

Categorical Data Analysis: Quiz 1

1. Let y_1, \dots, y_n be i.i.d. from a density $f(y; \theta)$, where θ is an unknown parameter.

(a) Write down the definition of the log-likelihood function L .

(b) Write down the definition of the score function s .

(c) Write down the definition of the information matrix (number) I .

(d) What is the definition of $\hat{\theta}$, the MLE of θ (use words, if you like to)?

(e) What is the asymptotic distribution of $\hat{\theta}$?

2. Let y_1, \dots, y_n be i.i.d. from a Poisson distribution

$$f(y; \mu) = P(Y = y; \mu) = \frac{\mu^y e^{-\mu}}{y!}.$$

On HW3, you showed that the MLE for μ is given by $\hat{\mu} = \bar{y} = \frac{1}{n} \sum_i y_i$.

(a) Find the score function $s(\mu)$ (show work).

(b) In general, the expected value of the score function equals what value?

(c) Show that the expected value of the score function for the Poisson equals zero.

- (d) Find the information matrix $I(\mu)$ (show work).
- (e) What is the asymptotic distribution of $s(\mu)$?
- (f) Find the score test statistic for testing $H_0 : \mu = \mu_0$ and indicate when to reject the null hypothesis.
- (g) Indicate how to find the score confidence interval for μ .