

Categorical Data Analysis: Quiz 2

1. Suppose our goal is to make inference about the mean μ of a Poisson random variable Y (which has pdf $P(Y = y) = \mu^y e^{-\mu} / y!$ as you remember). In a Bayesian approach, let the prior distribution for μ be the gamma distribution with parameters a and b and density given by

$$g(\mu; a, b) = \frac{b^a}{\Gamma(a)} \mu^{a-1} e^{-b\mu}, \quad 0 < a, b < \infty.$$

- (a) Suppose you just observed a Poisson variable y . Find the expression for the posterior distribution of μ .
- (b) The mean of the gamma distributed random variable above is a/b . Suppose you know from prior experience that $a = 9$ and $b = 2$ and your Poisson observation turned out to be 6. What is your Bayesian estimate of μ based on this observation?
2. Find the **exact** P-value for the test $H_0 : \pi = 0.1$ vs. $H_A : \pi < 0.1$ when observing $y_{\text{obs}} = 1$ success out of $n = 12$ Bernoulli trials. (I.e., you observation comes from a binomial($n = 12, \pi$) model.)
3. For a multinomial trial, suppose we want to test $H_0 : \pi_1 = 2\theta, \pi_2 = \theta$ and $\pi_3 = 1 - 3\theta$. Let n_1, n_2 and n_3 be the observed cell counts.
- (a) Write down the general expression of a multinomial log-likelihood. (You can drop the constant terms.)
- (b) Assume the above model on the multinomial probabilities holds (i.e., H_0 is true). Set up the equation that, when solved, yields the MLE for θ . (Let $\hat{\theta}$ denote the MLE.)
- (c) Indicate how you would find the Likelihood Ratio Statistic for testing H_0 and give its asymptotic distribution.