

WCMC Math Challenge Questions - Summer 2016

Instructions. Try as many of the following questions as you can. There is no time limit, but you should do these questions **on your own without any help from people, books, internet, or any other sources**. We are more interested in how you approach the questions and how you communicate your reasoning than in how many correct answers you obtain. For each solution you submit, please include a clear and complete explanation of your answer. Please typeset your answers. Send any questions you have to Allison.Pacelli@williams.edu.

1. There's a small village at the base of a mountain with a population of 8000. There are at least 2 people in the village that do not have dogs. Also, given any three people in the village, at least one of the three has a dog. Do we have enough information to determine exactly how many people in the village have dogs? Explain why not, or find the exact number. Explain your reasoning.

2. Let m and n be positive integers. If m has a digits and n has b digits, what are the possibilities for the number of digits that the product mn has? Explain your answer completely.

3. Find all integers M that satisfy all of the following properties. Explain your answer fully.

(i) If 2 divides M , then $40 \leq M \leq 70$.

(ii) If 6 does not divide M , then $40 \leq M \leq 57$.

(iii) 9 divides M .

4. Michael and his roommate Andy were known for throwing great dessert parties. At one of their cake-tasting events, five pairs of roommates were present (including Michael and Andy). The attendees were very friendly, and some shook hands with some of the other party-goers. Here's what we know: No one shook hands with themselves and no one shook hands with his or her roommate. Given these facts, a guest might not have shaken anyone's hand or might have shaken as many as 8 other people's hands. After the last crumbs were eaten, Michael gathered the crowd and asked the nine other people how many hands each of them had shaken. Much to Michael's delight, each person gave a different answer. That is, someone didn't shake any hands, someone else shook one person's hand, someone else shook two people's hands, someone else shook three people's hands, and so on. With this information, determine the exact number of people's hands that Michael's roommate Andy shook. Explain your answer fully.

5. There's a box of five hats: two blue and three white. Andy, Kate, and James each place a hat on his or her head, while blindfolded. One by one, each child removes his blindfold and (without using a mirror) gets one opportunity to guess the color of the hat on his own

head. If any of the three guesses correctly, everyone gets to go to the park! Andy, Kate, and James are each very logical, and know that the others are as well.

First, James removes his blindfold. He sees the hats that the others are wearing, but admits that he is unable to discern his own hat color.

Next, Kate removes her blindfold, and sadly reveals that she too is not able to determine the color of her own hat.

Finally, Andy pipes up and says “I can answer with my blindfold on! I know what color hat I am wearing.”

What color is Andy’s hat, and how does he know? Explain fully.

6. Andy encounters a strange island, where every creature has either green, purple, or blue hair on his head. He’s told by a reliable source that those with green hair always tell the truth, those with purple hair always lie, and those with blue hair make statements that are alternately true and false (though the order of which statements are true and which are false is unknown). One day, Andy meets three of the islanders, but each is wearing a very big hat which completely covers his hair. Andy asks each of the three the color of their hair, and the responses are as follows:

A: I have green hair.

B: I have purple hair.

C: 1. They are both lying. 2. I have blue hair.

Assuming that each of the three has a different color hair, determine the hair color of each of A, B, and C. Explain your answer fully. If we don’t assume that each has a different hair color, is it possible to determine the hair color of each islander? Explain fully.

7. Five pirates of different ages (A , B , C , D , and E oldest to youngest) have a treasure of 100 gold coins. On their ship, they decide to split the coins using this scheme:

The oldest pirate proposes how to share the coins, and ALL pirates (including the oldest) vote for or against it.

If 50% or more of the pirates vote for it, then the coins will be shared that way. Otherwise, the pirate proposing the scheme will be thrown overboard, and the process is repeated with the pirates that remain. As pirates tend to be a bloodthirsty bunch, if a pirate would get the same number of coins if he voted for or against a proposal, he will vote against so that the pirate who proposed the plan will be thrown overboard.

A) Assuming that all 5 pirates are intelligent, rational, greedy, and do not wish to die, (and are rather good at math for pirates) what will happen?

B) Come up with another mathematically interesting extension to this problem that you could investigate. You don't need to do the problem, only pose the question.