masses. As of 2019, by combining different lines of evidence, we know that the *sum* of these masses is between 0.06 and 0.15 eV/c^2 (see info on the "total neutrino mass" from the particle data group, https://pdg.lbl.gov/). However, we do not know the absolute masses of the neutrinos.

Neutrinos are produced, for example, in beta decay:



If we knew the masses of the other reactants and products in such a decay with sufficiently high precision, this would provide a way to determine the mass of the neutrino.

Accurate measurements of nuclear masses are important for this and other aplications.

Last week, we looked at the circular trajectories of charged particles in a magnetic field and used this to find the e/m ratio for the electron. This circular motion is sometimes called *cyclotron* motion. If we independently measured the electron's charge[1], we could combine this information with the e/m measurement to determine the electron's mass.

The paper we're reading this week uses a method somewhat analogous to last week's e/m measurement to compare the masses of different ions on cyclotron orbits in order to find the deuteron-to-proton mass ratio.[2] Rather than using the radius of the trajectories as we did, the experimenters use the *frequency* (= number of "orbits" per second) of the cyclotron motion, which they can measure much more precisely.

[1]This was first done by Millikan and Fletcher in 1909, which you'll read about next week.

[2]The deuteron is a deuterium nucleus, consisting of one proton and one neutron.

In the experiment, charged particles are stored in a *Penning trap*, which uses a magnetic field to create cyclotron motion and electric fields to confine the particles in a plane.

You can find a diagram of a Penning trap, and fields, in Brown and Gabrielse, Reviews of Modern Physics **58**, 233 (1986), figures 1 and 2.[3] (This trap is designed for holding negatively charged particles, the direction of the electric field would be switched for positive charges).

1. Go back to the derivation from last week's e/m lab and show that the cyclotron frequency (number of orbits per second) is

$$f_{\text{cyclotron}} = \frac{qB}{2\pi m} \tag{1}$$

   Hint: for a particle in uniform circular motion, the centripetal acceleration is v^2/r, where v is the tangential speed, and r is the radius of the circle.

Notice that $f_{\text{cyclotron}}$ doesn't depend on the particle speed. This is handy, because the experimentalists don't precisely control the speed of the ions they load into the trap.

$f_{\text{cyclotron}}$ can be measured very precisely using a Penning trap[4]; looking back at equation 1, this means that we can determine the mass m of a charged particle very precisely, *provided we also know the magnetic field very precisely.* This turns out to be a major limitation; to get around it, the typical procedure is to measure *ratios* of the cyclotron frequencies of two different particles in the same trap—thus, the ratios of the masses of two charged particles can be measured very precisely, even though their actual masses are not.

→ Locate the "in this paper"-style paragraph. Using the title, abstract, and introduction, write a sentence summarizing the main results of the paper. This may involve some terms that you don't fully understand yet; list these (and if their meaning doesn't become clear as you work through the assignment, ask about them!). In particular, what are the ions whose mass ratio (or, more correctly, cyclotron frequency ratio) is determined here?

2 Mass differences

The ions are generated by briefly turning on an electron beam in the trap, which can ionize atoms/ molecules in a collision. A problem with the H_2^+ ions generated, though, is that they are usually vibrating—and, amazingly, the experiment is so sensitive that the tiny mass change from this vibrational energy throws off their results!

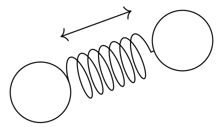
If the experimentalists wait long enough (∼months), the ions will eventually lose their vibrational energy through various mechanisms. In this setup, they took advantage of an effect

[3] `https://journals.aps.org/rmp/abstract/10.1103/RevModPhys.58.233`

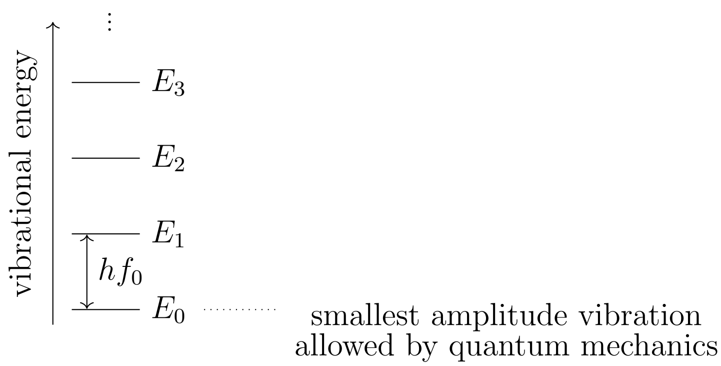
[4] Due to the electric field that is used to confine the particles in the plane, the trajectories are actually a bit more complicated, but still related to the cyclotron motion we saw in Lab 2

called Stark quenching, which helps the ion give up its excess energy by emitting a photon so that it can decay in a more reasonable time.

You can think of the H_2^+ ion as a spring with two masses attached. It has a natural frequency of oscillation, which we can call f_0 (which depends on masses of the hydrogen atoms and the "spring constant" of the bond between them).



Classically, the molecular ion could have *any* amplitude oscillation and any (positive) energy. However, at this small scale, we need to use quantum mechanics to describe the ion, and this means that the vibration is quantized—we end up with discrete allowed energies, $E_\nu = (\nu + \frac{1}{2})hf_0$, where ν is an integer ≥ 0 and $h \approx 6.626 \times 10^{-34}$ J/Hz is Planck's constant.



2. The natural frequency of oscillation of H_2^+ is 65.7 THz $= 65.7 \times 10^{12}$ Hz. (It has been measured much more precisely than this using spectroscopy, but for the following analysis, such precision isn't needed). What is energy difference in eV between the vibrational levels in the diagram?

3. Take a look at figure 1 in the paper. What is plotted? Can you determine the difference in mass between the two largest peaks (labeled $\nu = 0$ and $\nu = 1$)? (This requires some interpretation—what is plotted on the horizontal axis???... Also, if it's helpful, you can use the fact that the proton mass is ≈ 938 MeV.

4. Are the values you determined in questions 2 and 3 consistent? What is the mass difference from 3 as a fraction of the H_2^+ mass? This should give some sense how exquisitely precise their measurement is.

3 Uncertainty and systematics

The quantities reported have uncertainties, which are written in parentheses. The numbers in parentheses apply to the least significant digits—i.e., 1.274(38) means 1.274 ± 0.038. In precision measurements, the experimenters need to worry about how systematics will affect their results. There are different ways to cope with this, but some possibilities are to...

- design the experiment to eliminate or cancel systematics if possible
- use a theoretical calculation to correct for a systematic you understand
- try to measure the systematic effect directly and correct for it
- estimate an upper bound for the systematic (via theory or direct measurement)

5. Read the paper, identify two systematic effects the authors were worried about, and explain how they addressed them (look up unfamiliar terms if necessary!).

For further reading on the importance of nuclear mass measurements, see the News and Views article by J. Koelemeij: "Precise measurement of deuteron mass raises hopes of solving the nuclear-mass puzzle," Nature **585**, 35 (2020). `https://www.nature.com/articles/d41586-020-02474-3`.