Williams College Math Camp

Summer 2013

Math Challenge Questions

Instructions. Try as many of the following questions as you can. There is no time limit. We are more interested in how you approach the questions and how you communicate your reasoning than in how many correct answers you obtain. For each solution you submit, please include a clear and complete explanation of your answer. Send any questions you have to apacelli@williams.edu.

1. One thousand politicians attended a recent convention. You know two facts: (1) At least one of the politicians is honest, and (2) Given any two politicians at the convention, at least one of those two is a liar. Do you have enough information to determine exactly how many of the politicians are honest? Explain why or why not, and give the exact number of honest politicians if possible.

2. If 29 bakery staffers can decorate 29 cupcakes in 29 minutes, then how many bakery staffers are required to decorate 87 cupcakes in 87 minutes?

3. John McClane and his friend Zeus are trying to stop the bomb that Simon Gruber set! It will go off in 3 minutes unless you can help them defuse it! Before you are four glasses. One glass of indiscernible size contains nine ounces of water. The other three glasses are empty and unlabeled, but Simon told you that they hold exactly five, four, and two ounces of liquid respectively. The bomb will go off in 3 minutes unless you divide the nine ounces of water into three glasses containing exactly three ounces each. Be careful not to spill any water because there's no more nearby (close enough to access within the time limit). Good luck! And be sure to clearly write up what you did and why it worked when you're done defusing the bomb.

4. You're presented with two beautiful boxes, one purple and one yellow. Each is tied up with a gorgeous purple and yellow polka dotted bow. Each box contains either a secret treasure or a deadly disease. Each box has a note attached. The note on the purple box reads "*At least one of these boxes contains* $\$1,000,000$." The note on the yellow box reads "*The purple box contains a deadly plague which will kill you instantly."* You have also been told that either both notes are false, or both notes are true, and you must open one of the boxes. Which box do you open?

5. There are 100 doors, and 100 people lined up in a row. All the doors are closed. The first person goes through and opens all the doors. The second person goes

through and changes the position (open to closed or closed to open) of every second door. The third person goes through and changes the position of every third door. And so on. After all 100 people have gone through the doors, which doors are open and which are closed? Explain fully.

6. Andy encounters a strange island, where every creature has either green, purple, or blue hair on his head. He's told by a reliable source that those with green hair always tell the truth, those with purple hair always lie, and those with blue hair make statements that are alternately true and false (though the order of which statements are true and which are false is unknown). One day, Andy meets four of the islanders, but each is wearing a very big hat which completely covers his hair. Andy asks each for directions, and each islander makes two statements, but their answers are rather frustrating for Andy.

A: 1. I have either green or purple hair.

2. B has green hair.

B: 1. I have either green or blue hair.

2. A has blue hair.

C. 1. I have neither purple nor blue hair.

2. B claims falsely to have either green or blue hair.

D. 1. I have green hair, and A has purple hair.

2. C's second statement is false.

What is the hair color of each of the four islanders? Explain your answers fully.

Some of the puzzles above were taken from the book *The Heart of Mathematics*, by Burger and Starbird.