Categorical Data Analysis: Midterm 1, In-class

1. Let $y_1, \ldots, y_n$ be iid from a Poisson distribution with mean μ. (20 points)

    (a) Find the score statistics for testing the null hypothesis $H_0 : \mu = \mu_0$ and indicate its asymptotic null distribution.

    (b) Indicate how one would find a $100(1 - \alpha)$ confidence interval for μ based on the test in part (a).

    (c) In a Bayesian approach, let the prior distribution for μ be the gamma distribution with parameters a and b, i.e.,

    $$g(\mu) = \frac{b^a}{\Gamma(a)} \mu^{a-1} e^{-b\mu}, \quad a, b > 0.$$

    For a single Poisson observation y, find the posterior distribution for μ and briefly sketch how to construct a Bayesian interval for μ.

2. Assume Poisson sampling over the 4 cells of a 2×2 contingency table with means μ_{ij}. (20 points)

    (a) Write down the likelihood for the observed cell counts N_{11}, N_{12}, N_{21} and N_{22}

    (b) Let the row totals be $N_{i+} = N_{i1} + N_{i2}, i = 1, 2$. What is the distribution of N_{1+} and what is the distribution of N_{2+}?

    (c) Argue that N_{1+} and N_{2+} are independent.

    (d) Find the conditional distribution of the cell counts N_{ij} given the row margins $N_{1+} = n_{1+}$ and $N_{2+} = n_{2+}$.

    (e) This corresponds to what type of sampling, with what formula for the probabilities?

3. Let y be binomial with sample size n and success probability π and let $\hat{\pi} = y/n$. Find the asymptotic distribution of $\log(\hat{\pi})$. (10 points)

4. A binomial experiment tests $H_0 : \pi = 0.50$ versus $H_A : \pi > 0.50$ using significance level $\alpha = 0.05$. Only n $= 6$ observations are available. (15 points)

    (a) Find the exact P-value when $y = 5$ successes are observed.

    (b) Find the actual type I error for the exact binomial test.